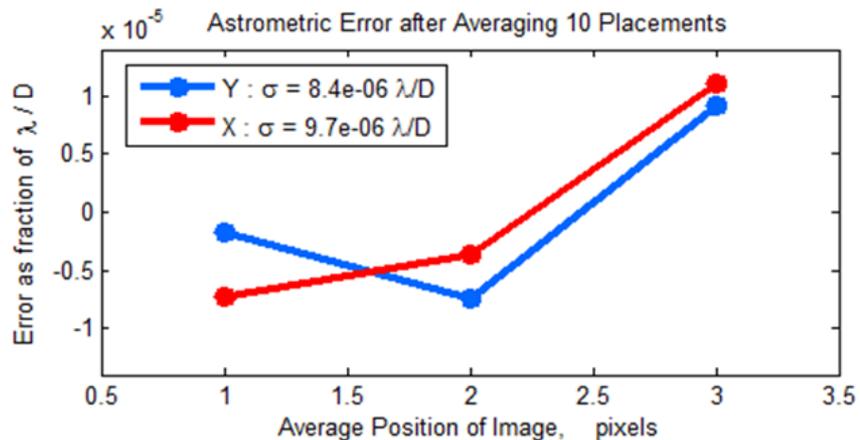




sub-pixel characterization for μas Astrometry with LUVOIR

And application to precision RV measurements.

M. Shao,



Pixel Position to 10^{-5} pix
Centroiding to 10^{-4} pixels
Nyquist sampling 12m telescope
Sampling ($2.5\text{pix}/(l/D)$)
Single shot 0.41 μas
dither 120
Single epoch 0.07 μas

Outline

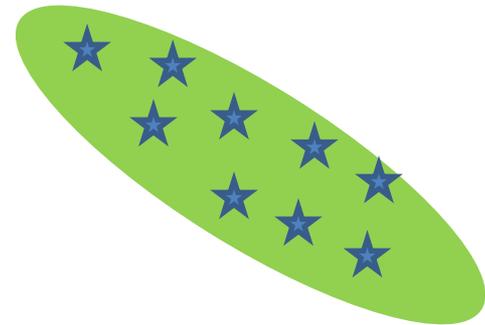
- Science cases
 - Exoplanets
 - Cosmology (dark matter, direct distance to 10's of megaparsec?)
- Random (phot noise) and systematic errors in astrometry
- Calibration of systematic errors
 - Detector errors (many types of detector errors)
 - Optical distortion, (in the optical design vs beam walk on imperfect optics)

Exoplanet Science

- What accuracy is needed to detect an Exoplanet?
 - SNR=6 is needed for a false alarm prob < 1%
 - SNR=signal/noise where signal is amplitude of the reflex motion. Noise = $0.07 \text{ uas} / \sqrt{\# \text{ hrs observed}}$
 - Assuming total # obs $\gg 10$, spread over $>$ (1 orbit period or 1 year)
 - If there are multiple planets orbiting the star the # of observations has to be $> 5 + N_{\text{plan}} * 6$.
- How many Exo-Earths can be found?
 - How many hours of LUVOIR time (not including slew) would it take to survey the easiest (nearby) **100 stars** for an EXO-Earth (1 Mearth, 1 AU scaled to Star's luminosity)? **Answer 150 hrs** not including slew time.
 - We sorted the Hipparcos catalog for all FGK stars < 30pc. (double stars where a HZ orbit was not stable were removed) We found 384 stars. It would take **~1500 hrs** or LUVOIR integration time to search all of them.
- How can astrometry help a direct imaging program?
 - Direct detection needs to image the star multiple times before it sees the exo-Earth outside the IWA. Then multiple detections to know the planet has a ~ 1 yr orbit (HZ).
 - Astrometry can inform the direct detection program which 90% of stars do not have an Earth-like planet.

Extra-galactic Parallaxes

- A technique used in radio VLBI astrometry can be used in the optical
- A number of O stars can be monitored, both with proper motion and radial velocity to measure the distance to the galaxy.
 - Proper motion of all the O stars are done at once.



Motion $\sim 100\text{km/sec}$
@ 50 Mpc is
0.4 $\mu\text{as/year}$
 $\sim 2 \mu\text{as}$ over 5 years

Direct distance measurement to Virgo

Dark matter, exoplanets (in that order) is the science rationale for THEIA (ESA) astrometry mission concept, using upixel detector calibration.

Multiple ground/lab searches for dark matter have “null” results.

Cold dark matter has been very successful at explaining the “soap bubble” geometry of galaxy clusters, but not very successful on galactic scale distances.

μas astrometry is emerging as a leading candidate to explore dark matter.

Solar System Science

- Does planet X (9) exist? (A several Earth mass object at 200AU exerts a significant perturbation on the planets in the outer solar system $\gg 1$ uas. (35uas in 1 year)
 - The problem with astrometry of planets is the center of light vs center of mass offset. The center of mass moves in a highly predictable way. The center of light not so much.
 - The solution is to look at 0.1~1.0 km (or smaller) moons of these planets. They orbit the center of mass of the planet.
 - These objects are faint, hence the need for high sensitivity along with high astrometric accuracy.

Getting to Sub uas Astrometry

- The easy part comes with a large $\sim 12\text{m}$ aperture.
 - (Note a 10m aperture only loses 20% from the systematic error point of view, 8m is 1.5X less accurate)
- With large apertures, photon noise is not an issue (photon noise of ref stars dominates over the target star)
- Two major sources of instrumental/systematic error
 - Detector
 - Optics
 - (everything that comes between photons from the stars and bits in the computer)

Random/Systematic Errors

- Random errors, primarily photon noise (and detector read noise) For exoplanet science, primarily from ref stars.
- Systematic (instrumental) errors
 - Detector imperfections
 - Photometric errors (non-lin, ghosts, non-uni subpix QE)
 - Geometric errors (pixels are not regularly spaced)
 - Optics imperfections (called optical distortion)
 - Most familiar with pin-cushion distortion. But a TMA telescope even if manufactured perfectly to picometer levels will have distortion, (but predictable).
 - Beam walk. (the starlight footprint on optics other than the primary is different for different stars, this beam walk coupled to imperfectly figured optics produces optics distortion errors.
- We attempt to calibrate all instrumental biases. The calibration will not be perfect, but we assume the calibration errors are “random” and decrease as \sqrt{N} for different measurements (at least for small $N < 1000$). If we change the pointing of the telescope by a few arcsec, we will sample a different instrumental error (both optical and detector)

ExoPlanet Astrometry Err Budget

Exoplanet Astrometry Error Budget
1hr Accur 0.068 uas

Target star error (1 hr)
0.056 uas
div sqrt(#dithers 120)

Ref Stars error (1hr)
0.039 uas
div sqrt(#dithers 120)

Target star (30s 1dither)
0.6 uas

Ref stars (30s 1dither)
0.4 uas

Target star photon
0.020 uas
7 mag star (30s)
err= $\lambda/(2D*\sqrt{Nphot})$

Target star instrument
0.61 uas

Ref star photon
0.180 uas
9~17 mag stars (30s)
i
err= $\lambda/(2D*\sqrt{Nphot})$

Ref stars instrument err
0.39 uas
for all ref stars ~600

Detector cal 0.43 uas
1e-4 pixel, 2.4pix per λ/D

per star err 0.056 uas

FOV 0.06 deg

Opt distort 0.43 uas
Assumed = detector err

Detector cal 0.43 uas
1e-4 pixel, 2.4pix per λ/D

Field distortic 0.43 uas

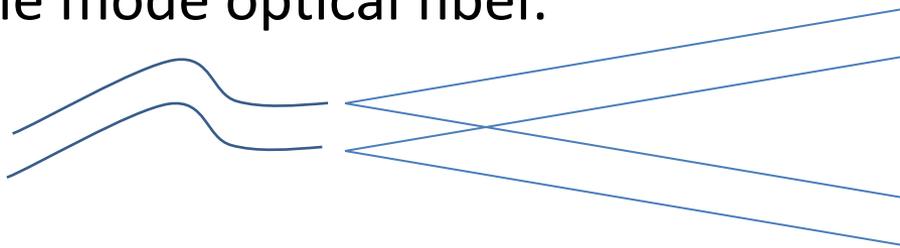
Target star 7 mag
Ref stars 13~18 mag
Dia 12m
Sampling 2.4 pix/ (λ/D)
120 dithers in 1 hr

Detector Err 1e-4 pix
Optical distortion 1e-4 pix
FOV ~0.06 deg
Average sky (for ref stars)
~20 stars < 18 mag

Light Source for Detector Calibration

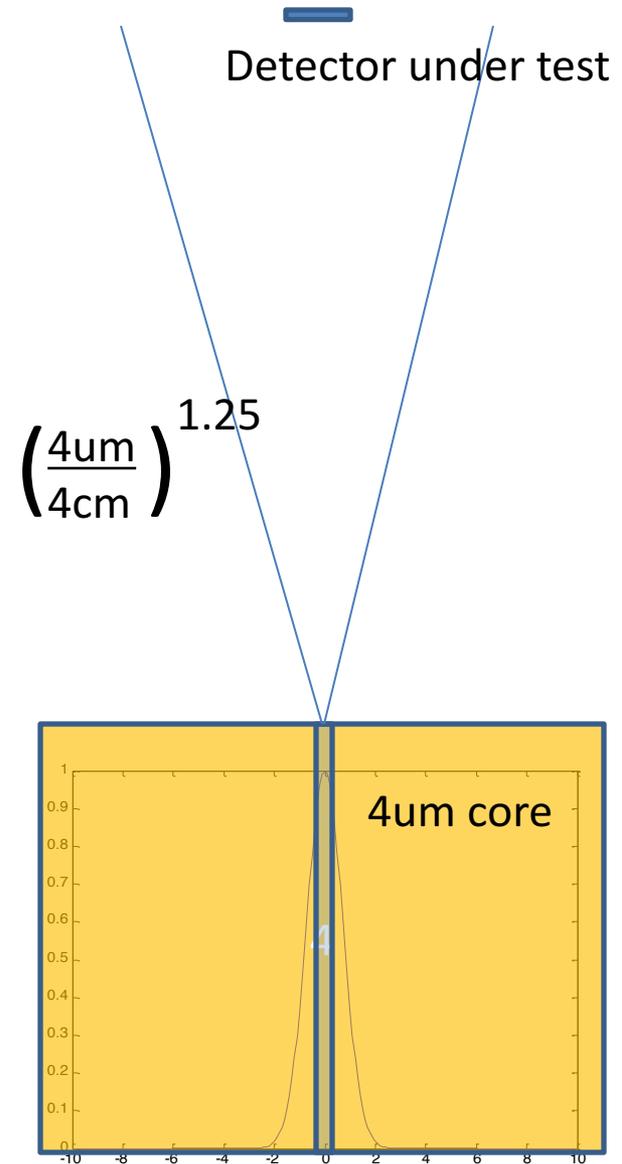
- What are the important properties of the light source for calibrating detectors?
- Dimensional precision (x,y)
- Photometric precision
- What source is closest to perfection?
- Single mode optical fiber.

Wavefront from fiber
Gaussian amplitude
spherical wavefront
from 2 fibers
form fringes



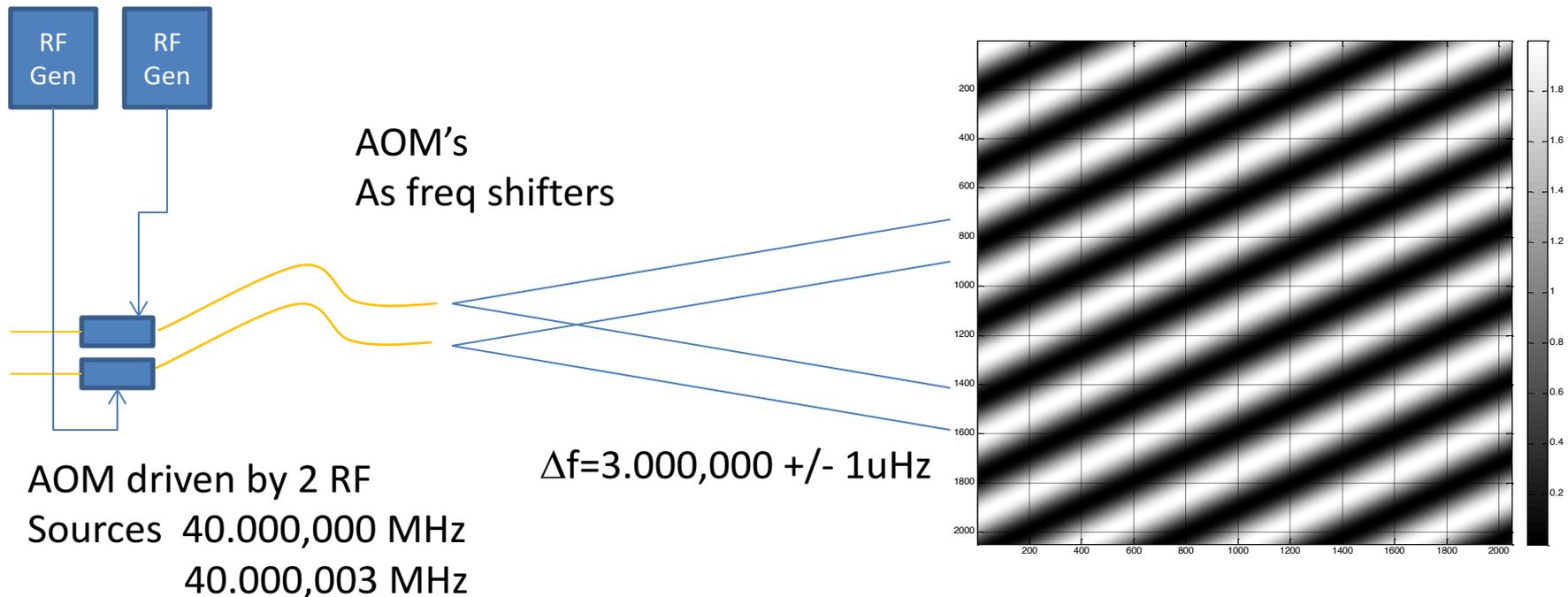
Wavefront from a Single Mode Fiber

- The wavefront from a single mode fiber is the closest thing to a perfect sphere made by Humans.
- Any error is in the flatness of the glass.
- Take 2 inch optic $\lambda/20$ p-v surface. And power spectrum
 - $\sim 1/f^{2.5}$
 - On a 4 μm scale the surface can potentially be flat to $\sim 10^{-5}\lambda$
- Interference between two spherical wavefronts produce hyperbolic fringes (that visually look like straight fringes)
- The intensity distribution is ideally a Gaussian. In reality also very close to a Gaussian. Deviations from a Gaussian are due to phase to amplitude conversion.



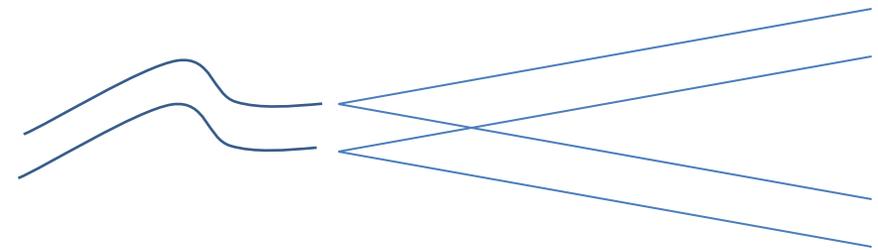
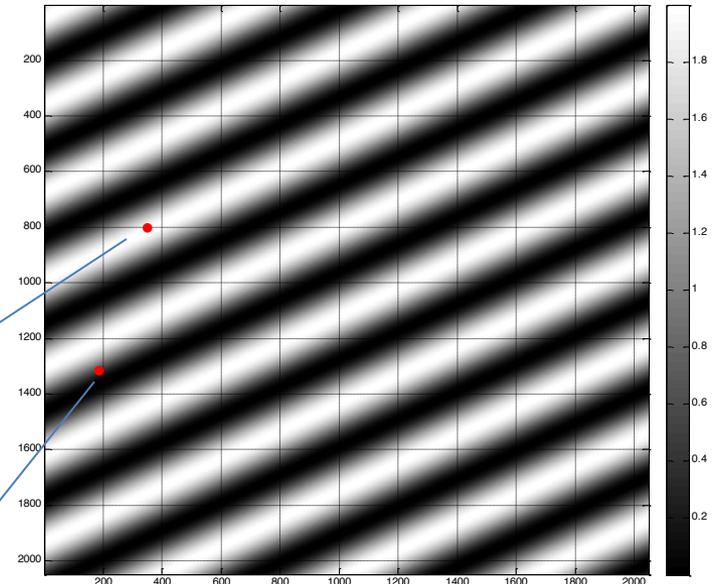
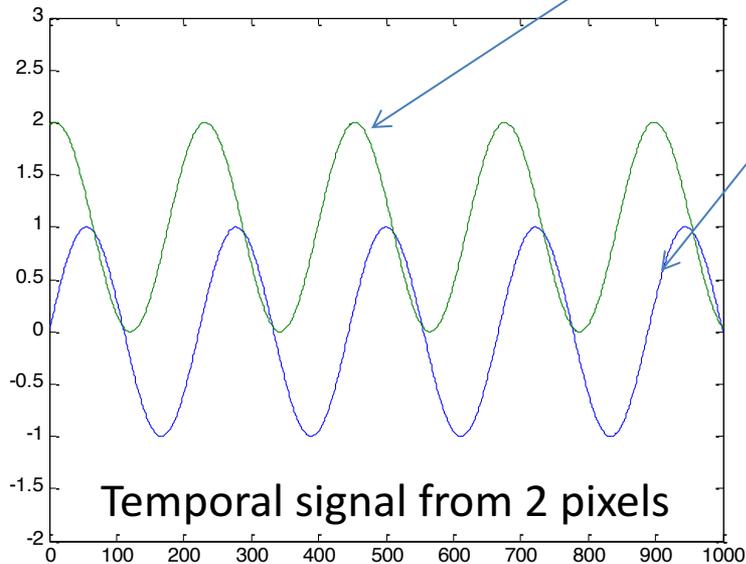
Fringes from Two Fibers

- There are multiple ways to “move the fringes across the detector”
- One common technique is to use AOM frequency shifters.
 - These devices shift the freq of the laser light by an amount by X hz, the RF signal driving the AOM
 - The accuracy of the freq shift of the laser light is the freq accuracy of the RF source.



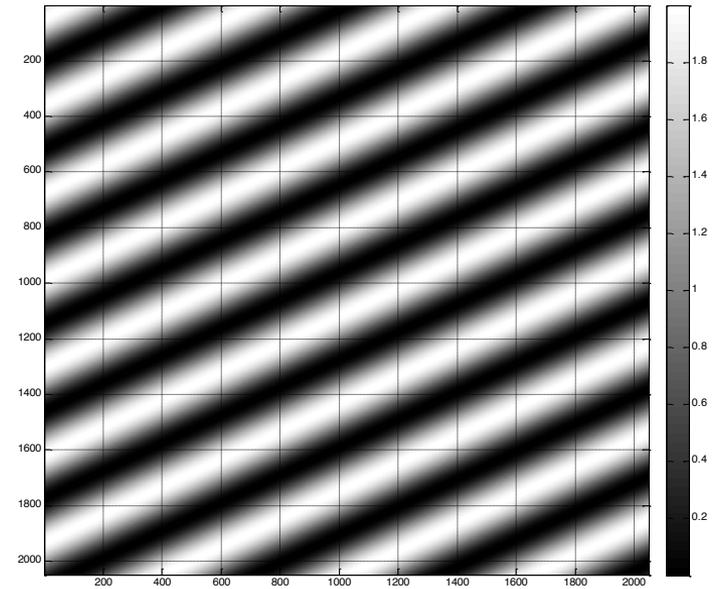
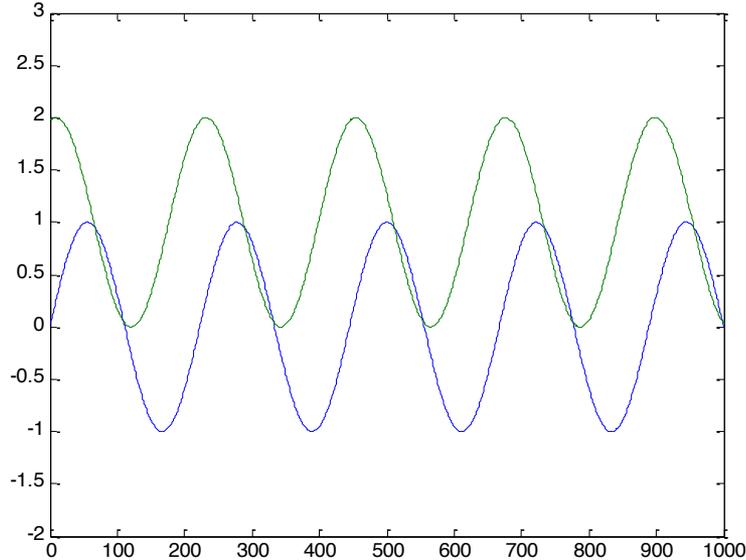
Fringes from 2 Fibers

- The dimensional accuracy of the fringes (across the whole detector) is directly related to the wavefront quality from the fibers.
- The fringes are caused to “move” either with an AOM or stretching the fiber. With an AOM the motion is as linear as the purity of the RF source driving the AOM.



Laser light in 1 fiber is freq shifted with respect to the other to provide “moving fringes” ~ a few hz the detector is read out with ~10 reads/ λ motion of the fringes

What are We Measuring?



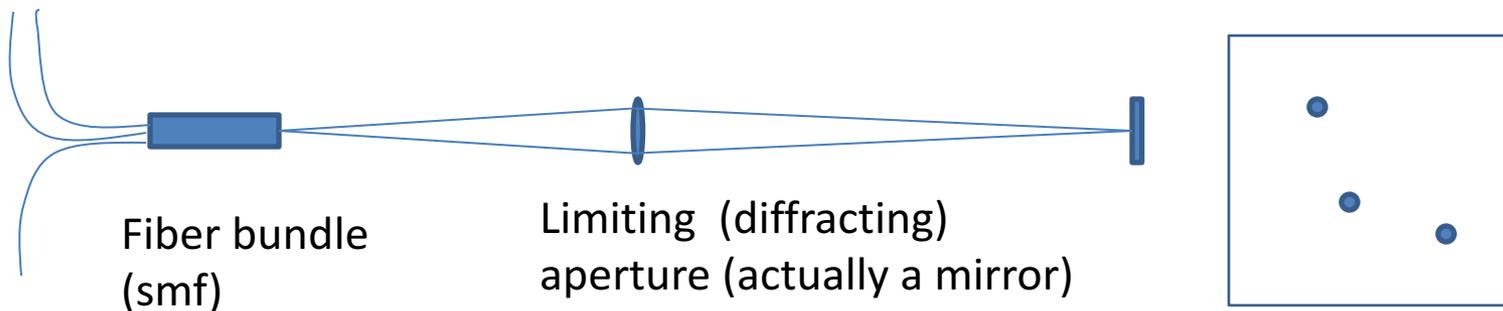
- The fringe is a near perfect sinusoid in space (x,y) and in time (t)
- $\text{Flux}(l,j,t) = A * \text{Vis} * \sin(k_x * X + k_y * Y + \omega * t + \phi)$
 - k_x, k_y spatial freq of fringe
 - ω is temporal freq of fringe
 - ϕ is the pixel position in the direction of fringe motion.

The detector only occupies a small part of the Gaussian beam from the fiber

$A(l,j)$ is the sum of the two Gaussians
 $\text{Vis}(l,j) = 2 * \sqrt{G1 * G2} / (G1 + G2)$

Projecting Point Source PSFs

- A point source diffracts off the focusing optic to produce an airy spot on the detector.



The wavefront from the fiber is near perfect. (geometric point)
There is **only 1 optic** that reimages the fiber to the detector

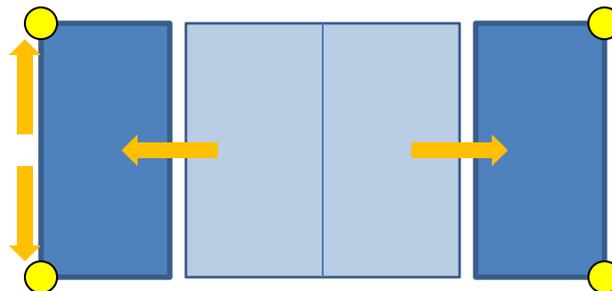
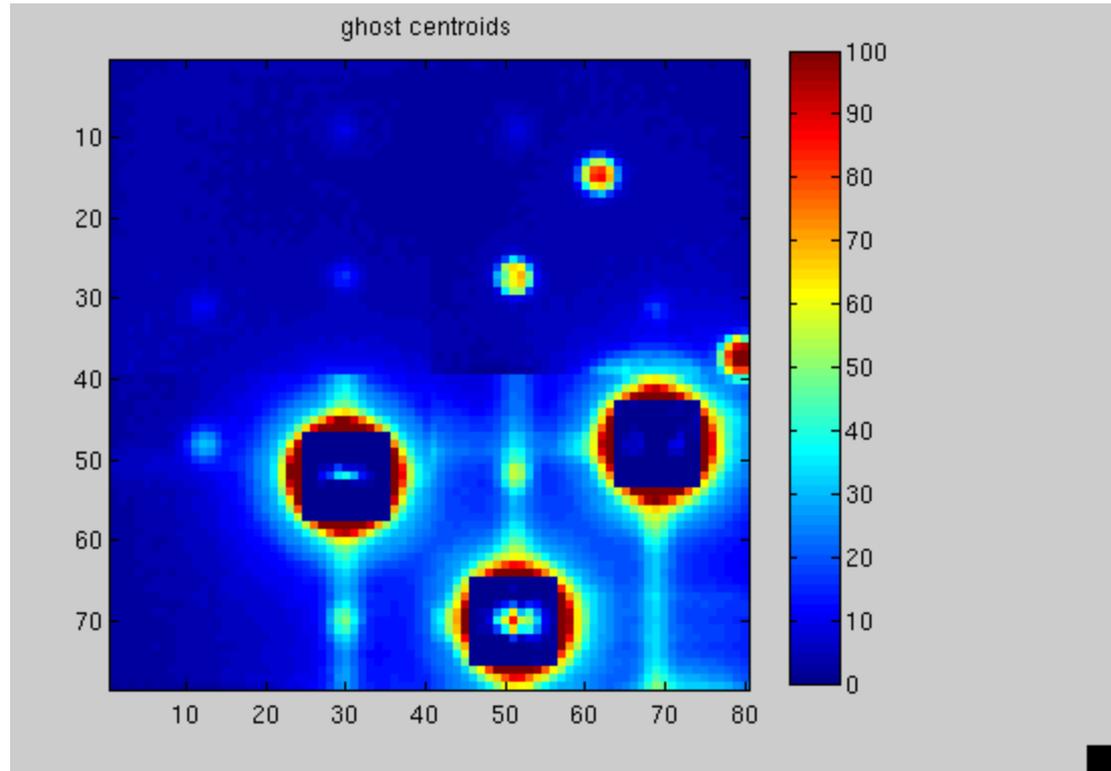
That optic is not perfect $\lambda/100$, the airy spot is not exactly the Bessel function(squared). But since all the images use the same part of the same focusing optic, **all the PSFs are identical.**

Ghost Images

We noticed ghost images in our setup. At first we looked for stray light reflections (egg window on the detector)

But when we looked at how the ghost images moved relative to the real images, we realized that these ghosts were the result of **electrical xtalk** between the **4 read amps** on the chip.

Our setup at JPL and our colleague's setup in Grenoble used the same E2V chip but totally different readout electronics. But saw similar ghosts.



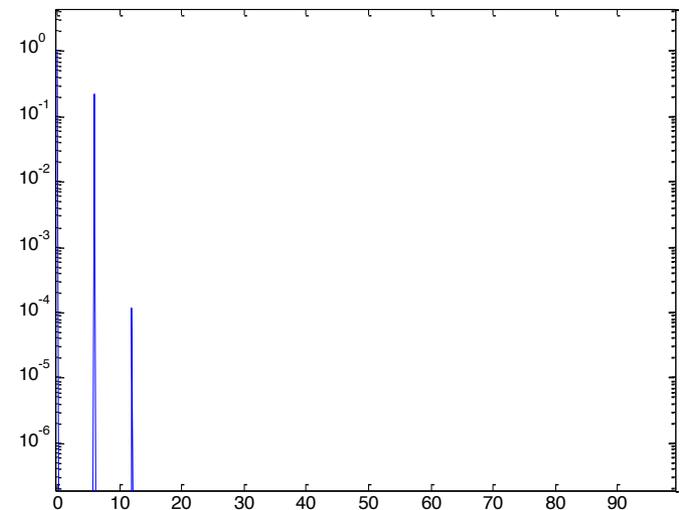
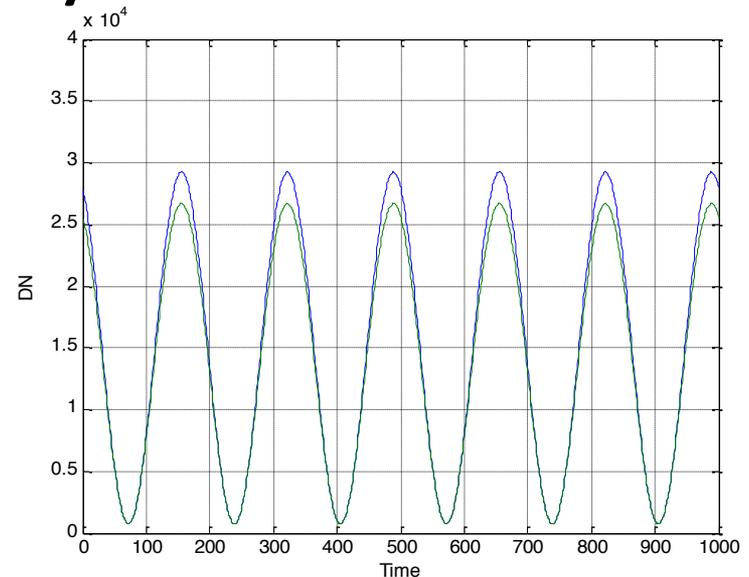
Chip
Rotated 90deg
Wrt movie

Photometric Linearity Calibration

- For each pixel's output, the laser fringes are a near perfect sinusoid in time. The phase of the sinusoid is as good as the RF generator driving the AOM.
- A deviation of a perfect sinewave is a measure of the non-linearity of the detector.

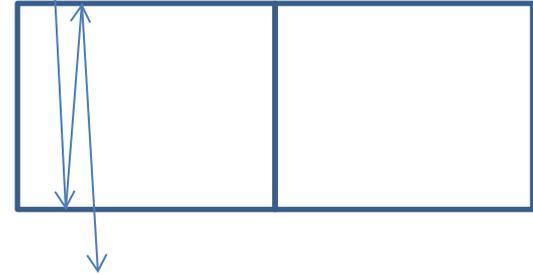
A quick test of photometric nonlinearity is the appearance of a 2nd harmonic. When taking A temporal FFT.

CCDs don't have (much) persistence. But with fiber illumination we can change the temporal freq without affecting the fringe amplitude. A change in amplitude with freq is a sign of persistence.



Internal Fringes from Laser Illumination

- With CCDs, at long wavelengths, the detectors are semi-transparent.
- A flat field measurement in laser light (especially at long λ) will exhibit “fringing” that is absent in a white light flat field at the same wavelength.
- The solution is to use a **tunable laser**, and repeat the detector calibration measurements across enough different wavelengths to average this effect away.



Cycle 17 WFC3 Instrument Handbook

documents.stsci.edu - 655 × 292 - Search by image

Figure 5.4: Fringe pattern of CCD Chip 2 with monochromatic flat-field illumination at 976 nm.

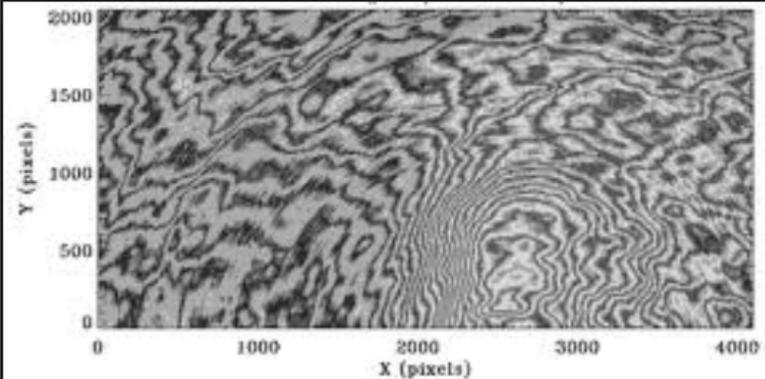
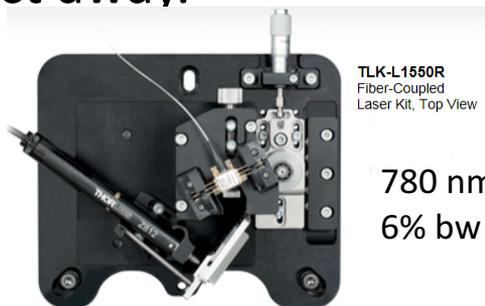
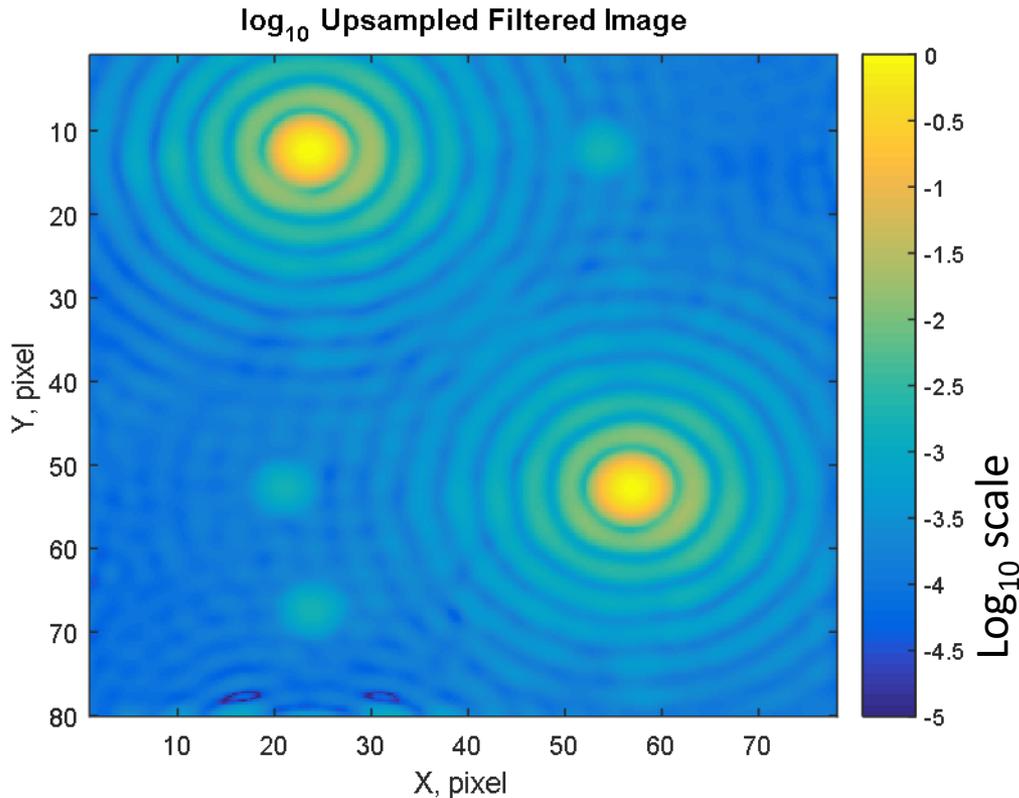


Image found via Google search



Photometric Errors from Ghosts

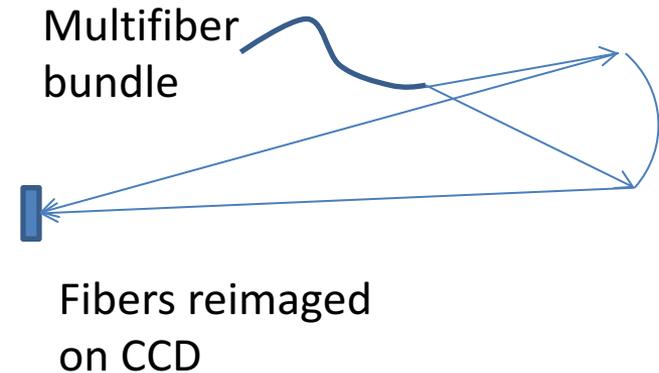
- Images were low pass filtered, and up sampled (to $16\text{pix}/(\lambda/D)$)
 - Image is taken $\sim 4\text{ pixels}/(\lambda/D)$
 - Source is white light $\sim 5\%$ bandwidth
 - Ghost images $\sim 10^{-3}$ from electrical Xtalk on chip



- Very high wavefront quality in test setup. (visually see 8th diffraction ring)
- Parameters fitted
 - X,Y center of image
 - A semi-major axis
 - B semi-minor axis
 - θ angle of semi-major axis
 - DC

Image Shape

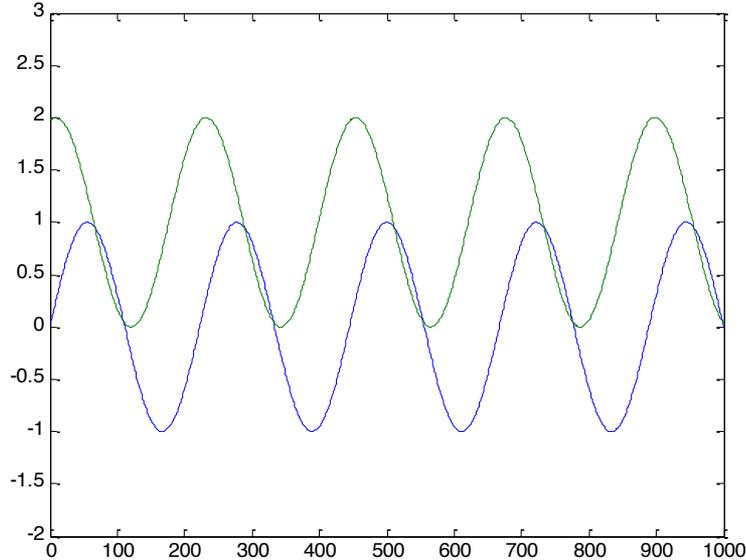
- The focusing optic was an off axis parabola. The circular aperture was slightly off axis, resulting in a slightly elliptical PSF.
- But both fiber images had exactly the same pupil aperture, and the two PSFs, and should therefore have almost the same ellipticity.



Parameter	Image 1(BR)	Image 2(UL)
$(1-b/a)$	0.05682	0.05554
$\text{Sqrt}(1-(b/a)^2)$	0.33228	0.32863
Rotation angle	1.48931	1.52428
Difference $(1-b/a)$	1.27×10^{-3}	
Sigma $(1-b/a)$	$\sim 0.8e-4$	$\sim 0.7e-4$

This exercise did not use the measured pixel offset errors. The images were moved across 3 pixels, the sigma represent the pixelation error in measuring ellipticity.

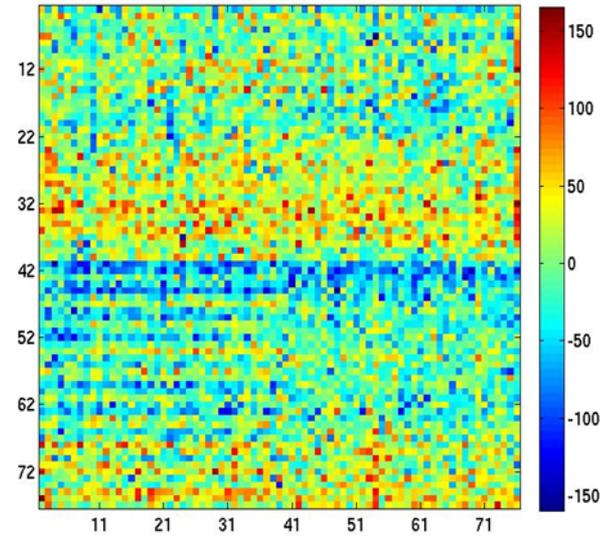
Pixels Geometry Errors



Δx_{nm}

Random
x,y err 40~50nm

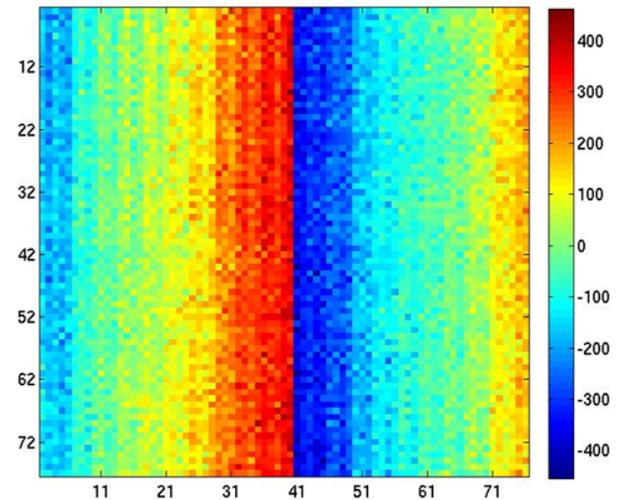
20101112r0486 pixel offset map,RMS=46.9nm



- The fringe is a near perfect sinusoid in space (x,y) and in time (t)
- $\text{Flux}(i,j,t) = A(1 + \text{Vis} \cdot \sin(k_x \cdot X + k_y \cdot Y + \omega \cdot t + \phi))$
 - K_x, k_y spatial freq of fringe
 - ω is temporal freq of fringe
 - ϕ is the pixel position in the direction of fringe motion.

Δy_{nm}

20101112r0474 pixel offset map,RMS=181nm

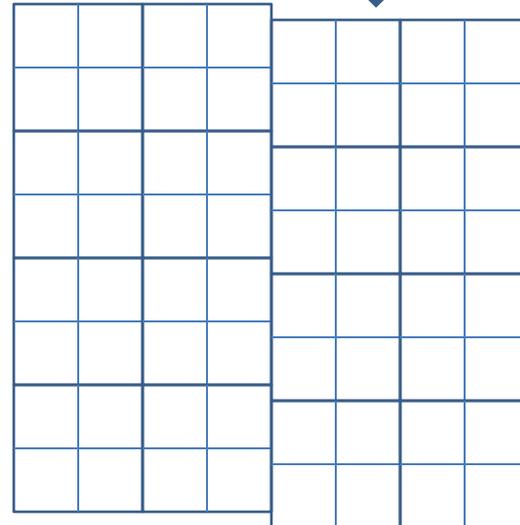
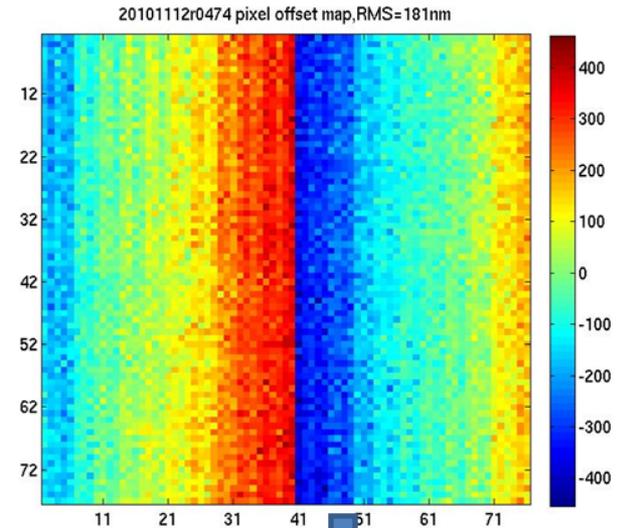


The color scale is in nanometers
the biggest offset is the 700nm slip between
the left ½ and right ½ of the chip.

$A(i,j)$ is the sum of the two Gaussians
 $\text{Vis}(i,j) = 2 \cdot \sqrt{G1 \cdot G2} / (G1 + G2)$

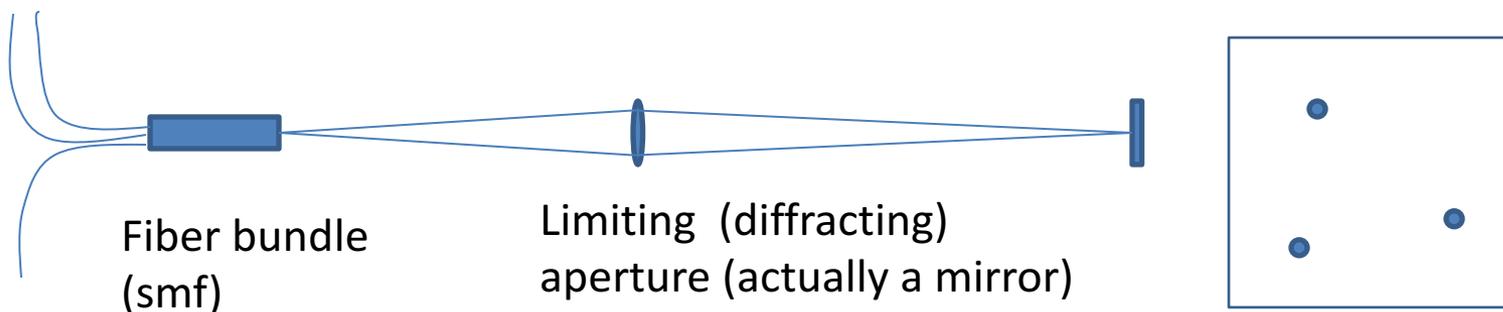
The Systematic Pixel Shift in E2V 39

- There is a an abrupt Y pixel shift as one moves across X in the detector
- Size of shift is $\sim 700\text{nm}$
- On top of that there are $30\sim 50\text{nm}$ random pixel position errors at every pixel



Projecting Point Source PSFs

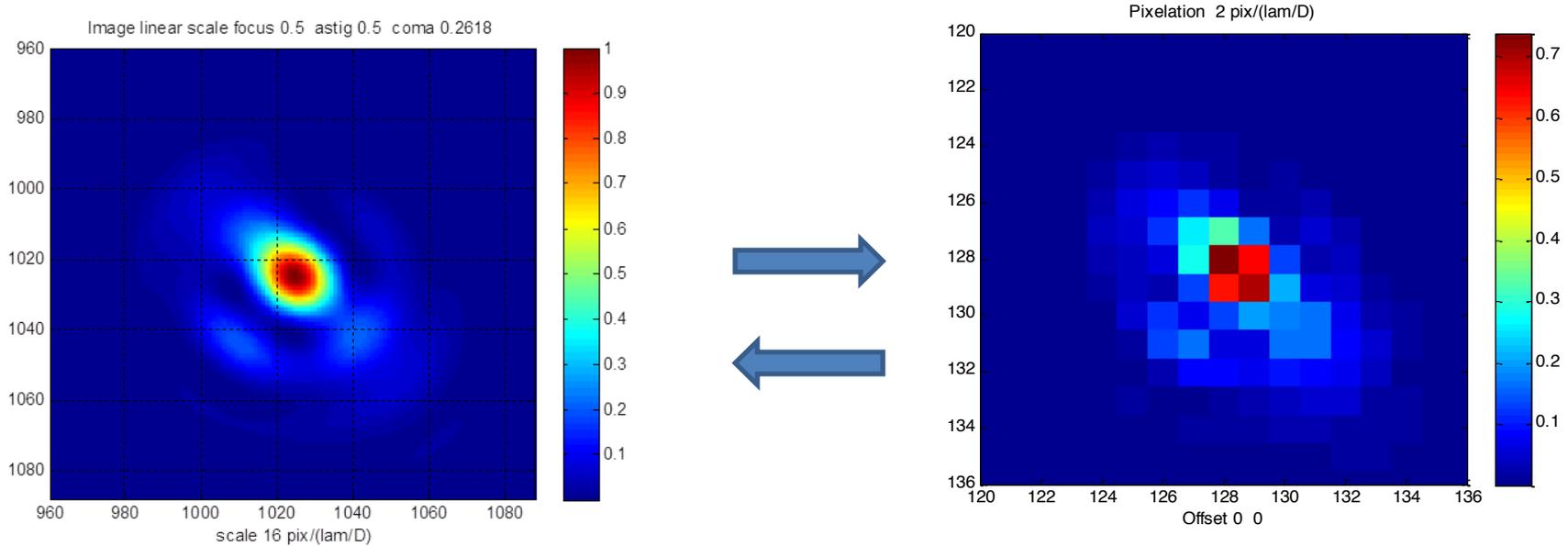
- A point source diffracts off the focusing optic to produce an airy spot on the detector.



The wavefront from the fiber is near perfect. (geometric point)
There is **only 1 optic** that reimages the fiber to the detector

That optic is not perfect $\lambda/100$, the airy spot is not exactly the Bessel function(squared). But since all the images use the same part of the same focusing optic, **all the PSFs are nearly identical.**

Accurate Astrometry Needs an Accurate Optical PSF



Original PSF (specified at high resolution)
Both images have the same information
content

Pixelated PSF (Nyquist sampled)
> 2 pixels per λ/D

For a Nyquist sampled images,
one can shift the pixelated image by 3.1416 pixels
without any computational error.
one can generate the original high resolution PSF from
the pixelated PSF without error.

Possible with a perfect
detector or a perfectly
calibrated detector.

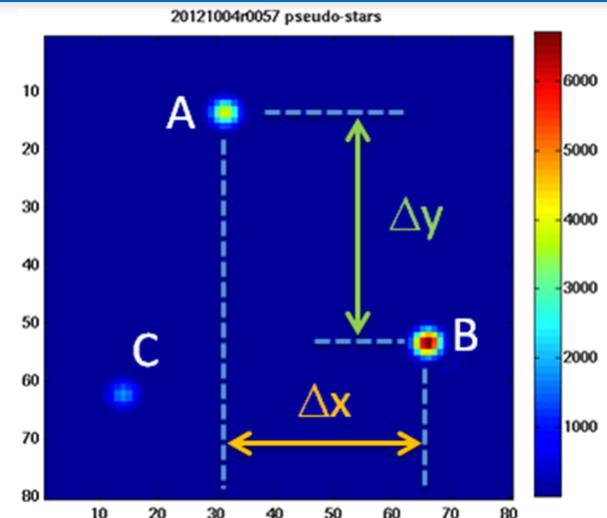
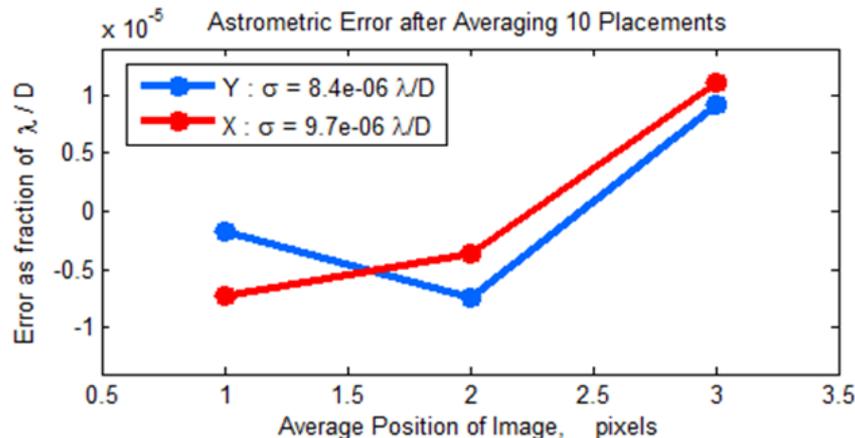
Centroiding at $10^{-5} \lambda/D$

- One of the errors in traditional CCD astrometry is a lack of knowledge of the true optical PSF.
 - But the true optical PSF can be measured with Nyquist sampled images (applying corrections with subpixel calibration).

We took the 3 airy spots and move them across 3 pixels of a CCD. Total of 30 positions. The separation between A,B was constant to $1.2e-4$ pixels at each of the 10 positions. After averaging 10 positions, the separation agreed to $\sim 10^{-5} \lambda/D$, $4e-5$ pixels.

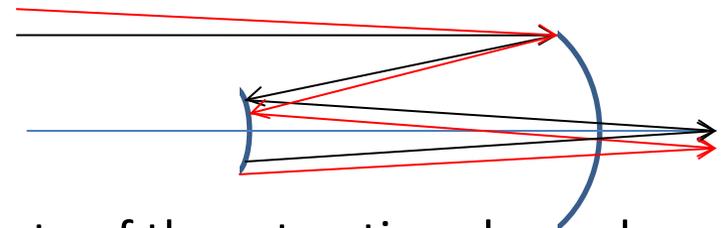
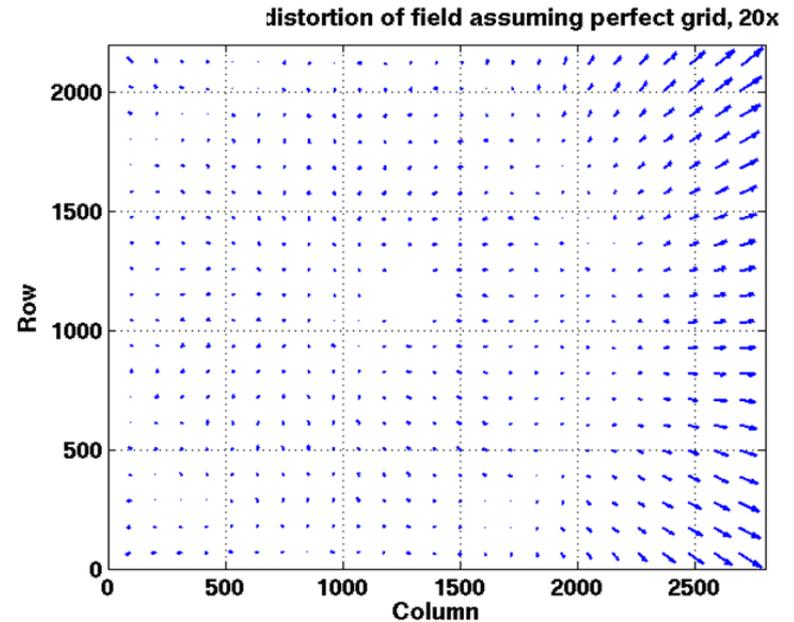
PSF oversampled $4\text{pix}/(\lambda/D)$

Achieved average error of $9e-6 \lambda/D$



Optical Distortion

- All telescopes have distortion, from the design. (pincushion shown)
 - But the distortion from the design is “known”
- But distortion can also arise from optics misalignment and beam walk.
- Light from all stars hit the primary mirror. But different stars use different parts of the secondary (tertiary etc.) optics.
- Imperfections in the fabrication or alignment of the optics will change distortion from the “design”.
- Simulations of a TMA (1.5m) telescope with state of the art optics, showed that distortion at the 1uas level implied need to calibrate every ~ 15 arcsec. Distortion can be modeled to ~ 1 uas over a 4 arcmin FOV with ~ 200 terms. For LUVVOIR, a higher number of terms may be needed.



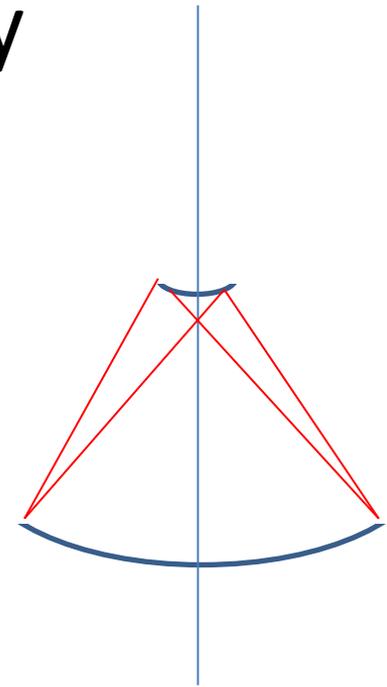
Optical Distortion

Stability and Calibration

- Optical distortion exists even for a telescope whose surfaces are perfect.
- HST is in a hostile thermal environment (Sunlight, night time) future observatories will be GEO/L2 orbits many orders of magnitude more thermally stable.
- Calibrate distortion by dithering on a dense star field (globular cluster)
 - What's important is the distortion doesn't change between more than errors from detector imperfections.
- Comparison with coronagraph.
 - Astrometry to $1e-4$ pixels => stability at $\sim 5 \times 10^{-5} \lambda$
 - Coronagraphy ($1e-10$) => correction to $\sim 3 \times 10^{-6} \lambda$
- Good thermal design, Choose calibration field and target field to only requires rotation along the Sun vector. (so solar illumination of the spacecraft is constant)

Maintaining Stability

- Stable optical system for astrometry
- Key points
 - Alignment stability
 - Optical figure of secondary/tertiary optics
 - (paradoxically stability of the primary is not critical, change in figure appears across the whole FOV)
- Operate optics near their Zero CTE temperature.
- Optical metrology to measure/maintain alignment at nm levels.



Optics stable to $10^{-4} \lambda$, $\sim 60\text{pm}$
CTE $\sim 10^{-8}/\text{K}$ a 2m optic is stable
60pm when the temp is stable to
3mK. (but uniform expansion of
60pm is not a problem)
Thermal gradients stable $\sim 10\text{mK}$.

This level of stability is **NOT**
Sufficient for coronagraphy

Summary

- There are two (and only 2) types of instrumental errors.
 - Detector errors (almost ready at $\sim 1e-4$ pixels)
 - Optical distortion errors. (not demonstrated in the lab yet, but simple calculations on thermal stability imply this is doable when the telescope is in HEO/GEO/L2)
- Major impact on exoplanet science and cosmology and solar system dynamics (planet X)
- **On orbit metrology** is part of the baseline design of the LUVOIR High definition camera. (along with Nyquist sampling of the PSF to $\sim 400\text{nm}$.)
- Detector calibration is also important for precision RV spectrometers that are aiming to get below $10\sim 20\text{cm/sec}$.