Picometer Wavefront Sensing using the Phase-Contrast Technique
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• Wavefront stability on the order of 10-40 picometer RMS required to maintain 1e-10 contrast
• Necessary to develop wavefront sensing techniques with:
  – Picometer sensitivity
  – High spatial-frequency resolution (~100 cycles per aperture)
  – Photon efficient
  – Computationally efficient

Analysis courtesy of Jeff Jewell (JPL)
Zernike Wavefront Sensor

- Zernike Wavefront Sensor (ZWFS) – phase contrast technique
  - Developed for microscopy of transparent cells (Zernike, F. (1942). *Physica*, 9(7), 686-698)
  - Modulates phase delays as intensity variations at an exit pupil
- Traditionally used for qualitative observations
  - Understanding electric field propagation required for quantitative measurements

Image credit: HTT Ltd. (2011)
ZWFS Operation

\[ u(x) = AP(x)e^{i\phi(x)} \]

\[ I(x) = u(x)u(x)^* \]
ZWFS Operation

- **ZWFS**: Common-mode interferometer
- Focal-plane dimple acts as spatial filter producing two interfering fields at a downstream pupil
  - $u_B(\mathbf{x})$: “Low-pass” and phase-shifted ($\pi/2$)
  - $u_A(\mathbf{x})$: “High-pass” un-shifted

\[
u(\mathbf{x}) = AP(\mathbf{x})e^{i\phi(\mathbf{x})}\\
\]

\[
u_B(\mathbf{x}) = L[u(\mathbf{x})]e^{i\pi/2} = iAb(\mathbf{x})e^{i\beta(\mathbf{x})}\\
\]

\[
v_A(\mathbf{x}) = H[u(\mathbf{x})] = AP(\mathbf{x})e^{i\phi(\mathbf{x})} - Ab(\mathbf{x})e^{i\beta(\mathbf{x})}\\
\]
ZWFS Operation

\[ u(x) = AP(x)e^{i\phi(x)} \]

\[ u_B(x) = L[u(x)]e^{i\pi/2} = iA\beta(x)e^{i\beta(x)} \]

\[ u_A(x) = H[u(x)] = AP(x)e^{i\phi(x)} - A\beta(x)e^{i\beta(x)} \]

\[ I(x) = |u_A(x) + u_B(x)|^2 \]

\[ \phi(x) - \beta(x) = \frac{\pi}{4} + \arcsin \left[ \frac{I(x) - I_P(x) - 2I_b(x)}{2\sqrt{2I_P(x)I_b(x)}} \right] \]
ZWFS Operation

\[ \phi(x) \]

\[ u(x) = AP(x)e^{i\phi(x)} \]

\[ u_B(x) = L[u(x)]e^{i\pi/2} = iAb(x)e^{i\beta(x)} \]

\[ u_A(x) = H[u(x)] = AP(x)e^{i\phi(x)} - Ab(x)e^{i\beta(x)} \]

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measured

modeled
Can a ZWFS detect picometer-level wavefront error changes?

\( u(x) = AP(x)e^{i\phi(x)} \)

\( u_B(x) = L[u(x)]e^{i\pi/2} = iAb(x)e^{i\beta(x)} \)

\( u_A(x) = H[u(x)] = AP(x)e^{i\phi(x)} - Ab(x)e^{i\beta(x)} \)

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ZWFS Testbed

- ZWFS Testbed established in the Precision Environment Test Enclosure (PETE)
  - Thermally/seismically/acoustically stable laboratory for in-air optical measurements
- 4D Twyman Green Interferometer (IFO) provides source (633nm) and independent wavefront measurement
- Andor iXon 897 EMCCD used as ZWFS Camera
  - Large pixel well depth and frame rate allow for high-speed acquisition
- 12x12 BMC Deformable Mirror (DM) used to induce wavefront errors


Steeves, J. et al., “Picometer Wavefront Sensing via the Phase-Contrast Technique”, in prep.
Data Acquisition

- DM actuators dithered at constant rate, multiple frames captured in ON or OFF states
- ZWFS operated in differential mode (wavefront error changes)
- Varying levels of averaging performed to eliminate stochastic effects
  - Air turbulence, testbed vibrations, thermal variations, etc.

\[
I_n^i = \frac{1}{M} \sum_{m=1}^{M} I_m^i \\
\Delta \phi_n = \phi(I_n^{ON}) - \phi(I_n^{OFF}) \\
\Delta \phi_N = \frac{1}{N} \sum_{n=1}^{N} \Delta \phi_n
\]

Steeves, J. et al., "Picometer Wavefront Sensing via the Phase-Contrast Technique", in prep.
ZWFS Calibration

- DM commanded to high-spatial frequency “checkerboard” pattern
  - Simultaneous measurement performed with 4D interferometer
- Strong agreement between the two techniques for wavefront changes up to 20 nm RMS
  - ZWFS signal inverts at $-\pi/4$ total wavefront error (static + change)
  - Techniques to extend dynamic range have been established (see vZWFS presentations by Doelman and Wallace)

Multi-valued problem: Intensity inversions occur when local phase exceeds $[-\pi/4, 3\pi/4]$ range

Steeves, J. et al., “Picometer Wavefront Sensing via the Phase-Contrast Technique”, in prep.
ZWFS Precision

- DM commanded with +/- 1 bit checkerboard pattern
- 60 picometer RMS measurement sensitivity demonstrated with strong SNR
  - Acquisition time <10sec for N =150
  - Measurement limited by testbed (DM LSB), not ZWFS
  - 0.6 picometer repeatability over 70 independent trials

Steeves, J. et al., "Picometer Wavefront Sensing via the Phase-Contrast Technique", in prep.
Performance Limitations

- Numerical models implemented to predict measurement repeatability
  - Camera noise parameters studied
- Sub-picometer repeatability predicted for single measurement (N=1)
- Experiments demonstrate shot-noise limited performance, \(1/\sqrt{N}\), for N > 150

\[
\sigma_{\Delta I(x)} = \sqrt{2\sigma_s^2 + \sigma_d^2 + \sigma_r^2} \propto \sqrt{N}
\]

\[
\sigma_{\Delta \phi(x)} = \frac{1}{N} \frac{\sigma_{\Delta I(x)}}{2\sqrt{2I_1(x)I_1,I_2(x)}} \propto \frac{1}{\sqrt{N}}
\]

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The Zernike Wavefront Sensor is a simple, robust method to detect spatially-varying wavefront errors
  – Common-mode interferometer
  – Requires only a focal plane mask and pupil-viewing camera

Strong agreement demonstrated between ZWFS and commercial IFO for nanometer-level wavefront errors

Picometer-level sensitivity demonstrated via in-air testbed

Repeatability performance agrees with numerical models

Applications
  – Out-of-band WFS for future coronagraphy missions (HabEx, LUVOIR)
  – In-band calibration/diagnostic tool
  – Dedicated WFS for DM drift monitoring
  – Laboratory-based interferometer for DM/active optic development
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