Instrument Modeling:
Direct Imaging Photometrics & SNR

Bijan Nemati
Tellus1 Scientific, LLC

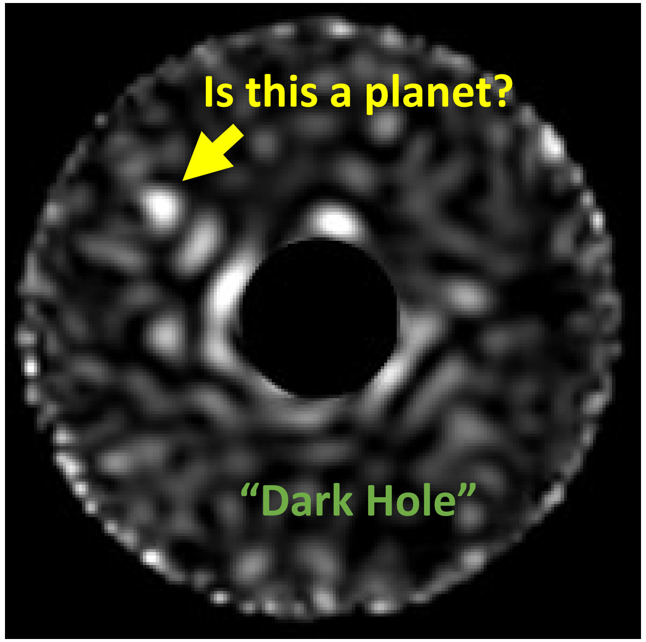
Exoplanet Science Yield Modeling Workshop
242nd AAS Meeting: 4–8 June 2023

Detecting a Planet

- Suppress the diffracted starlight within a limited “dark hole”
  - The planet is off-axis:
    - Its image undergoes relatively little suppression

- The challenges:
  - Starlight suppression is not perfect
    - Some starlight leaks out into the dark hole
      - This causes false positives
    - Planet light is also suppressed
      - But much less than the starlight
  - Mitigation:
    - Maximize signal efficiency
    - Minimize noise

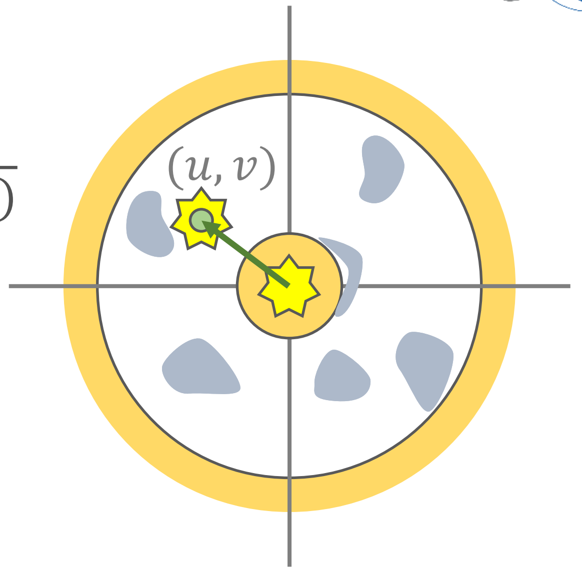
→ Maximize signal to noise ratio (SNR)



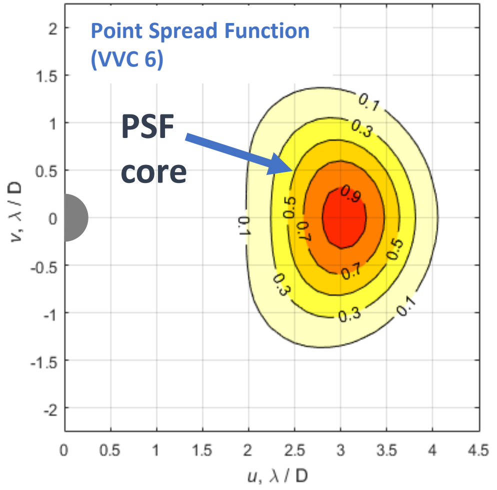
Key Coronagraph Attributes

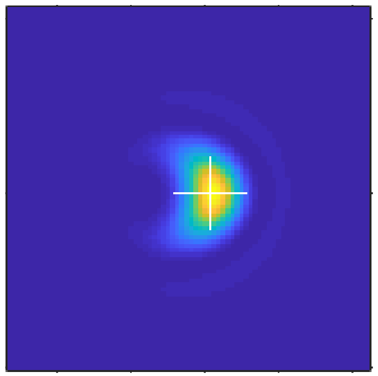
- Starlight Suppression
  - Contrast
    - ratio of throughput from (0,0) to throughput from (u, v)

$$C(u,v) \equiv \frac{\tau\,(0,0)}{\tau_{pk}\,(u,v)}$$

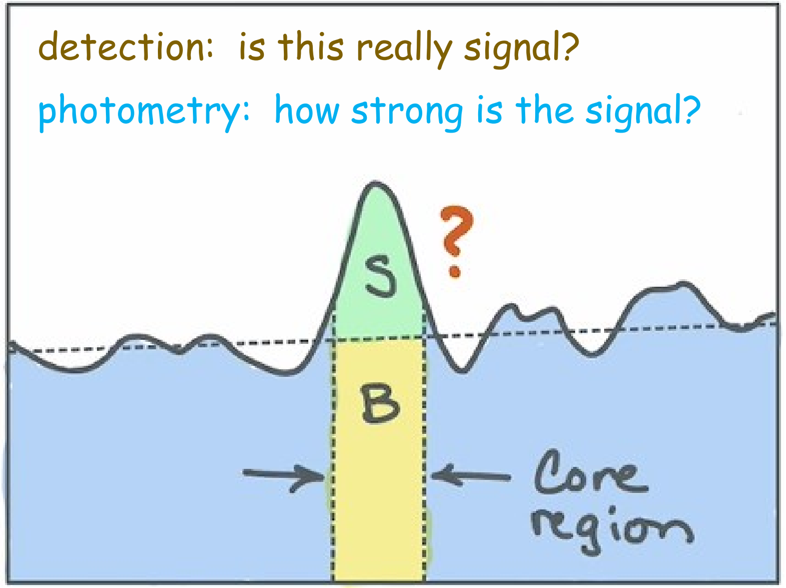


- Planet Light Acceptance
  - Core Throughput
    - fraction of planet light reaching focal plane & in core





Signal to Noise Ratio – Distinctions



Detection and spectroscopy are different statistical questions.

- For planet detection, we would be interested in detection SNR:
  - We are instead interested in the background's false positive probability

$$\mathrm{SNR}_{det} = \frac{S}{\sqrt{B}}$$

planet signal

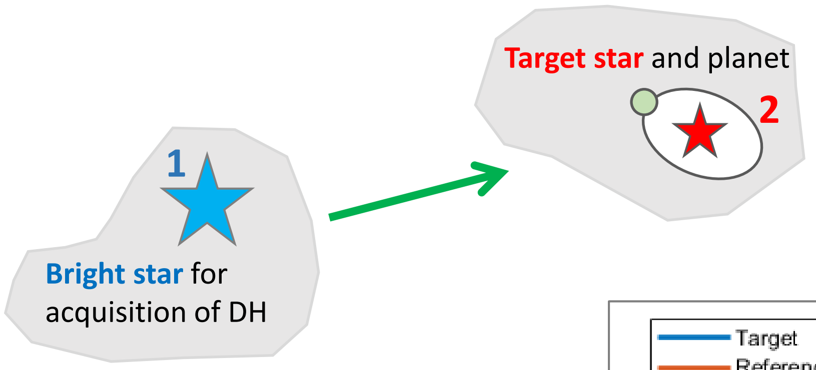
background

- For photometry and spectrometry, we are interested in the photometric SNR:

$$\mathrm{SNR}_{phot} = \frac{S}{\sqrt{S+B}}$$

A Simple Observing Scenario for Yield

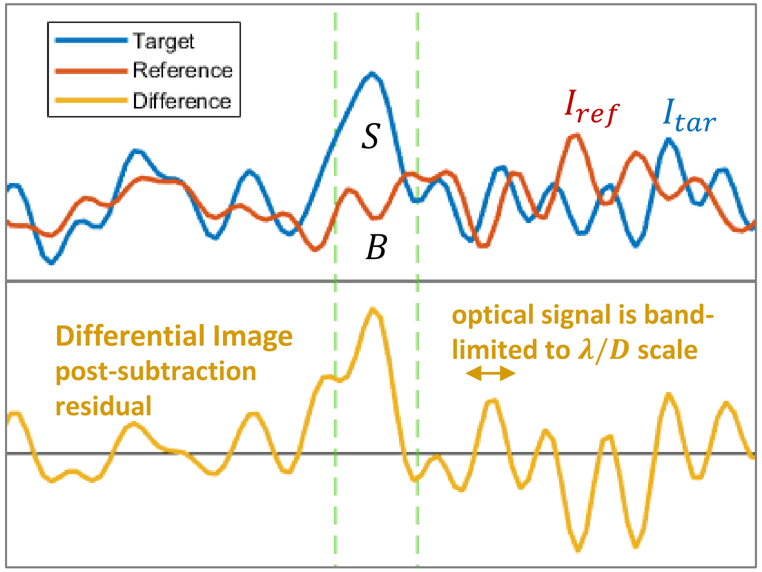
- We seek a simple analytical model of planet yield, via calculating the time to reach the desired SNR.



- Adopt some assumptions:

  1. We are doing **differential imaging**.
     - The SNR is for after differential imaging
     - For simplicity we assume we are doing **Reference Differential Imaging (RDI)**.

$$\text{Signal} = I_{tar} - I_{ref}$$
$$= (S + B_{tar}) - B_{ref}$$
$$= S + (B_{tar} - B_{ref})$$



Photometric SNR structure

- Analytical expression for SNR:

planet rate, time

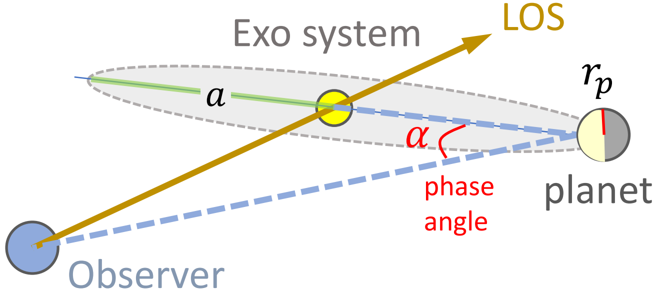
$$\mathrm{SNR} = \frac{r_{pl}\, t}{\sqrt{r_n t + \sigma_s^2}}$$

noise rate

Flux ratio, area, QE

$$r_{pl} = F_\lambda \Delta\lambda\, \xi_{pl}\, A\, \tau_{pl}\, \eta$$

flux, throughput



- Speckle Subtraction Error:
  - variance increases with time like signal!
  - This is reasonable to expect for the way speckle subtraction error scales with target star integration time

$$\sigma_s = r_{\Delta I}\, t$$

Planet
Flux Ratio

geometric albedo

$$\xi_{pl} = A_g \cdot \phi(\alpha) \cdot \left(\frac{r_p}{a}\right)^2$$

phase function

Phase function for a Lambert Sphere:

$$\phi(\alpha) = \frac{\sin(\alpha) + (\pi - \alpha)\cos(\alpha)}{\pi}$$

Random Noise Terms

planet speckle zodi

$$r_n = \underbrace{\left[F_\lambda \Delta\lambda\, \xi_{pl}\, \tau_{pl} + F_\lambda \Delta\lambda C_{CG} \tau_{pk} m_{pix} \tau_{sp} + \left(\frac{d\Phi_Z}{d\Omega}\, \Delta\Omega_{PSF} \right) \tau_Z \right] A_{PM}\, \eta}_{\text{photonic (shot noise) terms}}$$

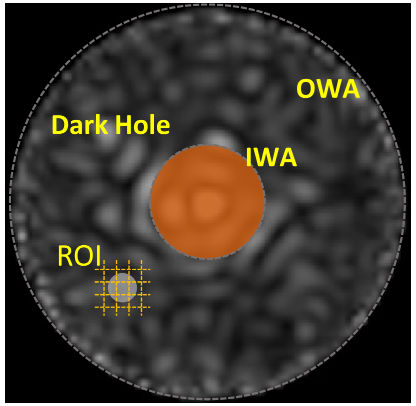
$\mathrm{ENF} \equiv$ EMCCD excess noise factor $\sim \sqrt{2}$

0-mag flux; zodi surface brightness, mag/as2

surface flux $\dfrac{d\Phi_Z}{d\Omega} = F^0 \Delta\lambda \cdot \dfrac{10^{-0.4\,\zeta}}{as^2}$

both local zodi and exo zodi need to be considered

m_{pix} pixels assumed under the signal ROI



SNR and time-to-SNR

- The signal to noise ratio for detection is given by:
  - where:
    - r_{pl} is the count rate for the planet
    - r_n is the count rate of the noise
    - $r_{\Delta I}$ is the residual speckle rate

$$\mathrm{SNR} = \frac{r_{pl}t}{\sqrt{r_n t + r_{\Delta I}^2 t^2}}$$

neglecting calibration errors

variance from random noise ($r_n t$); variance from residual speckle ($r_{\Delta I}^2 t^2$)

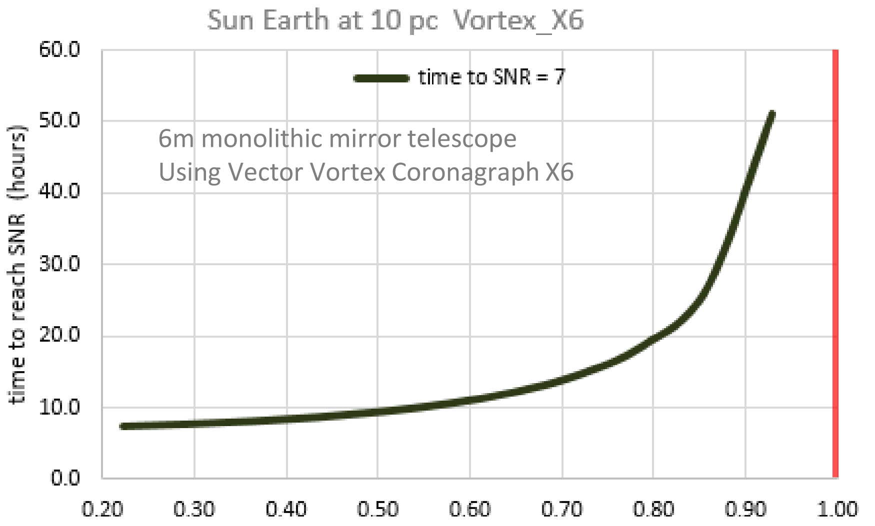
- Inverting this equation gives the integration time to get to a needed SNR:
  - Note: time goes to ∞ as the residual speckle rate approaches r_{pl}/SNR

$$t_{\mathrm{SNR}} = \frac{r_n}{\frac{r_{pl}^2}{\mathrm{SNR}^2} - r_{\Delta I}^2}$$

- Useful definition: Critical SNR

$$SNR_{crit} = \frac{r_{pl}}{r_{\Delta I}}$$

  - Given a planet rate, and given how well we can subtract speckles, the critical SNR is the asymptotic highest SNR achievable



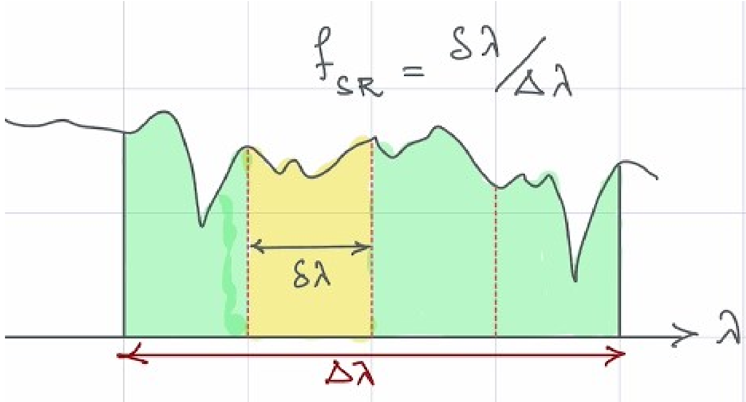
$$r_{\Delta I} \Big/ \left(\frac{r_{pl}}{\mathrm{SNR}}\right)$$

Modifications for Spectroscopy

- In spectroscopy, we are still doing photometry, but now per spectral element

$$r_{pl} = f_{SR}\, F_\lambda\, \Delta\lambda\, \xi_{pl}\, A\, \tau_{pl}\, \eta\, t$$

- Hence, we must account for the fact that only part of the light falls in the spectral element



$$f_{SR} = \begin{cases} 1 & \text{photometry} \\ \dfrac{1}{R \cdot BW} & \text{spectrometry} \end{cases}$$

\# of spec elements

- Often also the number of pixels per PSF core is different

$$R = \frac{\lambda}{\delta\lambda} \qquad BW = \frac{\Delta\lambda}{\lambda}$$

For Reference See:

Tellus1 scientific, LLC

The Analytical Performance Model and Error Budget for the Roman Coronagraph

Bijan Nemati[a,*], John Krist[b], Ilya Poberezhskiy[b], Brian Kern[b]
[a]Tellus1 Scientific, LLC, 8000 Madison Blvd., Ste. D102-265, Madison, AL
[b]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA

Abstract. The Nancy Grace Roman Space Telescope ("Roman"), under development by NASA, will investigate possible causes for the phenomenon of dark energy and detect and characterize extra-solar planets. The 2.4 m space telescope has two main instruments: a wide-field, infra-red imager and a coronagraph. The coronagraph instrument (CGI) is a technology demonstrator designed to help bridge the gap between the current state-of-the-art space and ground instruments and future high-contrast space coronagraphs that will be capable of detecting and characterizing Earth-like planets in the habitable zones of other stars. Using adaptive optics, including two high-density deformable mirrors and low- and high-order wavefront sensing and control, CGI is designed to suppress the star light by up to 9 orders of magnitude, potentially enabling the direct detection and characterization of Jupiter-class exoplanets. Contrast is the measure of starlight suppression, and high contrast is the chief virtue of a coronagraph. But it is not the only important characteristic: contrast must be balanced against acceptance of planet light. The remaining unsuppressed starlight must also have a stable morphology to allow further estimation and subtraction. To achieve all these goals in the presence of the disturbance and radiation environment of space, the coronagraph must be designed and fabricated as a highly optimized system. The CGI error budget is the top level tool used to guide the optimization, enabling trades of various competing errors. The error budget is based on an analytical model which enables rapid calculation and tracking of performance for the numerous and diverse questions that arise in the system engineering process. In this paper we outline the coronagraph system engineering approach and the error budget. We then describe in detail the analytical model for direct imaging and spectroscopy and show how it connects to the error budget. We introduce a number of useful ancillary metrics which provide insight into the capabilities of the instrument. Since models always need to be validated, we describe the validation approach for the CGI analytical model.

Keywords: space telescope, coronagraph, exoplanet, imaging, modeling, error budget.

*Bijan Nemati, bijan.nemati@tellus1.com

this paper has been submitted to: