Jet Propulsion Laboratory
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Picometer Wavefront Sensing using the Phase-Contrast Technique

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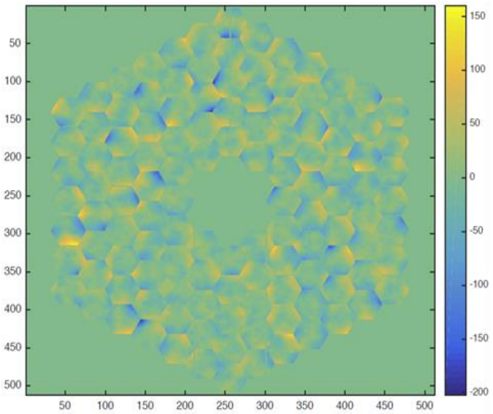
ExEP Workshop on Advanced Wavefront Sensing for Coronagraphs

05/01/2020

Picometer Wavefront Errors

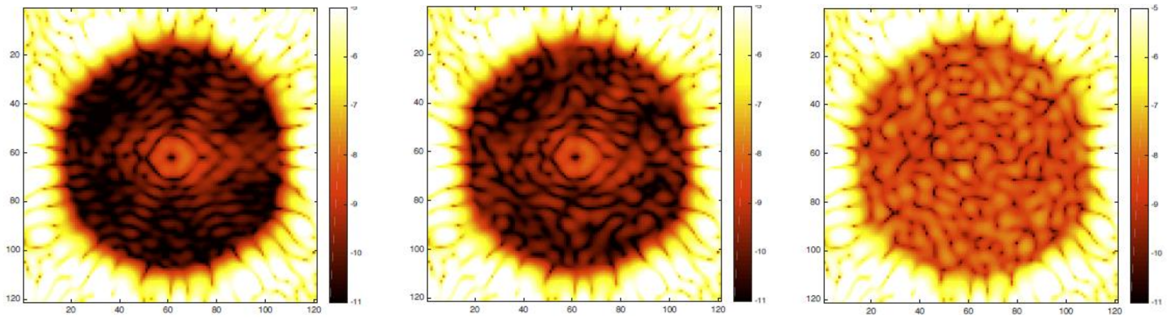
High spatial frequency wavefront errors at the picometer level



0 pm RMS
Contrast: 8.84e-11

10 pm RMS
Contrast: 1.38e-10

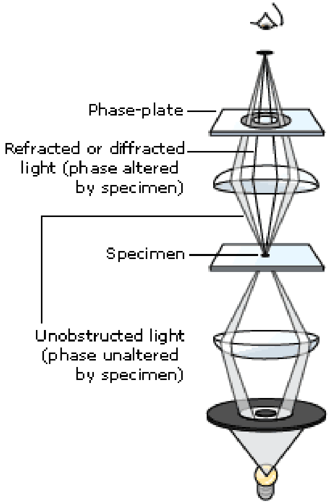
100 pm RMS
Contrast: ~1e-9



Analysis courtesy of Jeff Jewell (JPL)
Steeves J., et al. (2017). "Active Mirrors for High-Contrast Imaging". (URS269467)

- Wavefront stability on the order of 10-40 picometer RMS required to maintain 1e-10 contrast
- *Necessary to develop wavefront sensing techniques with:*
  - *Picometer sensitivity*
  - *High spatial-frequency resolution (~100 cycles per aperture)*
  - *Photon efficient*
  - *Computationally efficient*

Zernike Wavefront Sensor



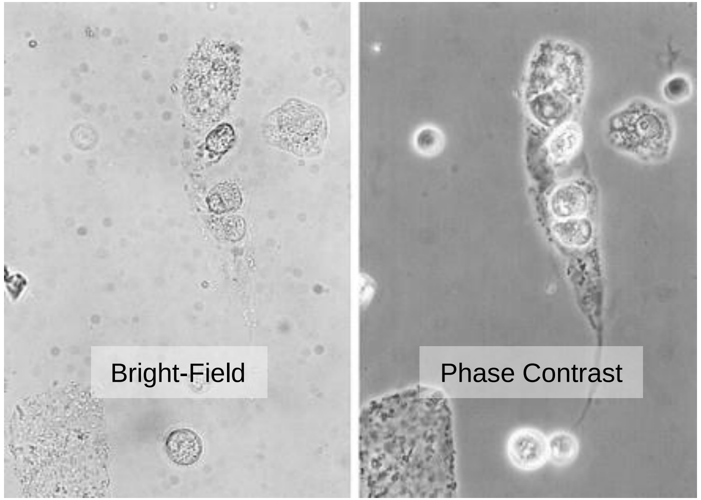
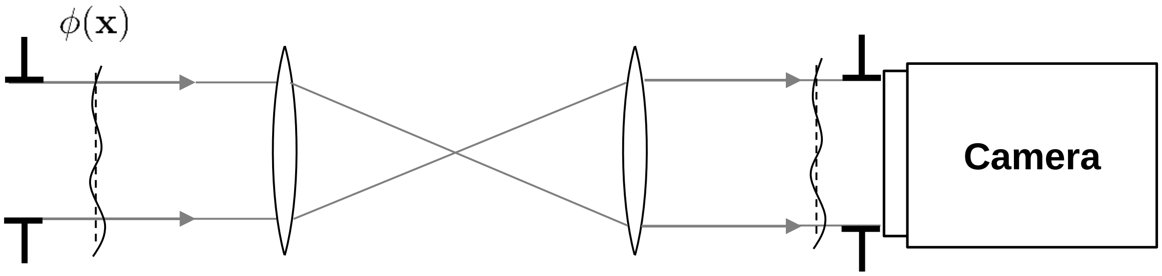


Image credit: HTT Ltd. (2011)

- Zernike Wavefront Sensor (ZWFS) – phase contrast technique
  - Developed for microscopy of transparent cells (Zernike, F. (1942). *Physica*, *9*(7), 686-698)
  - Modulates phase delays as intensity variations at an exit pupil
- Traditionally used for qualitative observations
  - Understanding electric field propagation required for quantitative measurements

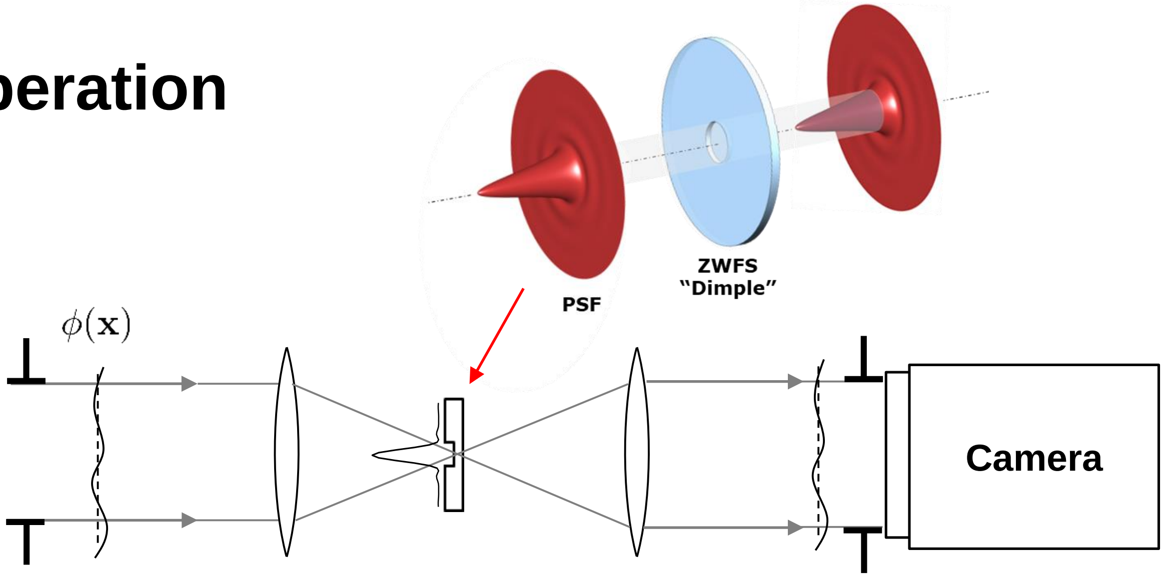
ZWFS Operation



$$u(\mathbf{x}) = AP(\mathbf{x})e^{i\phi(\mathbf{x})}$$

$$I(\mathbf{x}) = u(\mathbf{x})u(\mathbf{x})^*$$

ZWFS Operation



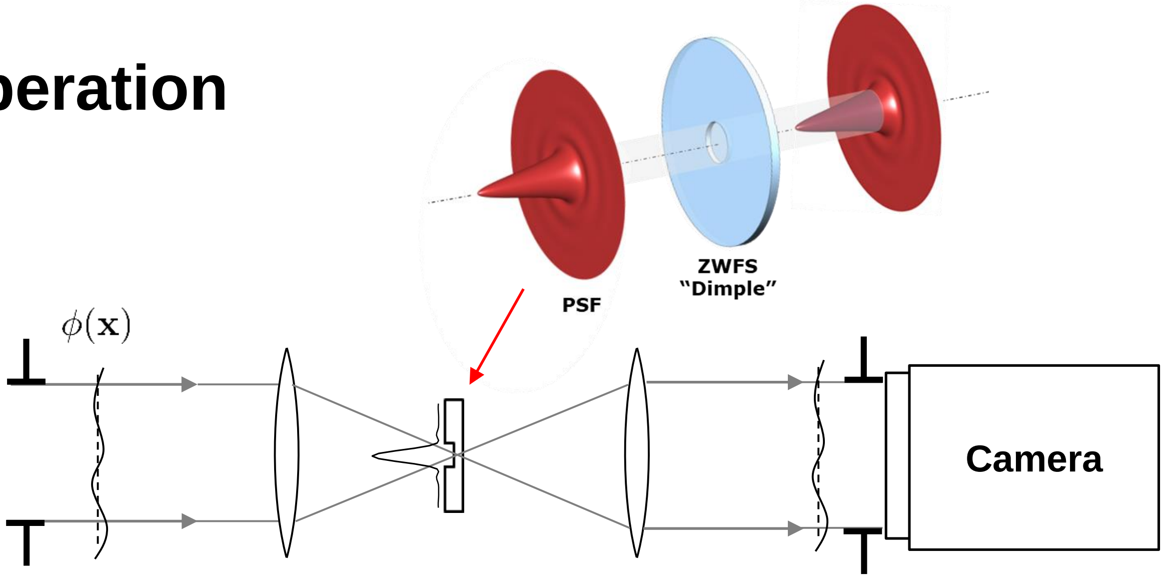
$$u(\mathbf{x}) = AP(\mathbf{x})e^{i\phi(\mathbf{x})}$$

$$u_B(\mathbf{x}) = L[u(\mathbf{x})]e^{i\pi/2} = iAb(\mathbf{x})e^{i\beta(\mathbf{x})}$$

$$u_A(\mathbf{x}) = H[u(\mathbf{x})] = AP(\mathbf{x})e^{i\phi(\mathbf{x})} - Ab(\mathbf{x})e^{i\beta(\mathbf{x})}$$

- ZWFS: Common-mode interferometer
- Focal-plane dimple acts as spatial filter producing two interfering fields at a downstream pupil
  - $u_B(\mathbf{x})$: "Low-pass" and phase-shifted (π/2)
  - $u_A(\mathbf{x})$: "High-pass" un-shifted

ZWFS Operation



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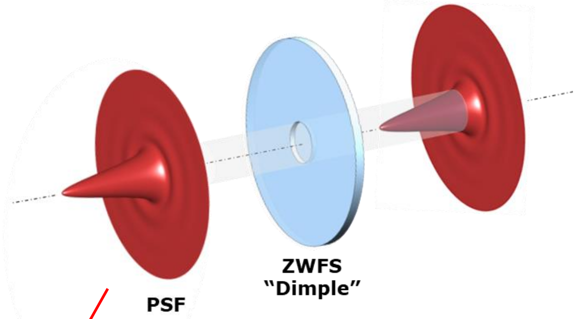
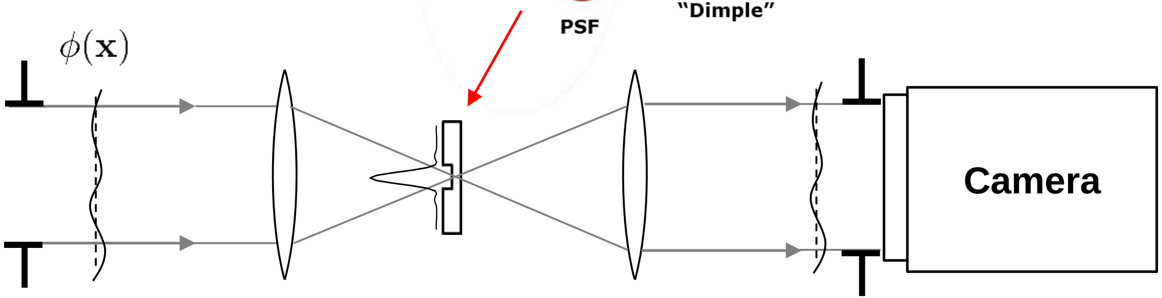
$$I(\mathbf{x}) = |u_A(\mathbf{x}) + u_B(\mathbf{x})|^2$$

$$\phi(\mathbf{x}) - \beta(\mathbf{x}) = \frac{\pi}{4} + \arcsin\left[\frac{I(\mathbf{x}) - I_{P(\mathbf{x})} - 2I_{b(\mathbf{x})}}{2\sqrt{2I_{P(\mathbf{x})}I_{b(\mathbf{x})}}}\right]$$

ZWFS Operation





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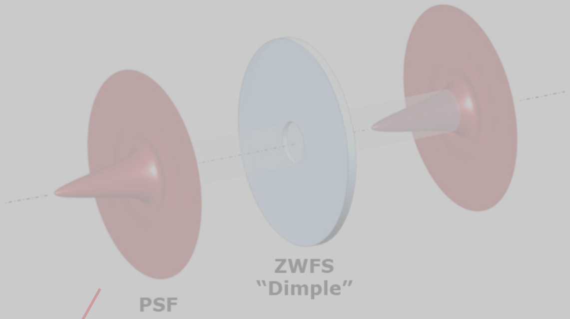
measured

modeled

ZWFS Operation



$\phi(\mathbf{x})$

Can a ZWFS detect picometer-level wavefront error changes?

$$u(\mathbf{x}) = AP(\mathbf{x})e^{i\phi(\mathbf{x})}$$

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measured

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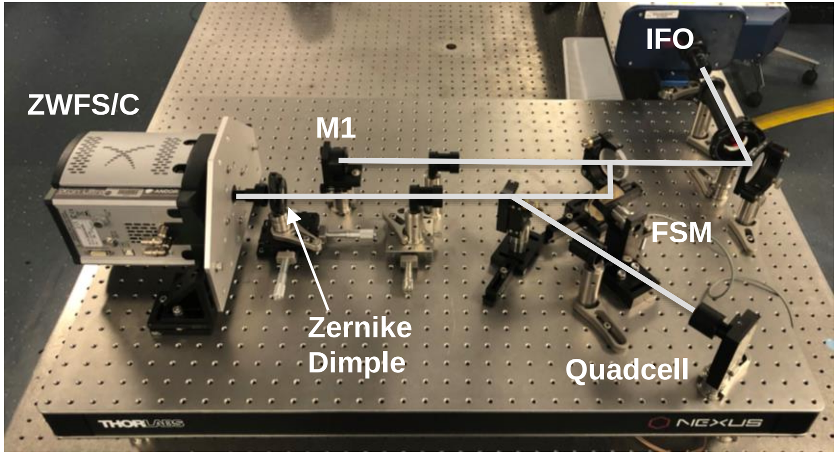
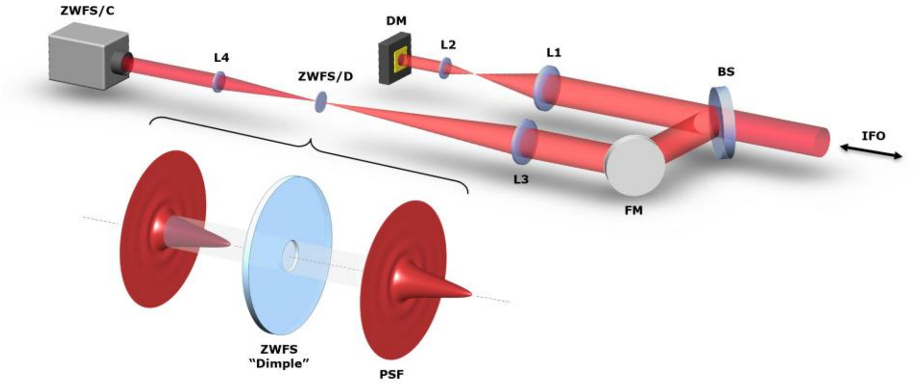
modeled

ZWFS Testbed



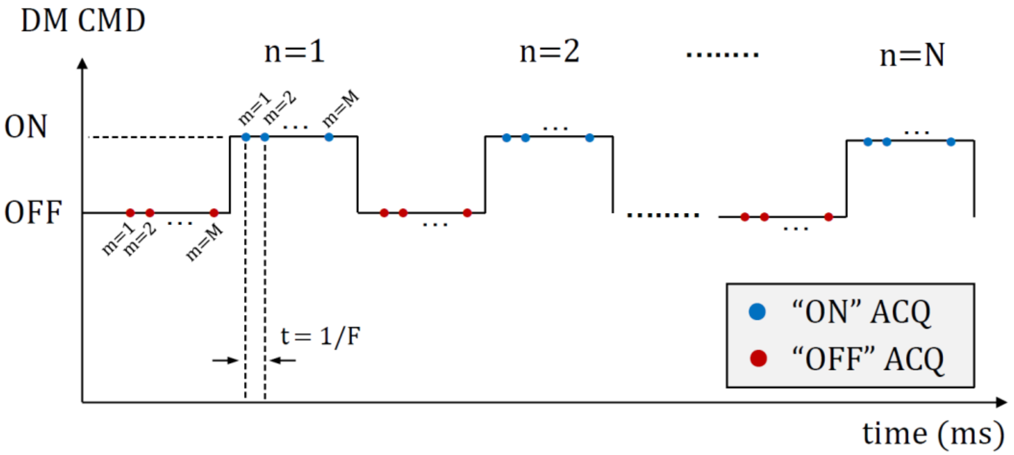
Steeves J., et al. (2017). “Active Mirrors for High-Contrast Imaging”.
JPL RTD Poster Session (URS270081)



Steeves, J. *et al.*, “Picometer Wavefront Sensing via the Phase-Contrast Technique”, in prep.

- ZWFS Testbed established in the Precision Environment Test Enclosure (PETE)
  - Thermally/seismically/acoustically stable laboratory for in-air optical measurements
- 4D Twyman Green Interferometer (IFO) provides source (633nm) and independent wavefront measurement
- Andor iXon 897 EMCCD used as ZWFS Camera
  - Large pixel well depth and frame rate allow for high-speed acquisition
- 12x12 BMC Deformable Mirror (DM) used to induce wavefront errors

Data Acquisition



$$I_n^i = \frac{1}{M} \sum_{m=1}^{M} I_m^i$$

$$\Delta\phi_n = \phi(I_n^{ON}) - \phi(I_n^{OFF})$$

$$\Delta\phi_N = \frac{1}{N} \sum_{n=1}^{N} \Delta\phi_n$$

- DM actuators dithered at constant rate, multiple frames captured in ON or OFF states
- ZWFS operated in differential mode (wavefront error changes)
- Varying levels of averaging performed to eliminate stochastic effects
  - Air turbulence, testbed vibrations, thermal variations, etc.

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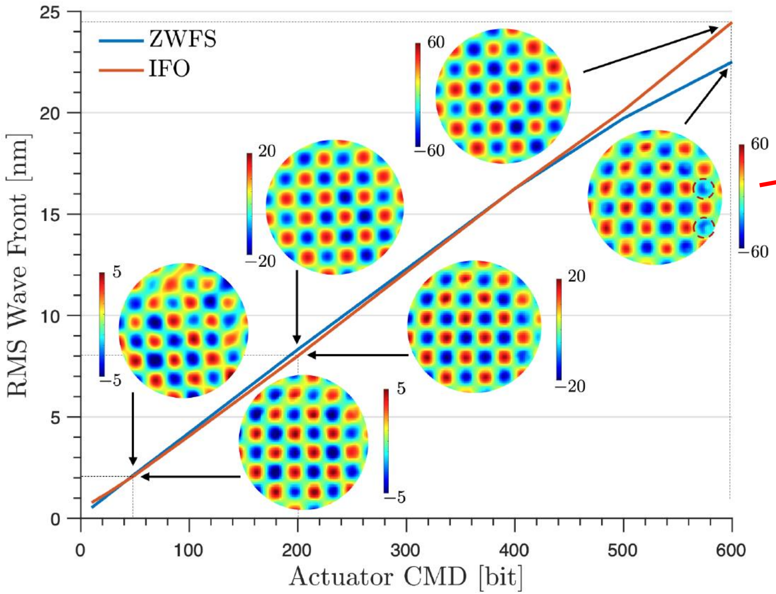
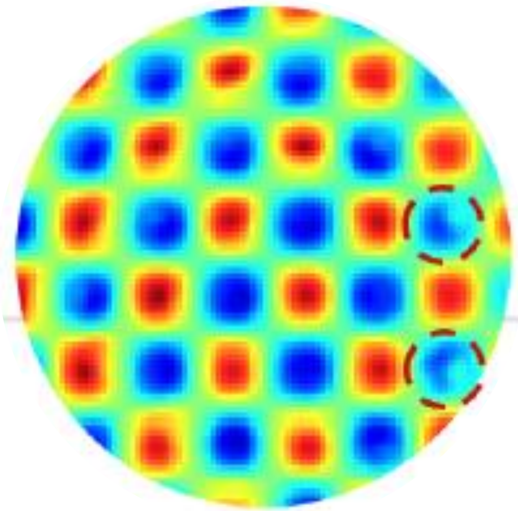
ZWFS Calibration





Multi-valued problem: Intensity inversions occur when local phase exceeds [-π/4, 3π/4] range

- DM commanded to high-spatial frequency "checkerboard" pattern
  - Simultaneous measurement performed with 4D interferometer
- Strong agreement between the two techniques for wavefront changes up to 20 nm RMS
  - ZWFS signal inverts at –π/4 total wavefront error (static + change)
  - Techniques to extend dynamic range have been established (see vZWFS presentations by Doelman and Wallace)

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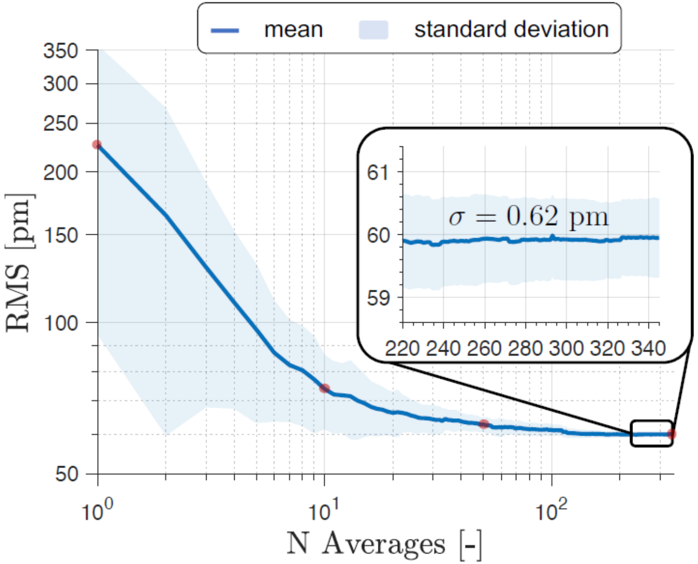
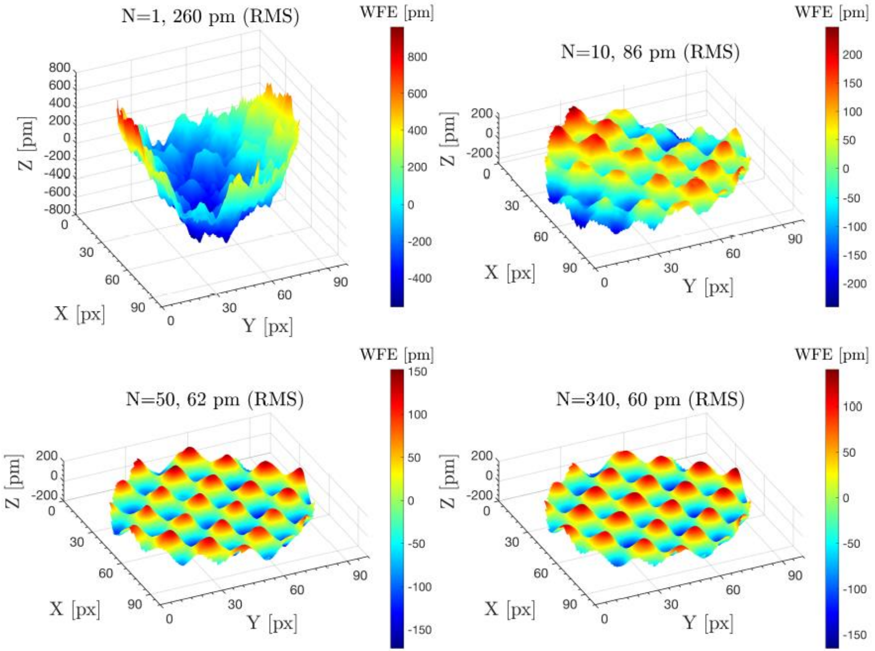
ZWFS Precision





- DM commanded with +/- 1 bit checkerboard pattern
- 60 picometer RMS measurement sensitivity demonstrated with strong SNR
  - Acquisition time <10sec for N =150
  - Measurement limited by testbed (DM LSB), not ZWFS
  - 0.6 picometer repeatability over 70 independent trials

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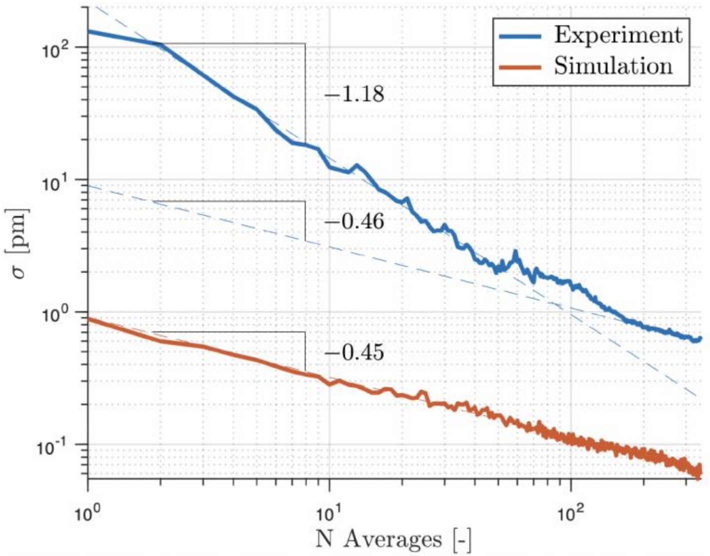
Performance Limitations



Parameter	Value	Unit	Symbol
Photon Flux	42 200 000	photons/sec	Φ
Quantum Efficiency	0.95	e-/photon	QE
Read Noise	247	e-'s/pixel, rms	N_r
Dark Current	1000	e-'s/sec/pixel	I_d
Integration Time	4.2	millisec	τ
Pupil Array Size	97 x 97	pixels	
Wavelength	632.8	nanometers	λ
Zernike Dimple Size	1.0	λ/D	

$$\sigma_{\Delta I(\mathbf{x})} = \sqrt{2(\sigma_s^2 + \sigma_d^2 + \sigma_r^2)} \propto \sqrt{N}$$

$$\sigma_{\overline{\Delta\phi(\mathbf{x})}} = \frac{1}{N} \frac{\sigma_{\Delta I(\mathbf{x})}}{2\sqrt{2I_1(\mathbf{x})I_{1,lf}(\mathbf{x})}} \propto \frac{1}{\sqrt{N}}$$

- Numerical models implemented to predict measurement repeatability
  - Camera noise parameters studied
- Sub-picometer repeatability predicted for single measurement (N=1)
- Experiments demonstrate shot-noise limited performance, $(1/\sqrt{N})$, for N > 150

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Summary

- The Zernike Wavefront Sensor is a simple, robust method to detect spatially-varying wavefront errors
  - Common-mode interferometer
  - Requires only a focal plane mask and pupil-viewing camera
- Strong agreement demonstrated between ZWFS and commercial IFO for nanometer-level wavefront errors
- Picometer-level sensitivity demonstrated via in-air testbed
- Repeatability performance agrees with numerical models

- Applications
  - Out-of-band WFS for future coronagraphy missions (HabEx, LUVOIR)
  - In-band calibration/diagnostic tool
  - Dedicated WFS for DM drift monitoring
  - Laboratory-based interferometer for DM/active optic development

Acknowledgements

Technical Support: David Redding, Frank Loya, Michael Randolph

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