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Method for deriving optical telescope stability specifications for Earth-detecting coronagraphs

Bijan Nemati

Center for Applied Optics – University of Alabama in Huntsville

Also: Philip Stahl (MSFC), Mark Stahl (MSFC), Leah Sheldon (UAH), Garreth Ruane (JPL)

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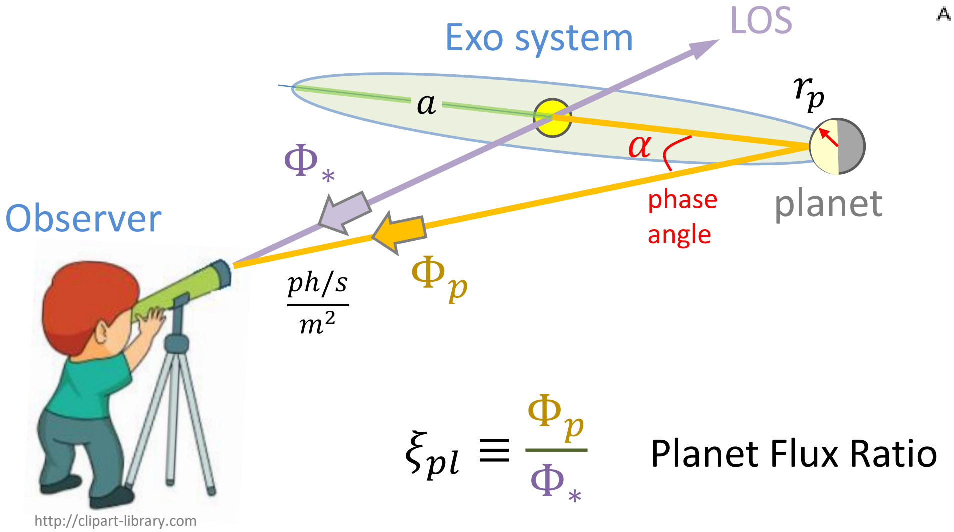
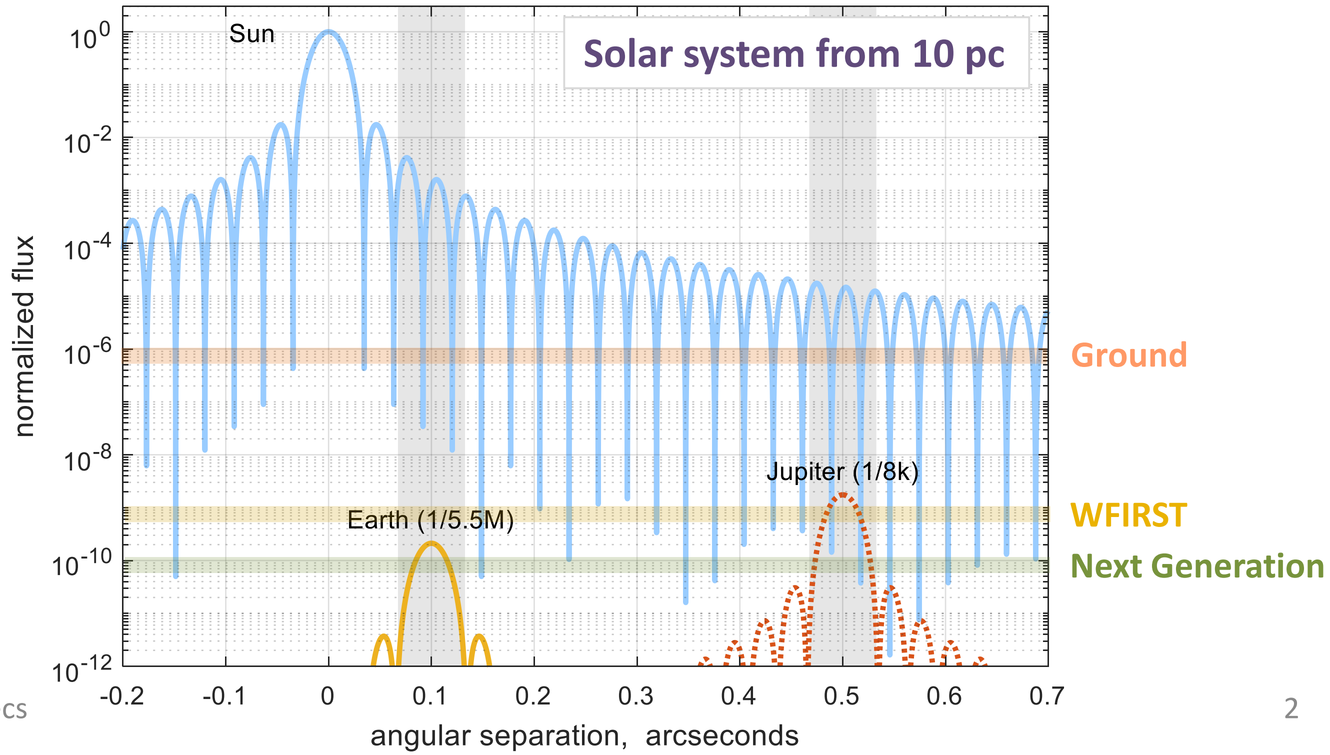
What does it take to see an exo-Earth?

- The flux ratio of the earth, relative to the sun, is $\sim 2.1 \times 10^{-10}$
- If we could look at our solar system
    - in the visible band
    - from 10 pc away
    - using a 4-meter telescope
    - without suppressing starlight,
  - the Earth would be buried under an airy ring of the sun,
    by a factor of >5 million
- Need to divert diffracted starlight
  - We can do this with a coronagraph

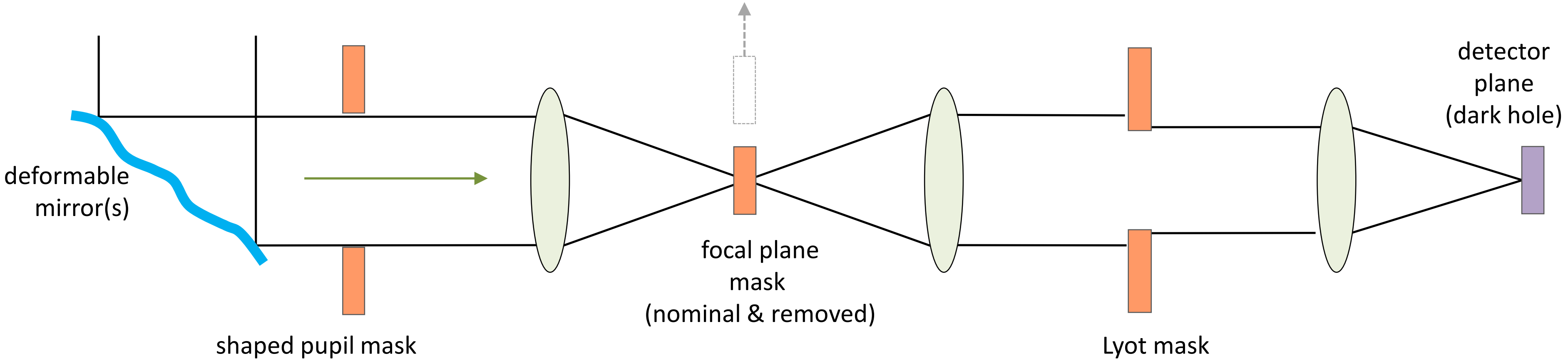


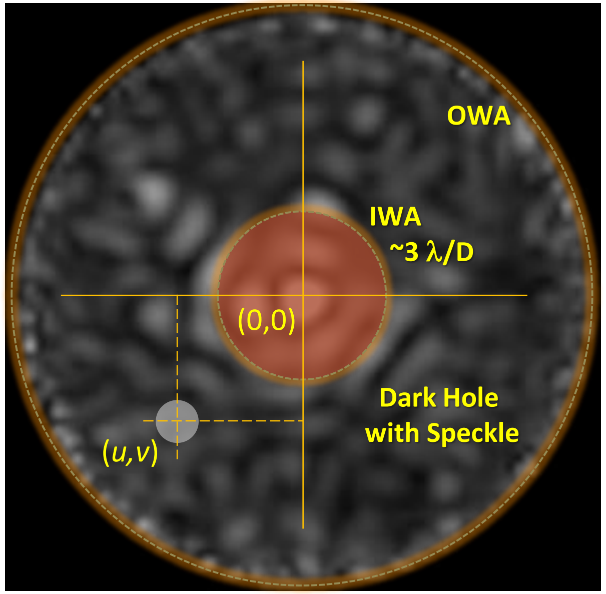
http://clipart-library.com

$$\xi_{pl} \equiv \frac{\Phi_p}{\Phi_*}$$ Planet Flux Ratio



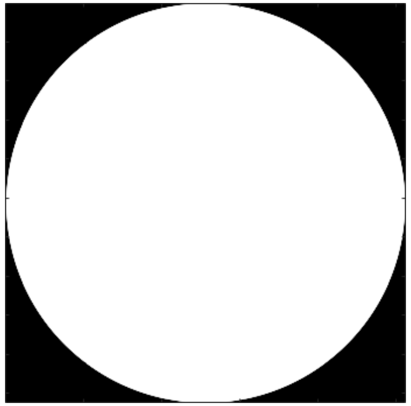
Coronagraphs for Exoplanet Detection



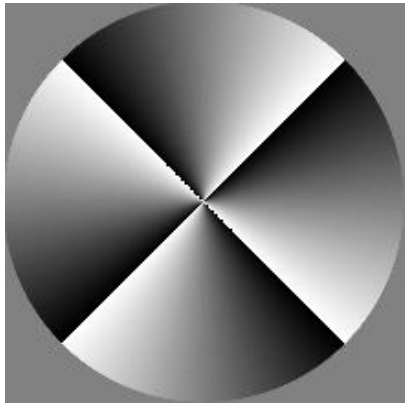


Architectures Studied – HabEx-Relevant

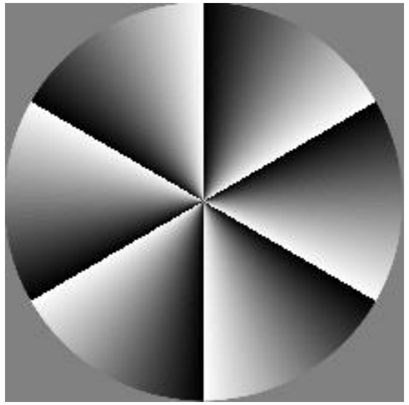
Unobscured Monolithic (4m)



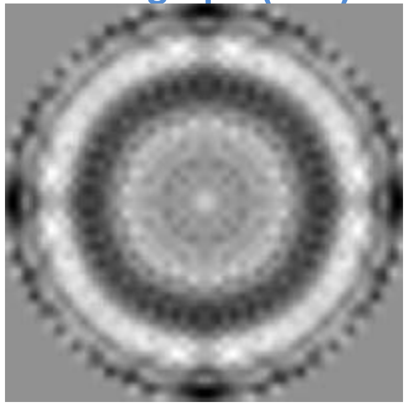
Vector Vortex Coronagraph, Charge 4 (VVC-X4)



Vector Vortex Coronagraph, Charge 6 (VVC-X6)

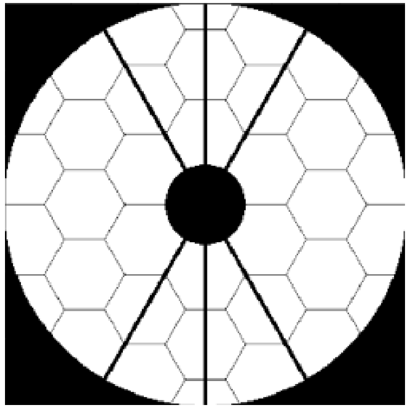


Hybrid Lyot Coronagraph (HLC)

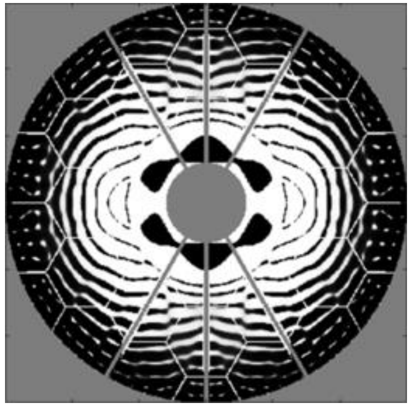


Segmented (6m)

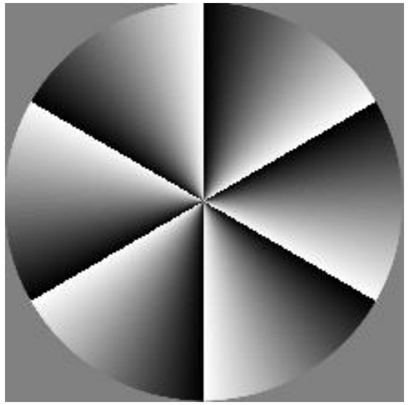
Obscured



Apodized Pupil Lyot Coronagraph (APLC)



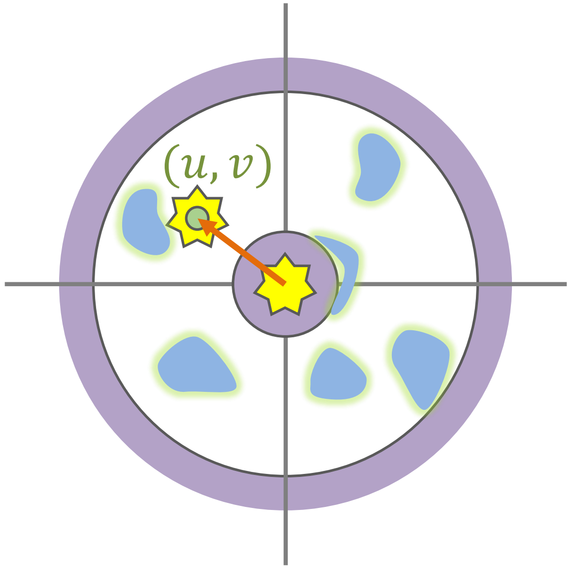
VVC-X6 with off-axis segmented



Key Coronagraph Attributes

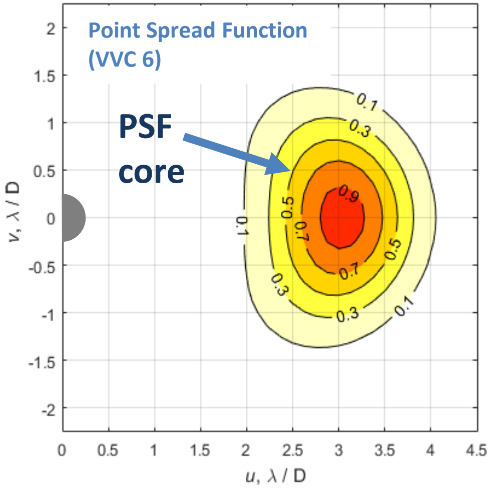
- Starlight Suppression
  - Contrast
    - ratio of throughput from (0,0) to throughput from (u, v)

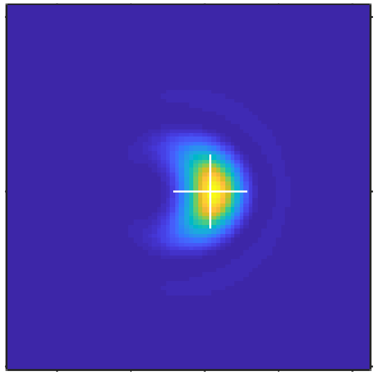
$$C(u,v) \equiv \frac{\tau\,(0{,}0)}{\tau\,(u,v)}$$



- Suppression Stability
  - Residual Speckle
    - measured as the spatial standard deviation of temporal difference of mean speckle

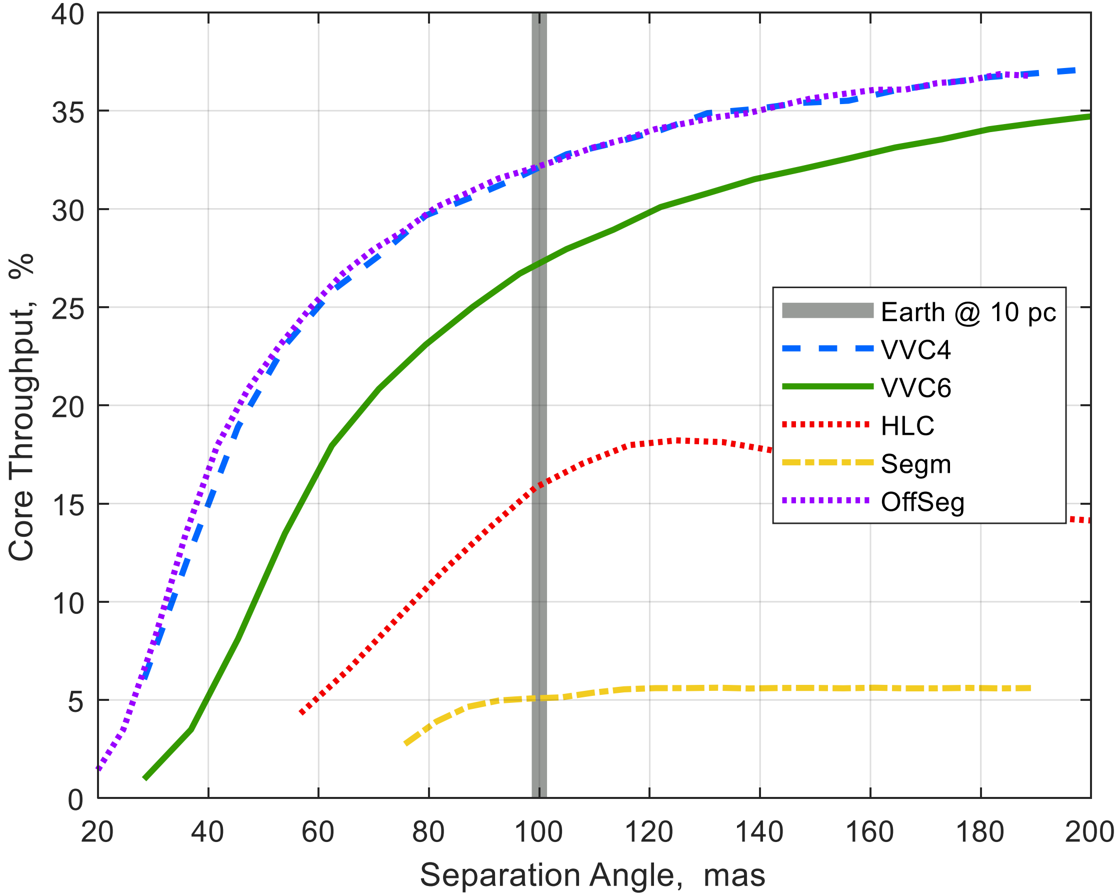
- Planet Light Acceptance
  - Core Throughput
    - fraction of planet light reaching focal plane & in core





Comparison of Core throughput

- The separation for an Exo-Earth at 10 pc (100 mas) is indicated with the vertical line



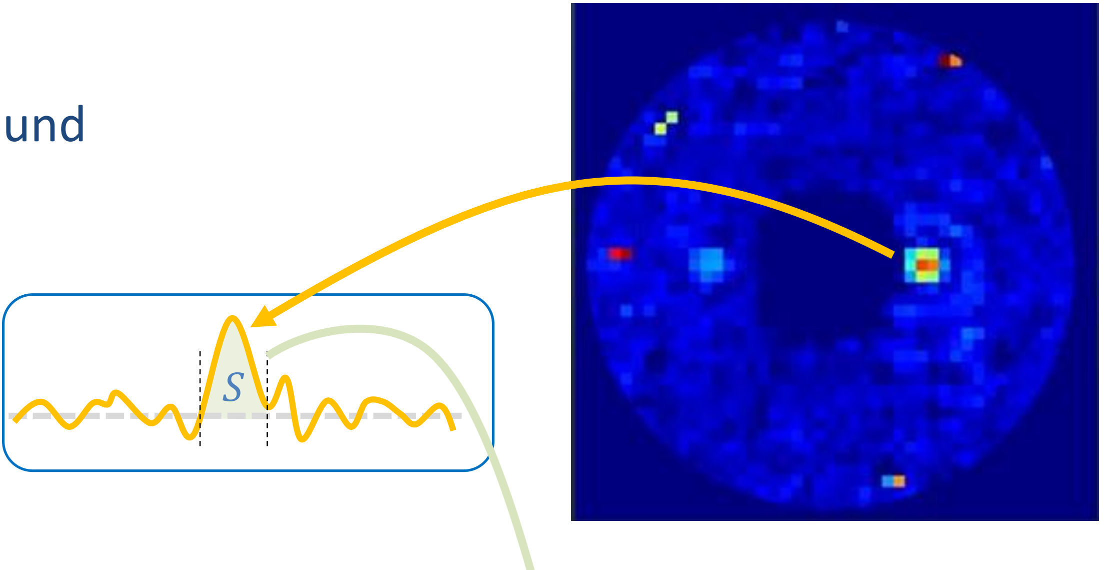
Note: 6m aperture has approximately 2X more collecting area than 4m aperture. Thus 'comparative' throughput @ 100 mas would be ~10%.

Setting Up An Error Budget

… to track Flux Ratio Noise

Flux Ratio Noise

- In direct imaging
  - We are detecting a signal, against some background
  - We set up the observation and integrate



- This integration leads to a signal S
  - The signal is accompanied with noise δS
  - So the measurement yields: $S \pm \delta S$

- The signal is proportional to flux ratio:

counts (e^-) detected attributed to signal →

$$\underset{\text{Signal}}{S} = \underset{\text{flux}}{F_\lambda} \Delta\lambda \, \overset{\text{planet flux ratio}}{\xi_{pl}} \, \underset{\text{area}}{A} \, \underset{\text{throughput}}{\tau_{pl}} \, \overset{\text{QE}}{\eta} \, \overset{\text{time}}{t}$$

- The conversion from signal to flux ratio is:

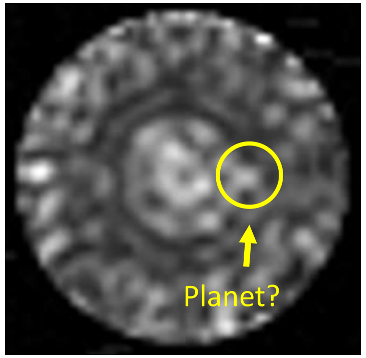
$$\xi_{meas} = \underbrace{\left(\frac{1}{F_\lambda \, \Delta\lambda \, A_{col} \, \tau_{pl} \, \eta \, t}\right)}_{\kappa} \cdot S$$

- Noise that accompanies the signal becomes noise in the flux ratio measurement:

$$\underset{\text{flux ratio noise}}{\delta\xi} = \kappa \cdot \underset{\text{measurement counts, } e^-}{\delta S}$$

Minimizing δS

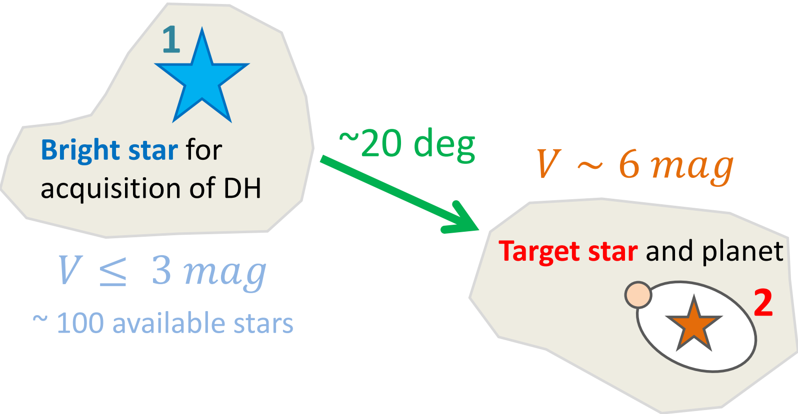
- Generally, there are two types of error:
  - **Random noise** from shot noise and detector noise
    - goes down with integration time
  - **Confusion** due to the speckled nature of the background
    - **may not go down with integration time**



simulated noisy image before post processing

- The speckle background can be suppressed via **differential imaging**:
  - Calibrate/model the speckle and subtract it

- For example, WFIRST incorporates two types:
  - **Reference Differential Imaging (RDI)**
    - Obtain a reference speckle pattern from another star, and subtract
  - **Angular Differential Imaging (ADI)**
    - Roll the telescope about the line of site, and demodulate planet vs. speckle
    - Speckle, to first order, will be stationary on the focal plane



Bright star for acquisition of DH

$V \leq 3\,mag$

~ 100 available stars

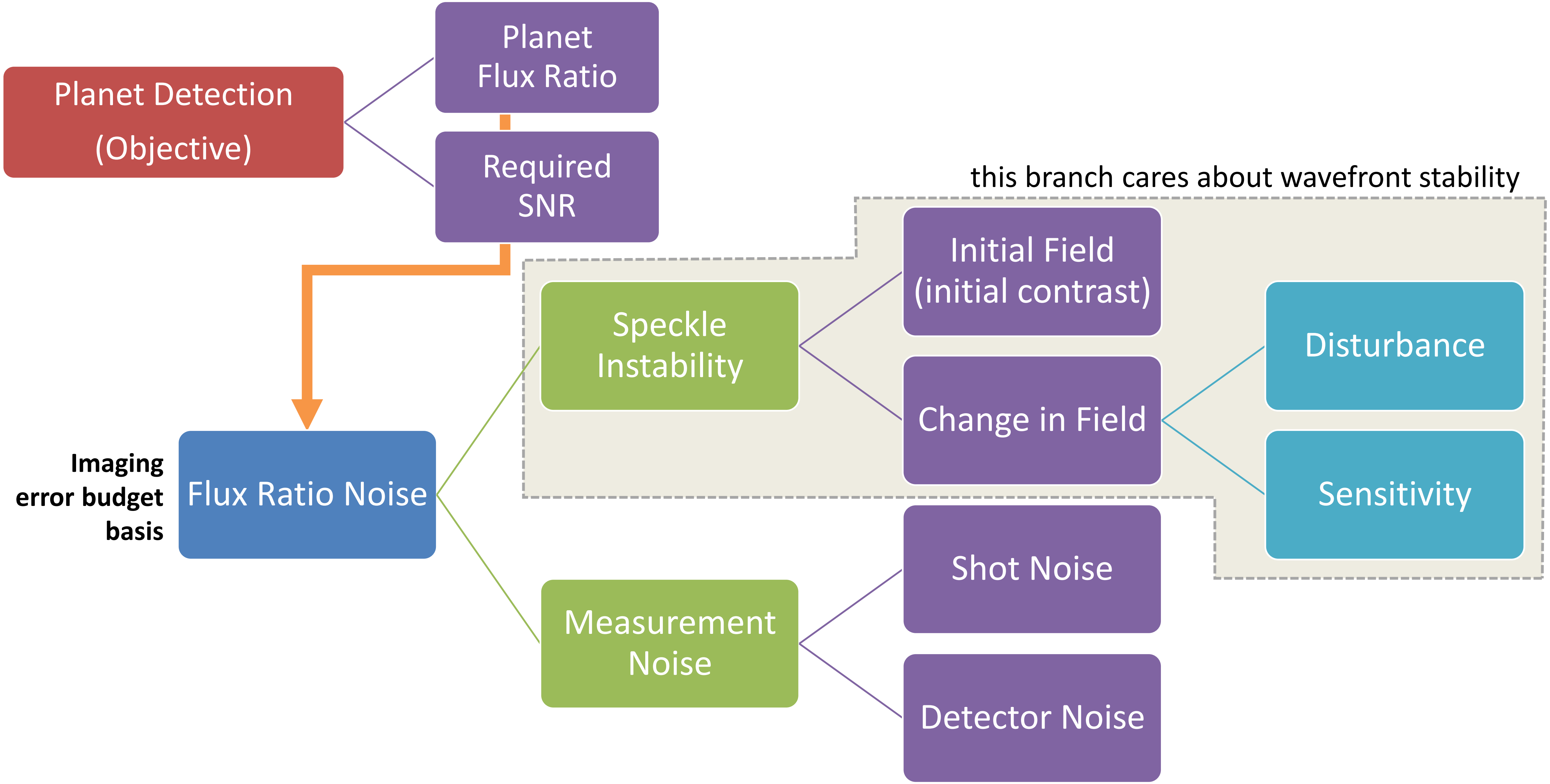
~20 deg

$V \sim 6\,mag$

Target star and planet

- ***These methods require speckle stability***

Error Budget Structure


Planet Detection (Objective)

Planet Flux Ratio

Required SNR

this branch cares about wavefront stability

Imaging error budget basis

Flux Ratio Noise

Speckle Instability

Measurement Noise

Initial Field (initial contrast)

Change in Field

Shot Noise

Detector Noise

Disturbance

Sensitivity

SNR and time-to-SNR

- The signal to noise ratio for detection is given by:
  - where:
    - r_{pl} is the count rate for the planet
    - r_n is the count rate of the noise
    - $r_{\Delta I}$ is the residual speckle rate

$$\mathrm{SNR} = \frac{r_{pl}t}{\sqrt{r_n t + r_{\Delta I}^2 t^2}}$$

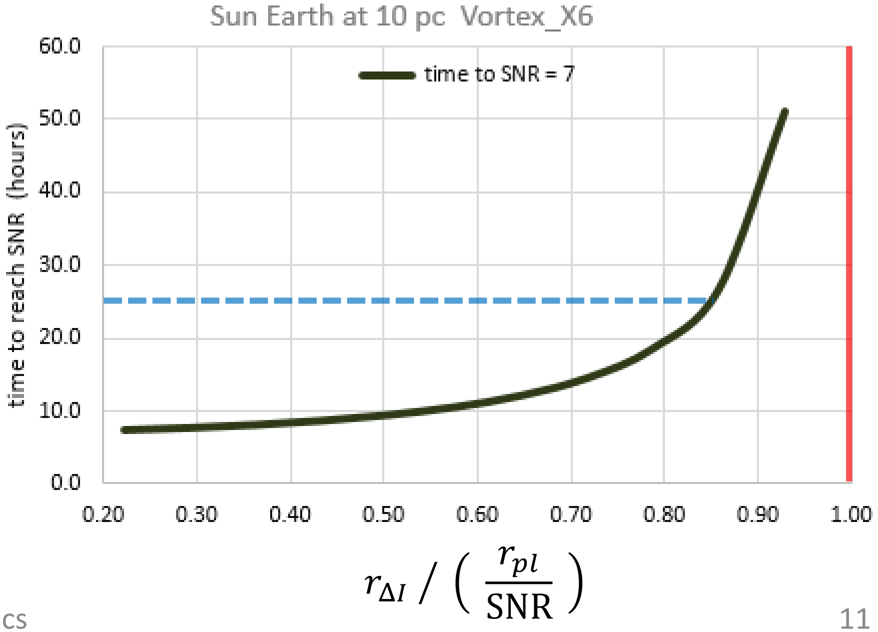
variance from random noise

variance from residual speckle

- Inverting this equation gives the integration time to get to a needed SNR:
  - Note: time goes to ∞ as the residual speckle rate approaches r_{pl}/SNR

$$\boxed{t_{\mathrm{SNR}} = \frac{\mathrm{SNR}^2\, r_n}{r_{pl}^2 - \mathrm{SNR}^2\, r_{\Delta I}^2}}$$

- Time is a scarce resource for a space mission
  - Allocations need to be made for engineering, dark hole, etc.
  - Net result, typically < 3000 hrs per year of actual integration time
- For a given desired SNR, the larger the residual speckle, the longer we need to integrate

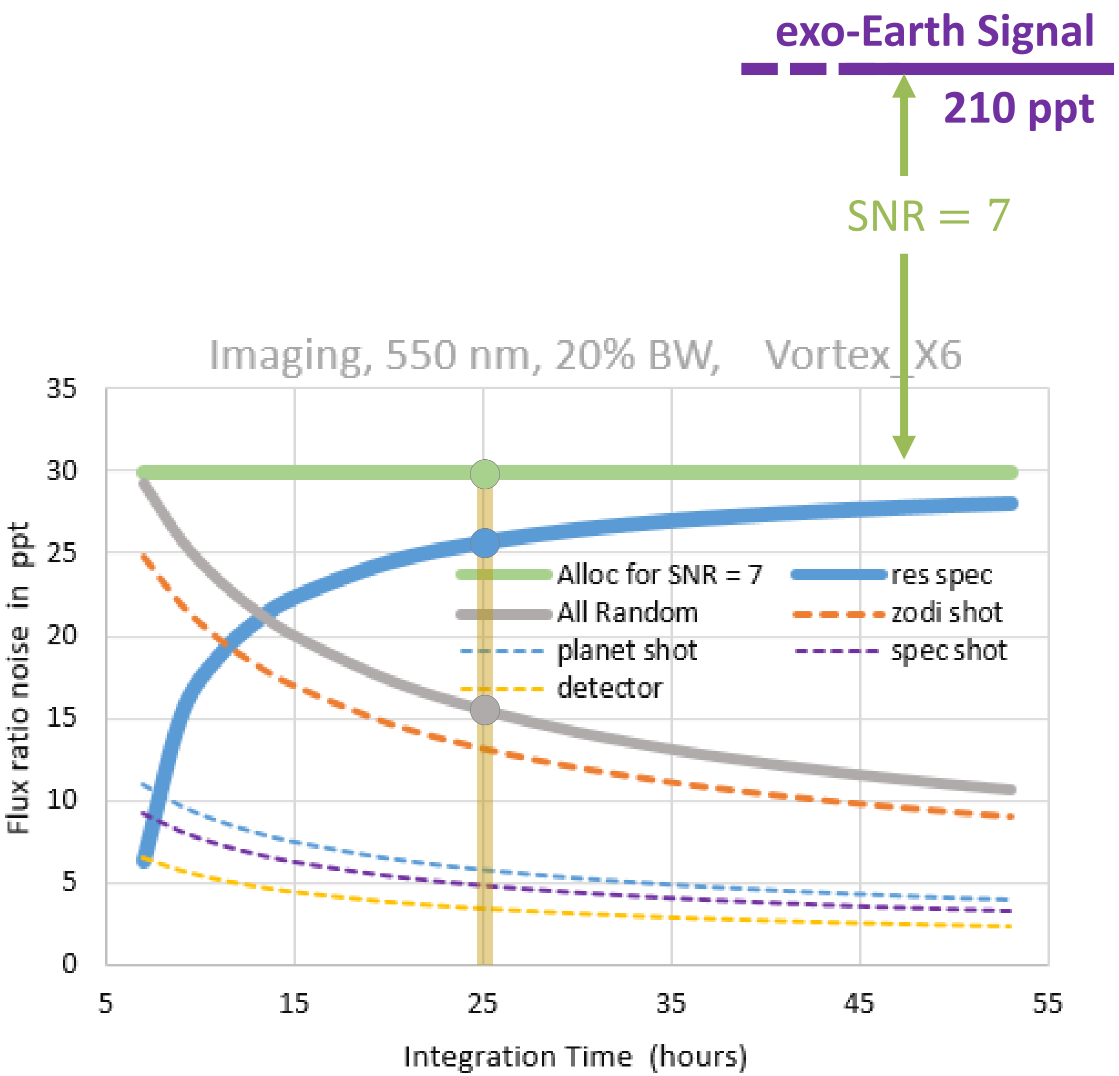


Balancing the Errors – Random vs. Systematic

- Start with the exo-Earth flux ratio (210 ppt)
- Settle on desired SNR (e.g. 7)
- The allowable maximum flux ratio noise is 210 ppt / 7 = 30 ppt

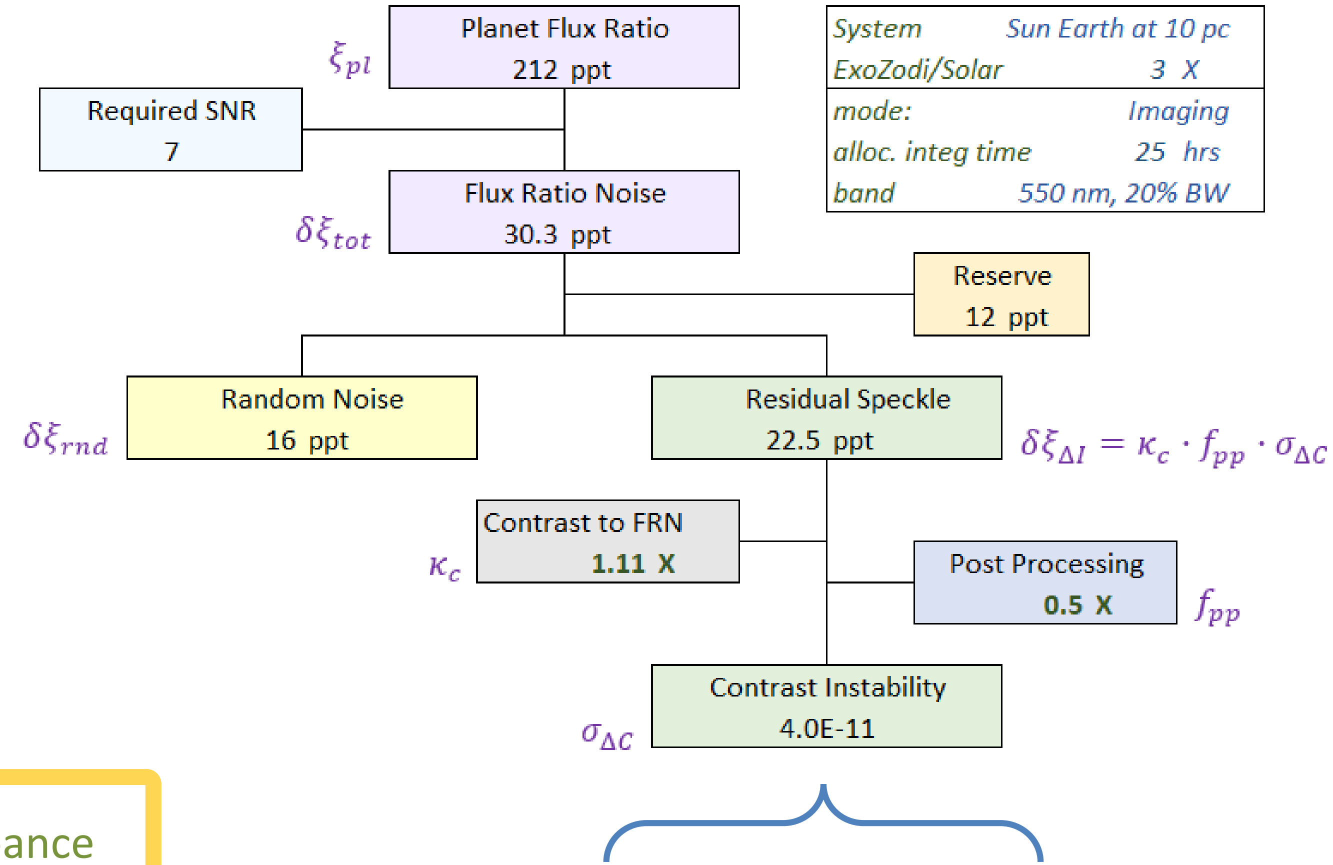
- Choose an integration time that makes the random errors small fraction of 30 ppt
  - Small in rss sense is good enough

- Allocate the remainder to:
  - residual speckle
  - reserve



Exo-Earth Direct Imaging – Basic Error Budget

- Post processing using ancillary data can be used to predict the changes in speckle within differential imaging
- This information can be used to further reduce the residual speckle, alleviating the requirements on speckle stability
- A factor of 2X (additional) is reasonable to assume
- This process leads to a top level allocation of 40 ppt (4×10^{-11}) total to **speckle instability**
  - Disturbances and sensitivities must add up to less than this quantity



System	*Sun Earth at 10 pc*
ExoZodi/Solar	*3 X*
mode:	*Imaging*
alloc. integ time	*25 hrs*
band	*550 nm, 20% BW*

ξ_{pl} Planet Flux Ratio 212 ppt

Required SNR 7

$\delta\xi_{tot}$ Flux Ratio Noise 30.3 ppt

Reserve 12 ppt

$\delta\xi_{rnd}$ Random Noise 16 ppt

Residual Speckle 22.5 ppt $\delta\xi_{\Delta I} = \kappa_c \cdot f_{pp} \cdot \sigma_{\Delta C}$

κ_c Contrast to FRN **1.11 X**

Post Processing **0.5 X** f_{pp}

$\sigma_{\Delta C}$ Contrast Instability 4.0E-11

sub-allocate this error

allocation

disturbance

$$\epsilon_i = \left(\frac{\partial \epsilon}{\partial x_i}\right) \cdot \delta x_i$$

sensitivity

Sensitivities to Contrast Instability

Wavefront Instability and Raw Contrast

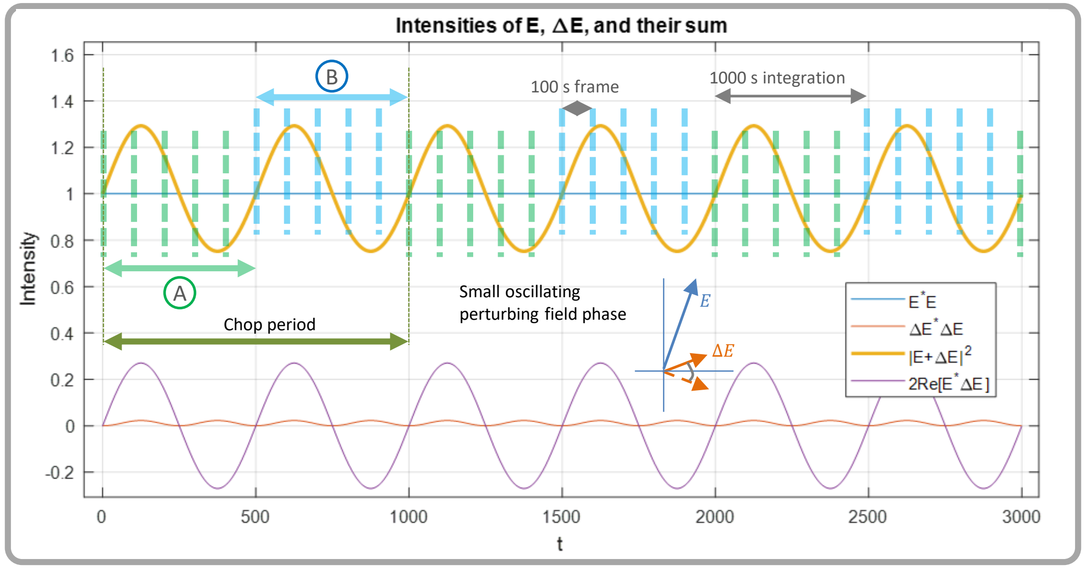
- How does the error affect the speckle pattern?

$$C \propto |E + \Delta E|^2 = |E|^2 + |\Delta E|^2 + 2\,\mathcal{R}\{E^*\Delta E\}$$

existing field (static field)

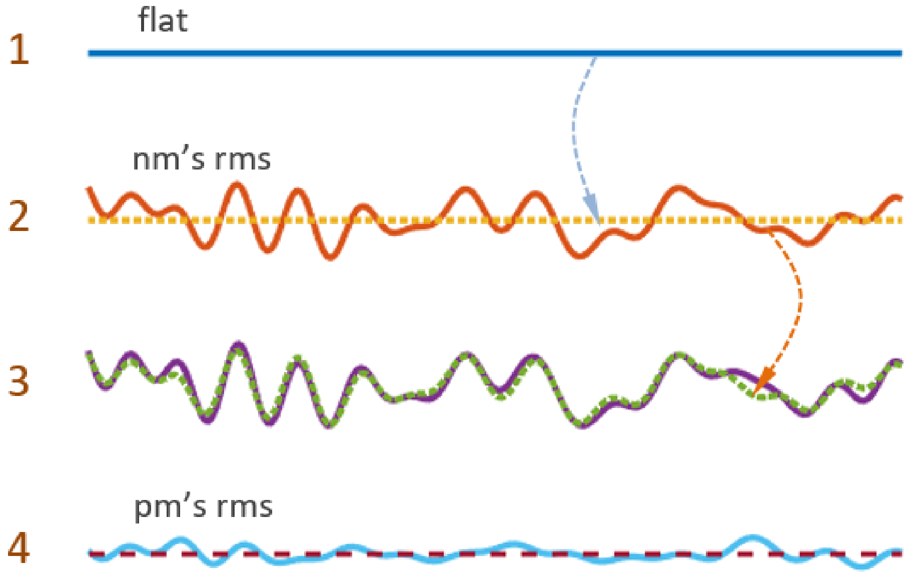
perturbation (instability)

Cross term



→ The sensitivity to wavefront change depends on the initial raw contrast

Contrast vis Effective Surface Error



the WFE or DM surface shape assumed by the design

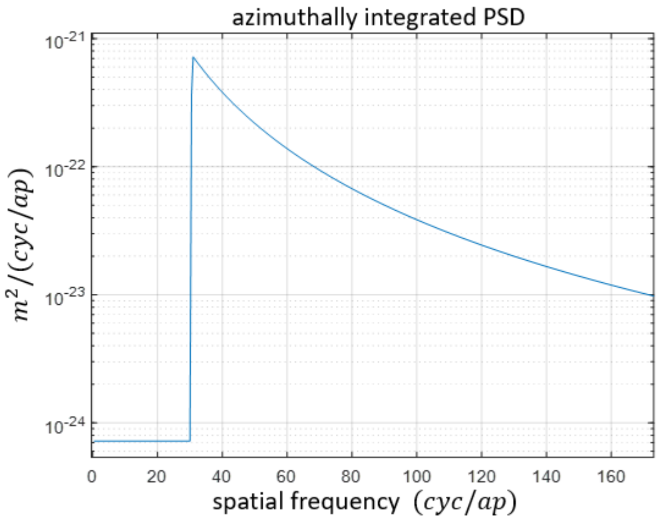
the practical limit of DM surface fidelity to ideal

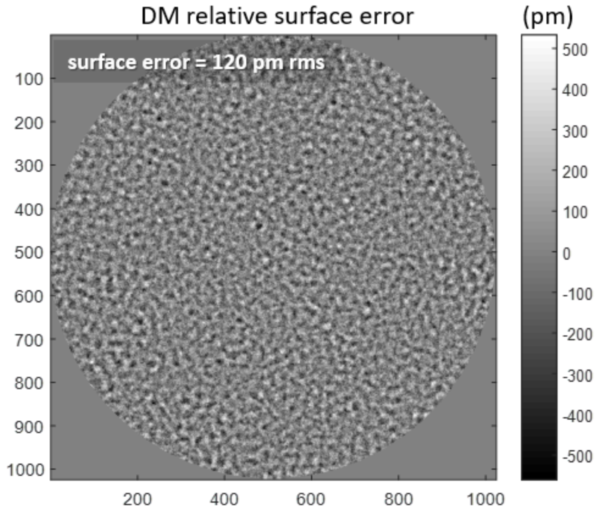
the high-contrast solution near this part of the DM phase space

equivalent relative error with same dark hole E field

In this example, a 64 x 64 DM is assumed to control the wavefront out to ~ 30 cyc / aper

Correspondingly, the PSD of the effective post-DM error has very small error below this spatial frequency

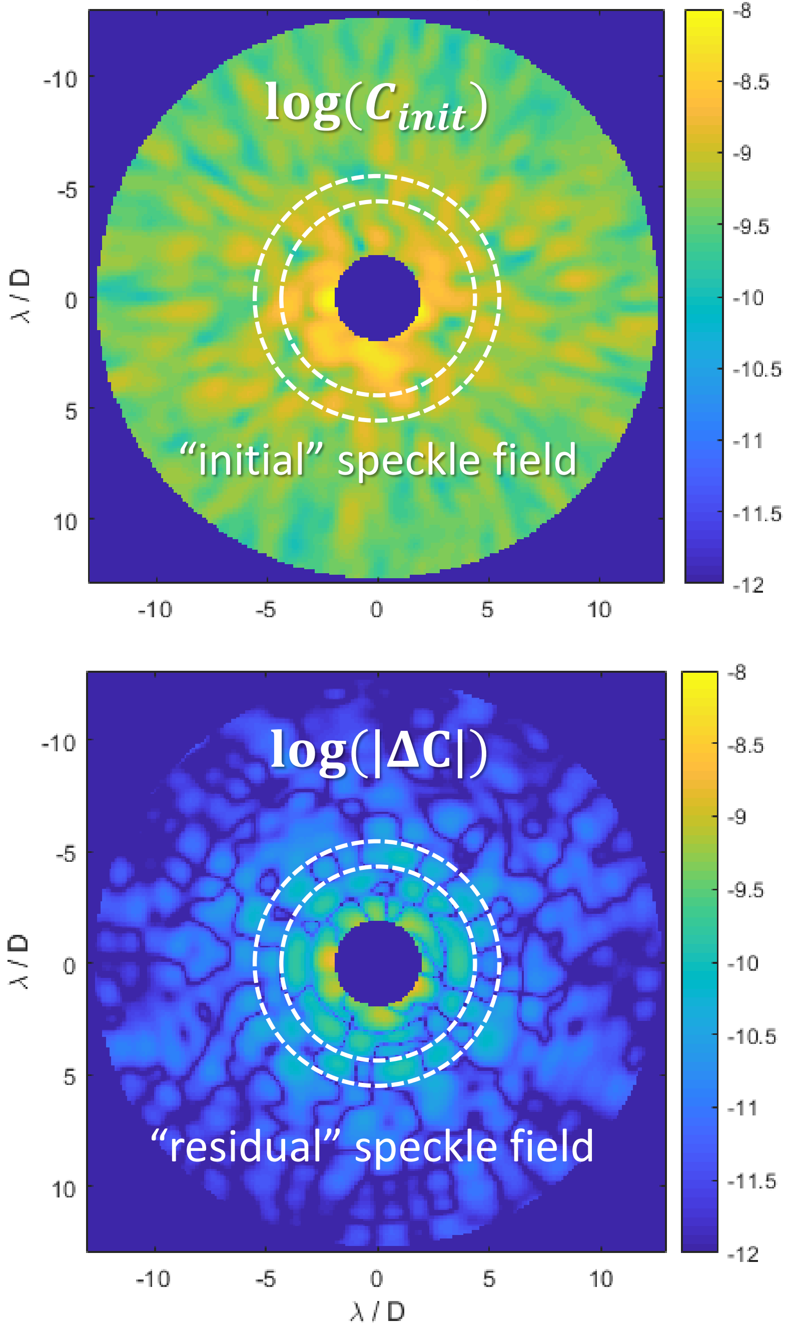




Modeling Contrast Sensitivity

- Start with some initial field and add the perturbing field
  - The initial field sets the initial speckle in the dark hole
  - The perturbed field causes the changed contrast.
    - Subtracting the perturbation-added speckle map from the original speckle map gives **the residual speckle map**
- We are interested in the structure of the residual contrast map as quantified by its spatial standard deviation within the dark hole
  - We are specifically interested in
    - the spatial standard deviation of
      - the difference of
        - temporally averaged contrast maps
  - We do this for each annular slice
- The sensitivity is the ratio of this quantity over strength of the perturbation.

VVC6 Secondary Astigmatism

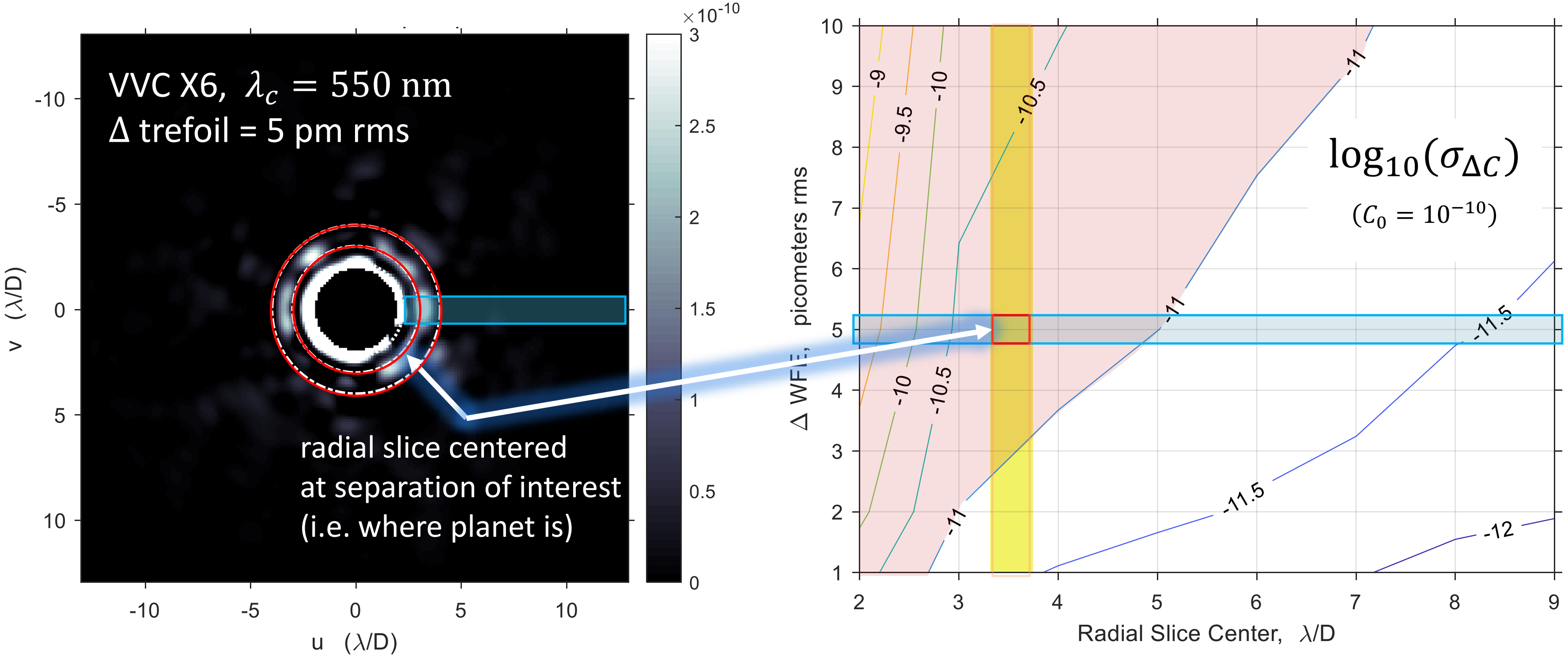


How residual speckle depends on radial slice

- The PSF, and the residual speckle, fall off as we get farther from the star.
- On Left we see the residual speckle in the case of 5 pm (PV) of trefoil added to the wavefront error between the reference and target star observations (VVC-6 case).
- In this example, the pink-shaded region exceeds an allocation of 1e-11 to the residual contrast standard deviation over the radial slice.

trefoil mode error introduced between reference and target star observations

CG = VVC6



An algorithm for distributing errors

1. Start with
   1. an assumed **initial contrast** (requirement) $C_0 = 1 \times 10^{-10}$
   2. the total allowed **FRN for contrast instability** $\epsilon_{tot} = 40\text{ ppt}$
2. Compute **sensitivities** from annular standard deviations: $S_i = \frac{\sigma_{\Delta C}}{\Delta w_i} \equiv \frac{\partial \epsilon}{\partial x_i}$
3. Create **weights** for allocations such that
   1. are proportional to the sensitivities
   2. have a limited dynamic range

$$w_i = \max(S_i, 1\% \cdot \max(S_i)\ )$$

4. Use the total allocation and the weights to determine **individual allocations**
   - Normalize the weights to RSS to 1.0

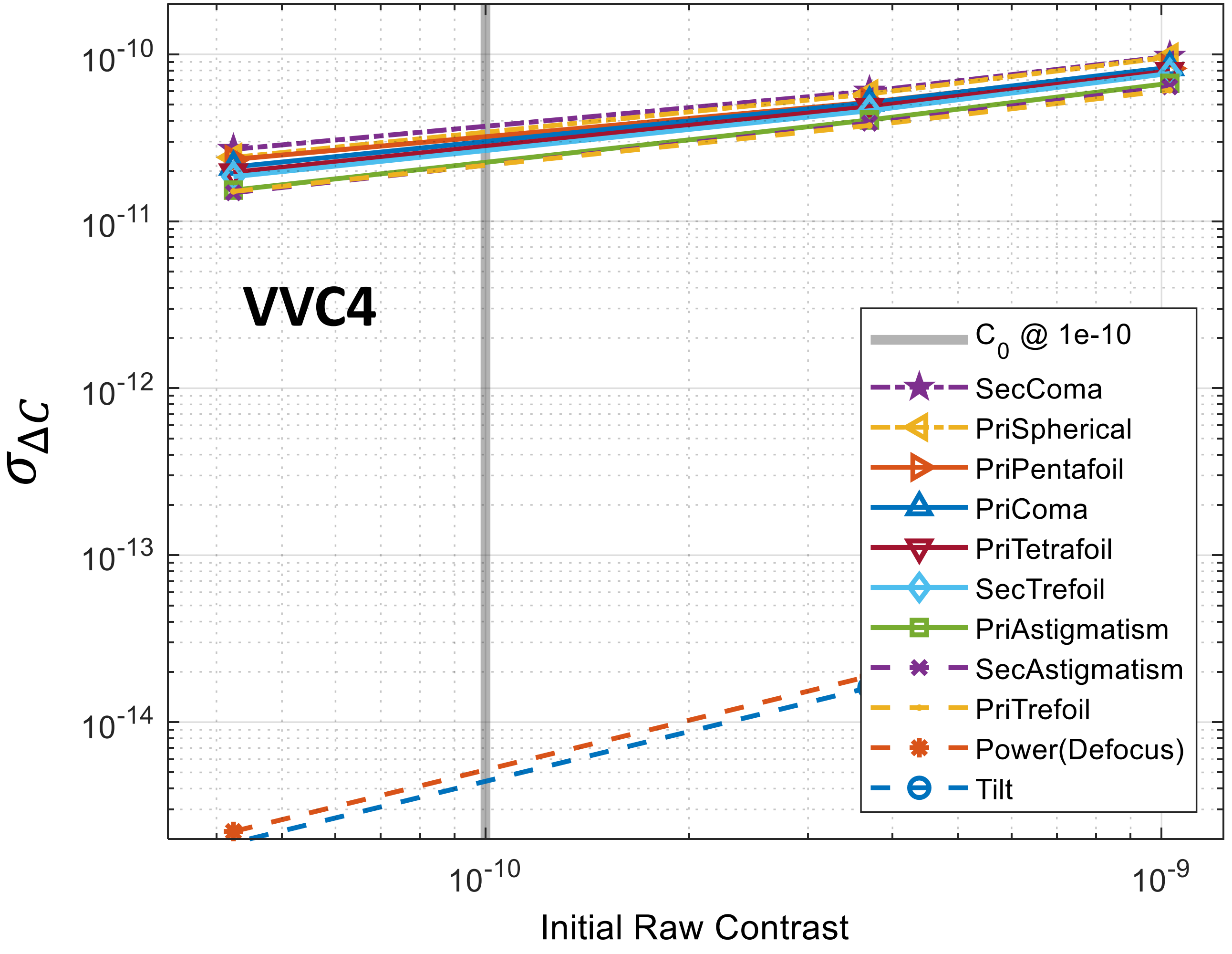
$$\epsilon_i = w_i \cdot \left(\frac{\epsilon_{tot}}{\sqrt{\sum_i w_i^2}} \right)$$

5. From the sensitivities and allocations, get the **tolerances**:

$$\underset{\text{tolerances}}{\delta x_i} = \frac{\overset{\text{allocations}}{\epsilon_i}}{\underset{\text{sensitivities}}{S_i}}$$

sensitivities are usually the challenging step!

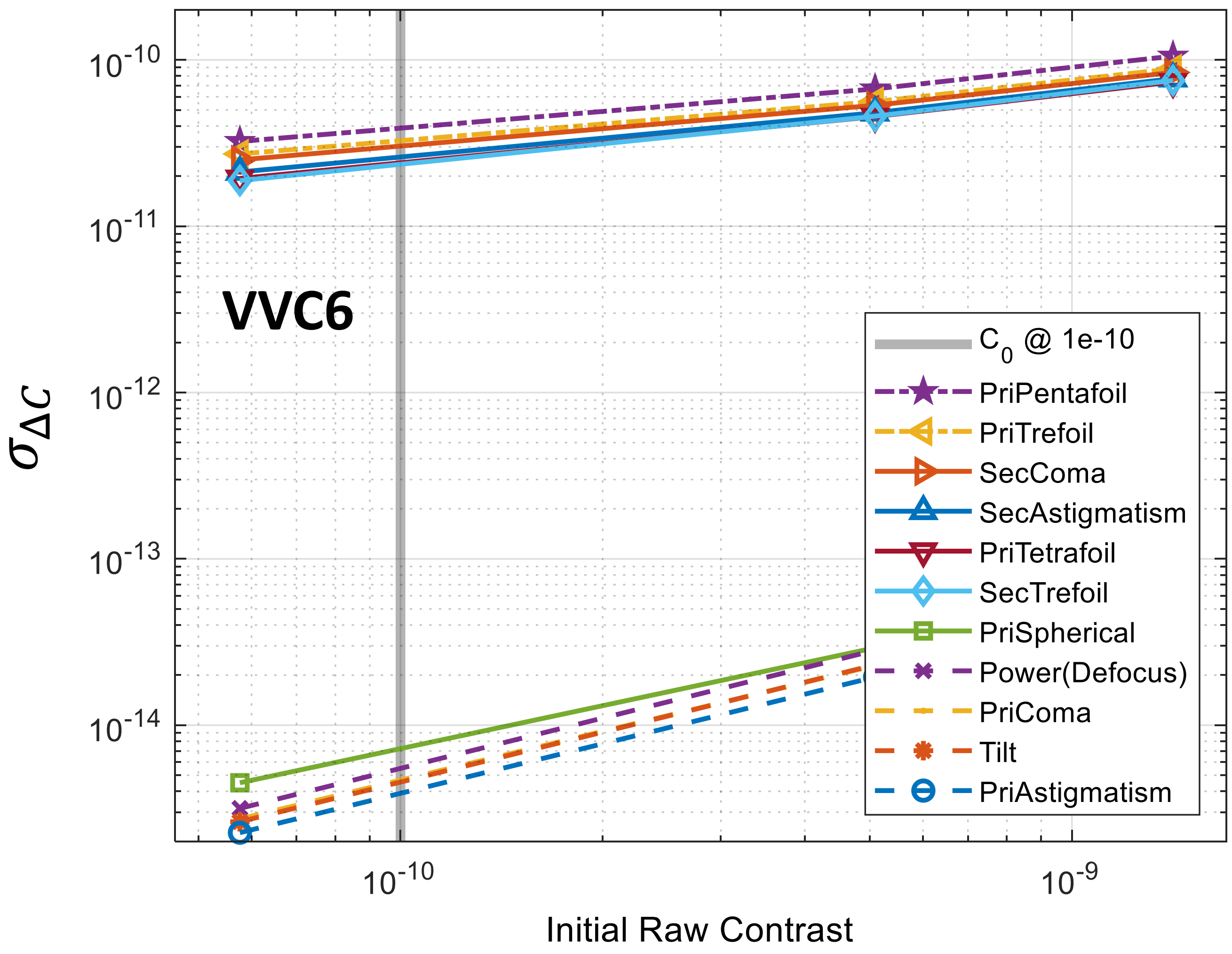
Sensitivities to 5 pm rms ΔWFE – for VVC4



4-m Off-Axis Monolithic with VVC-4

Perturbation Mode	Sensitivity	Allocations (auto)	Tolerance
	ppt/pm	**40.00**	**pm**
Tip/Tilt	0.00088	0.17	195.7
Defocus	0.0010	0.17	167.4
Astigmatism	4.08	10.5	2.6
Coma	5.44	14.0	2.6
Trefoil	3.92	10.1	2.6
Spherical	6.17	15.8	2.6
Secondary Atigmatism	3.93	10.1	2.6
Tetrafoil	5.09	13.1	2.6
Secondary Coma	6.72	17.2	2.6
Secondary Trefoil	4.80	12.3	2.6
Pentafoil	5.83	14.9	2.6

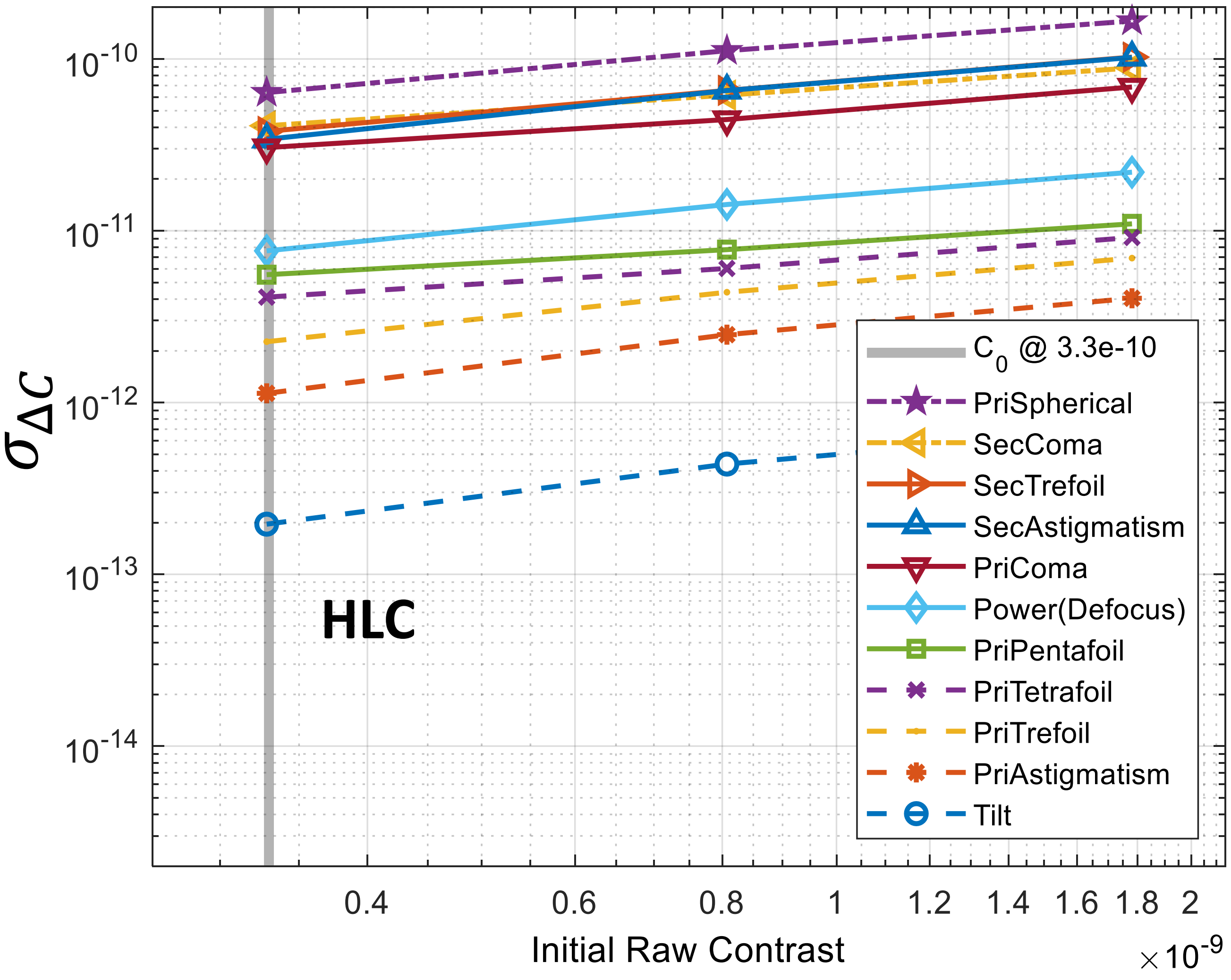
Sensitivities to 5 pm rms ΔWFE – for VVC6



4-m Off-Axis Monolithic with VVC-6

Perturbation Mode	Sensitivity	Allocations (auto)	Tolerance
	ppt/pm	40.00	pm
Tip/Tilt	0.00091	0.21	235
Defocus	0.0011	0.21	194
Astigmatism	0.00	0.2	276
Coma	0.00	0.2	231
Trefoil	6.06	18.0	3.0
Spherical	0.00	0.2	156
Secondary Atigmatism	4.82	14.3	3.0
Tetrafoil	4.44	13.2	3.0
Secondary Coma	5.62	16.7	3.0
Secondary Trefoil	4.34	12.9	3.0
Pentafoil	7.21	21.4	3.0

Sensitivities to 5 pm rms ΔWFE – for HLC



4-m Off-Axis Monolithic with Hybrid Lyot

Perturbation Mode	Sensitivity	Allocations (auto)	Tolerance
	ppt/pm	**40.00**	**pm**
Tip/Tilt	0.03935	0.26	6.69
Defocus	1.5326	3.16	2.06
Astigmatism	0.23	0.5	2.06
Coma	6.11	12.6	2.06
Trefoil	0.45	0.9	2.06
Spherical	12.78	26.3	2.06
Secondary Atigmatism	6.83	14.1	2.06
Tetrafoil	0.82	1.7	2.06
Secondary Coma	8.18	16.9	2.06
Secondary Trefoil	7.60	15.7	2.06
Pentafoil	1.11	2.3	2.06

Sensitivities to 5 pm rms ΔWFE – For Off-Axis Segm.

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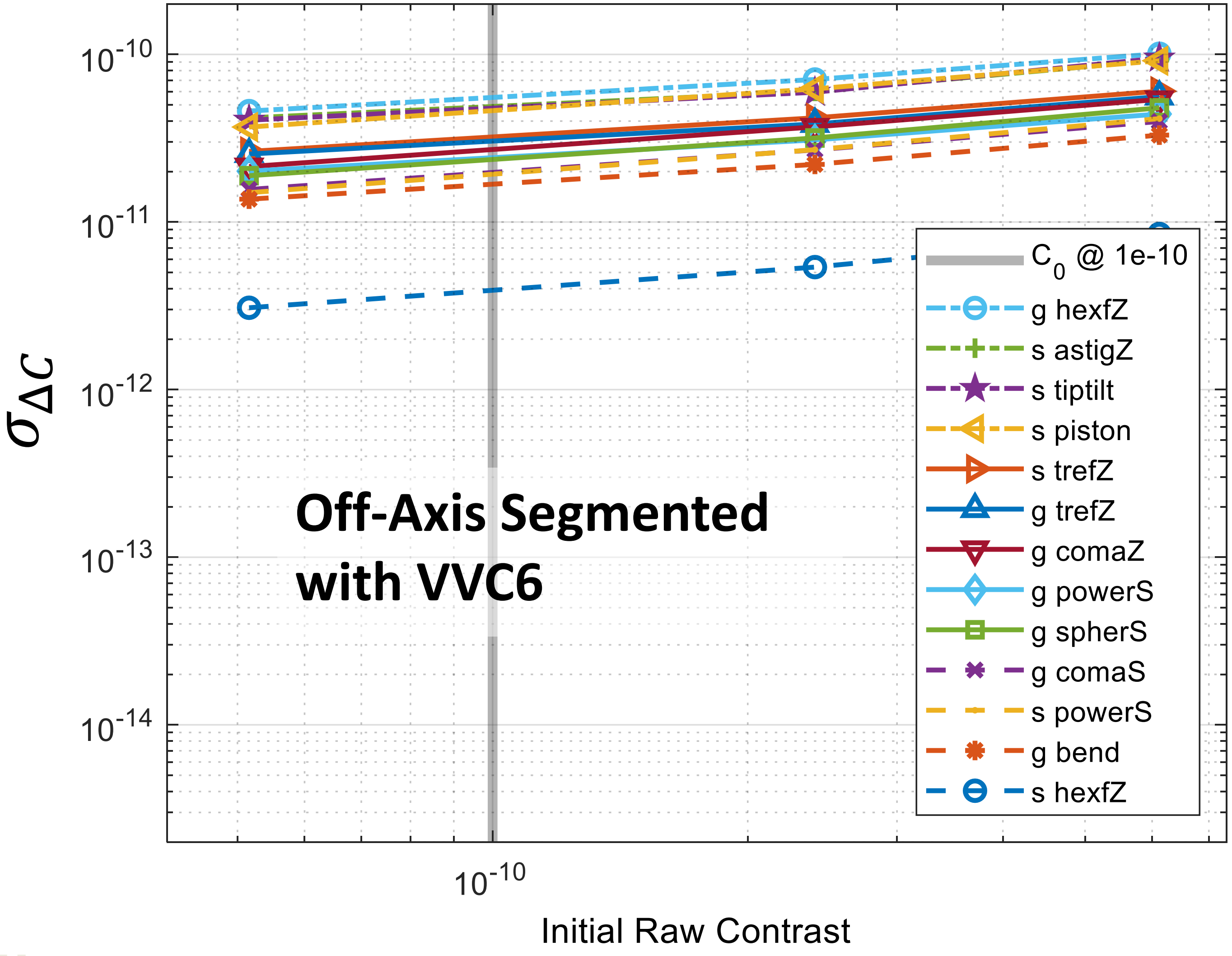

6-m Off-axis Segmented with VVC6

Perturbation Mode	Sensitivity	Allocations (auto)	Tolerance
Global	**ppt/pm**	**40.00**	**pm**
Bend	3.21	5.6	1.75
Defocus	4.64	8.1	1.75
Spherical	4.51	7.9	1.75
Coma	5.19	9.1	1.75
Trefoil	5.82	10.2	1.75
Hexafoil	10.61	18.5	1.75
Segmented			
Piston	8.84	15.4	1.64
Tip/Tilt	9.09	15.9	1.64
Power	3.68	6.4	1.64
Astigmatism	9.33	16.3	1.64
Trefoil	6.16	10.8	1.64
Hexafoil	0.74	1.3	1.64

Sensitivities to 5 pm rms ΔWFE – for On-Axis Segmented

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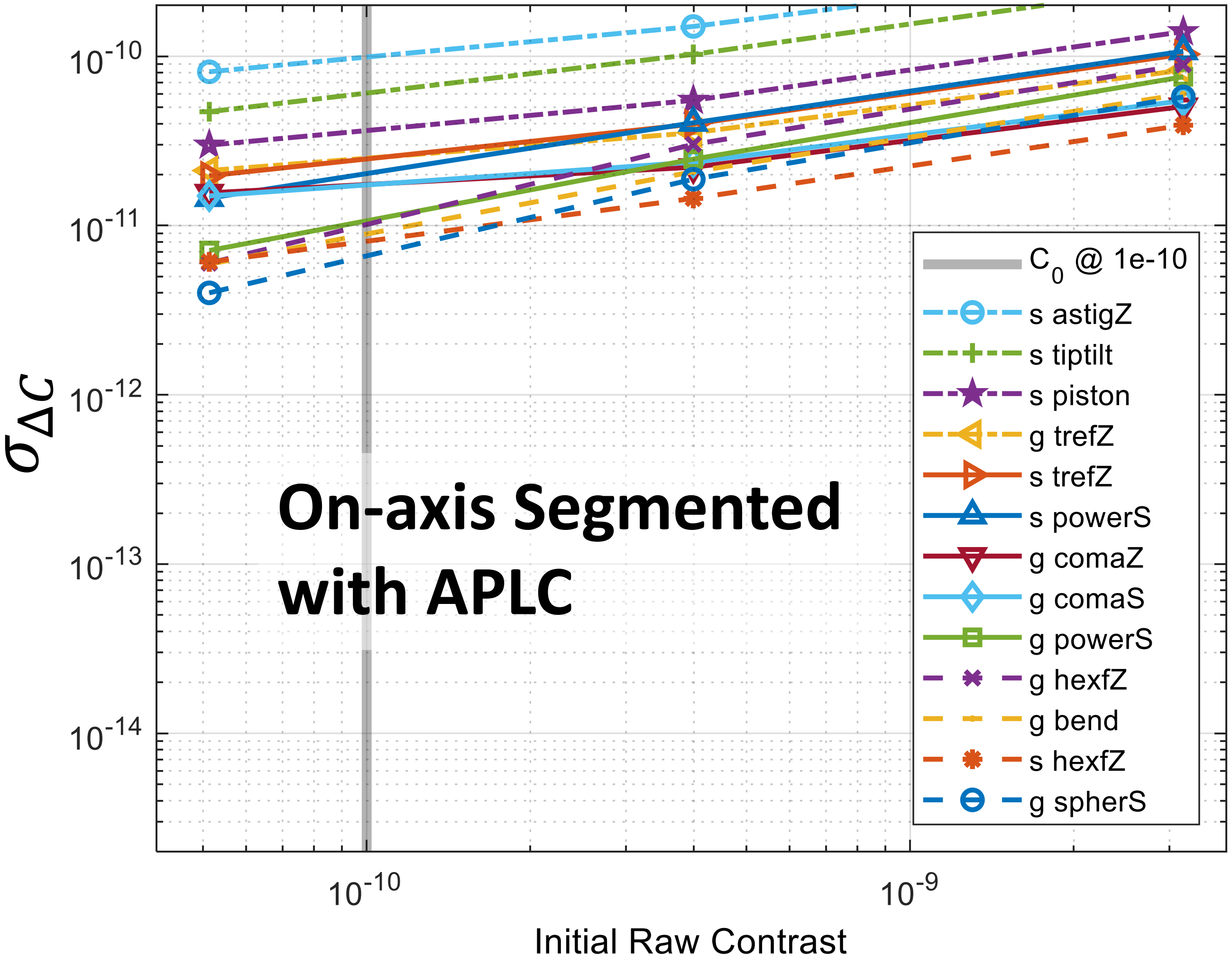


6-m On-axis Segmented with APLC			
Perturbation Mode	Sensitivity	Allocations (auto)	Tolerance
Global	**ppt/pm**	**40.00**	**pm**
Bend	1.65	2.7	1.65
Defocus	1.96	3.2	1.65
Spherical	1.26	2.1	1.65
Coma	3.31	5.5	1.65
Trefoil	4.66	7.7	1.65
Hexafoil	1.95	3.2	1.65
Segmented			
Piston	6.74	11.1	1.64
Tip/Tilt	11.12	18.4	1.64
Power	3.71	6.1	1.64
Astigmatism	18.33	30.3	1.64
Trefoil	4.55	7.5	1.64
Hexafoil	1.48	2.4	1.64

Tolerances for Error Modes Studied

Mono – VVC 4

4-m Off-Axis Monolithic with VVC-4

	tiptilt	defocus	astigmatism	coma	trefoil	spherical	secAstig	tetrafoil	secComa	secTrefoil	pentafoil
Sensitivities (ppt/pm)	0.00088	0.0010	4.08	5.44	3.92	6.17	3.93	5.09	6.72	4.80	5.83
Allocations (ppt)	0.17	0.17	10.5	14.0	10.1	15.8	10.1	13.1	17.2	12.3	14.9
Tolerances (pm)	195.7	167.4	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6

Mono – VVC 6

4-m Off-Axis Monolithic with VVC-6

	tiptilt	defocus	astigmatism	coma	trefoil	spherical	secAstig	tetrafoil	secComa	secTrefoil	pentafoil
Sensitivities (ppt/pm)	0.000909	0.001102	0.000775	0.000924	6.06	0.00137	4.82	4.44	5.62	4.34	7.21
Allocations (ppt)	0.214	0.214	0.214	0.214	18.0	0.214	14.3	13.2	16.7	12.9	21.4
Tolerances (pm)	235	194	276	231	2.96	156	2.96	2.96	2.96	2.96	2.96

Mono – HLC

4-m Off-Axis Monolithic with Hybrid Lyot

	tiptilt	defocus	astigmatism	coma	trefoil	spherical	secAstig	tetrafoil	secComa	secTrefoil	pentafoil
Sensitivities (ppt/pm)	0.0393	1.53	0.23	6.11	0.45	12.78	6.83	0.82	8.18	7.60	1.11
Allocations (ppt)	0.26	3.16	0.47	12.6	0.9	26.3	14.1	1.7	16.9	15.7	2.3
Tolerances (pm)	6.69	2.06	2.06	2.06	2.06	2.06	2.06	2.06	2.06	2.06	2.06

Segm – APLC

6-m On-axis Segmented with APLC

	g_bend	g_powerS	g_sphericS	g_comaZ	g_trefZ	g_hexfZ	s_piston	s_tiptilt	s_powerS	s_astigZ	s_trefZ	s_hexfZ
Sensitivities (ppt/pm)	1.65	1.96	1.26	3.31	4.66	1.95	6.74	11.12	3.71	18.33	4.55	1.48
Allocations (ppt)	2.72	3.25	2.08	5.47	7.70	3.23	11.1	18.4	6.13	30.3	7.52	2.44
Tolerances (pm)	1.65	1.65	1.65	1.65	1.65	1.65	1.64	1.64	1.64	1.64	1.64	1.64

Segm – VVC 6

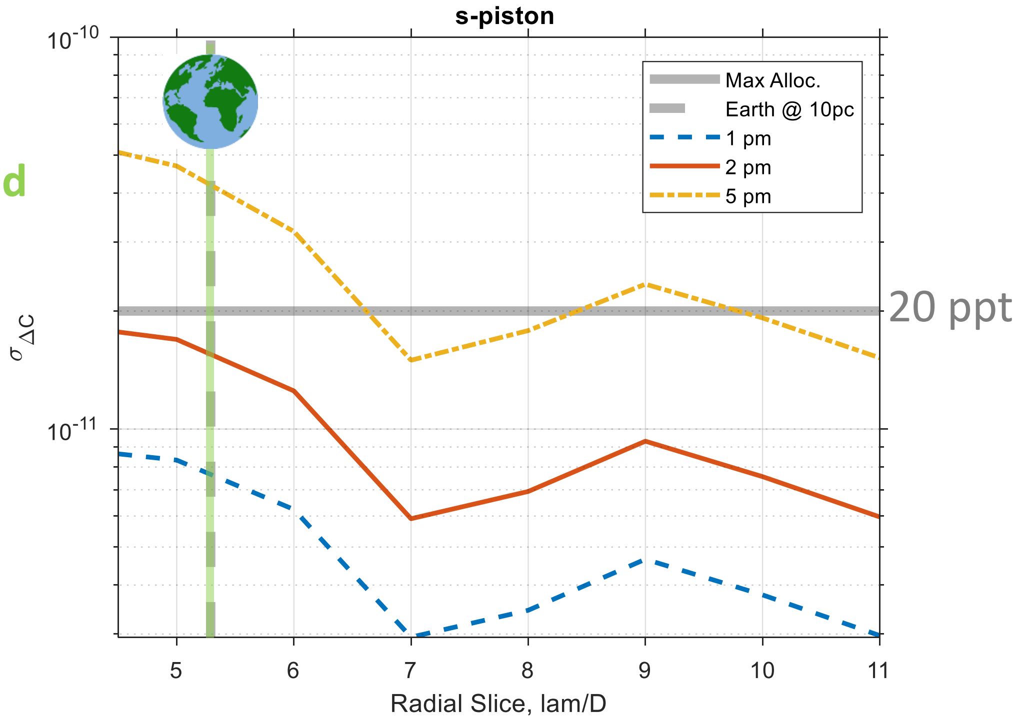
6-m Off-axis Segmented with VVC6

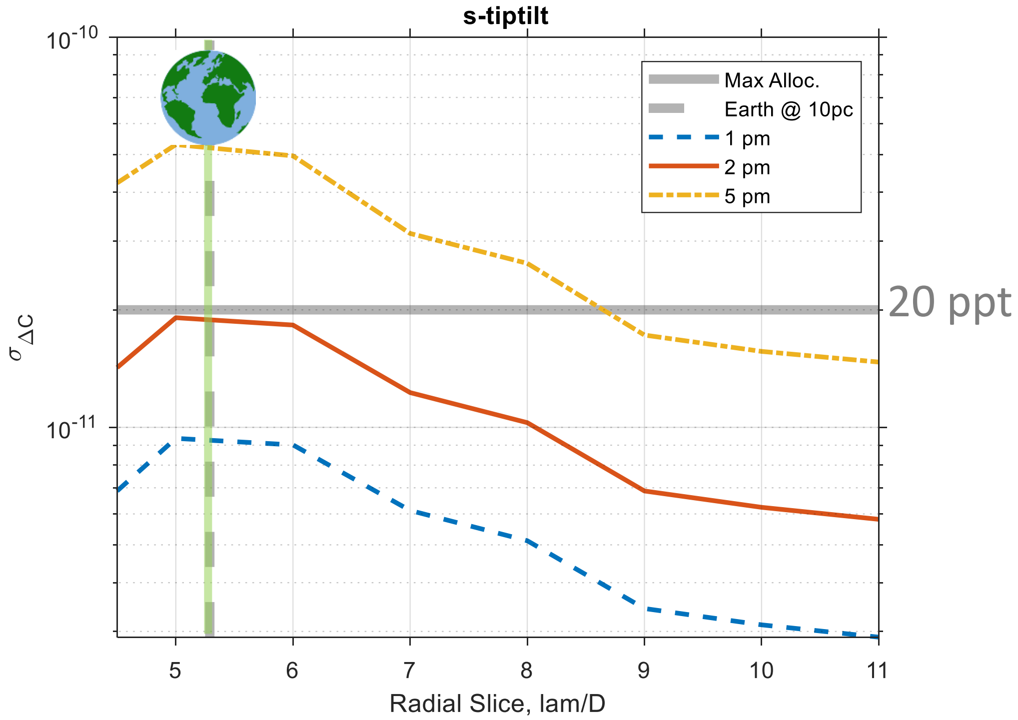
	g_bend	g_powerS	g_sphericS	g_comaZ	g_trefZ	g_hexfZ	s_piston	s_tiptilt	s_powerS	s_astigZ	s_trefZ	s_hexfZ
Sensitivities (ppt/pm)	3.21	4.64	4.51	5.19	5.82	10.6	8.84	9.09	3.68	9.33	6.16	0.74
Allocations (ppt)	5.60	8.10	7.86	9.05	10.2	18.5	15.4	15.9	6.43	16.3	10.8	1.30
Tolerances (pm)	1.75	1.75	1.75	1.75	1.75	1.75	1.64	1.64	1.64	1.64	1.64	1.64

Summary – Wavefront stability is very important

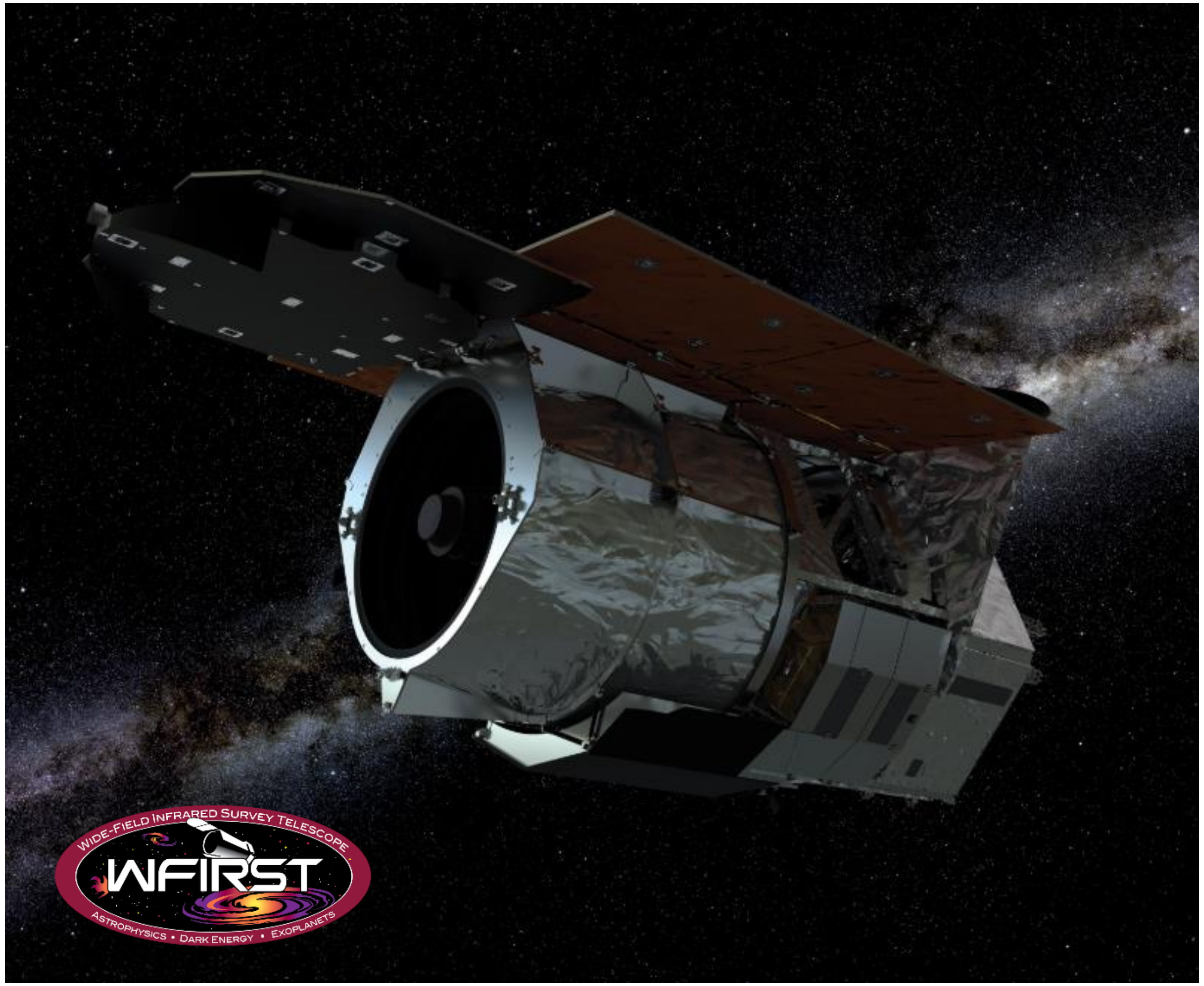
- When it comes to instability, the number that is important is **flux ratio / SNR**
- For an Exo-Earth detection with SNR of 7, the total allowable error is $3 \cdot 10^{-11}$ (30 ppt)
  - This is the number that limits the allocations
- Of particular note are segment piston and tip tilt in the segmented case
  - Here is a different visualization of the challenge:

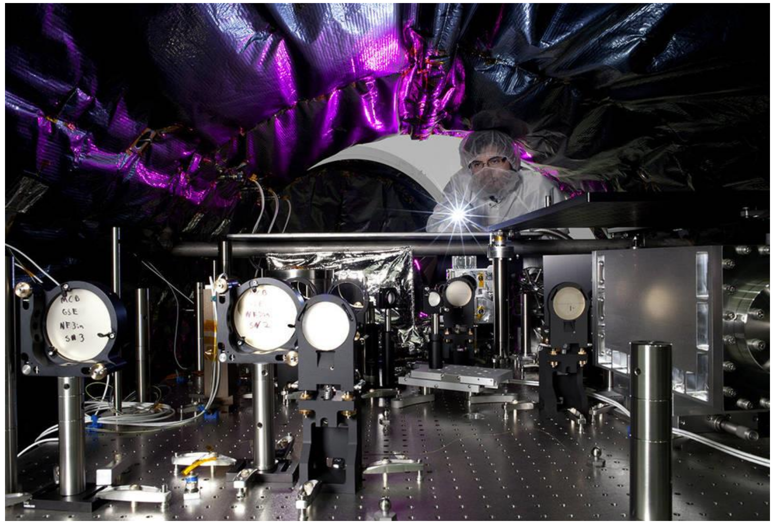
6-meter segmented w/ APLC

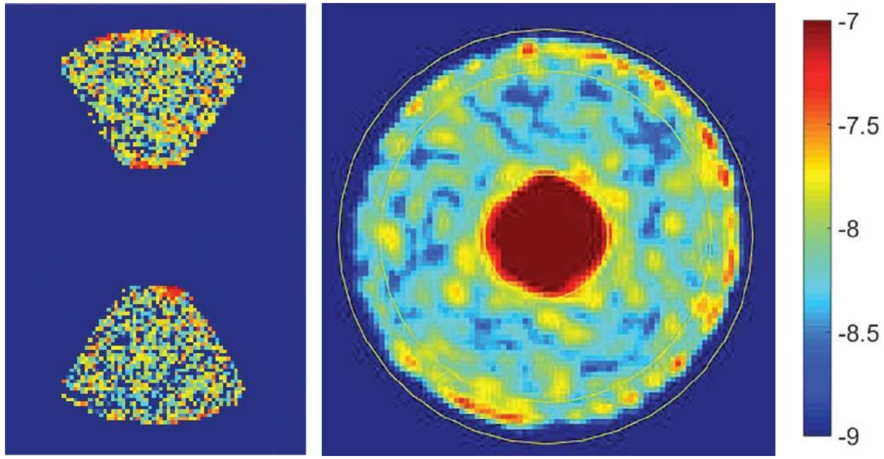




Paving the Way – The WFIRST Coronagraph Instrument

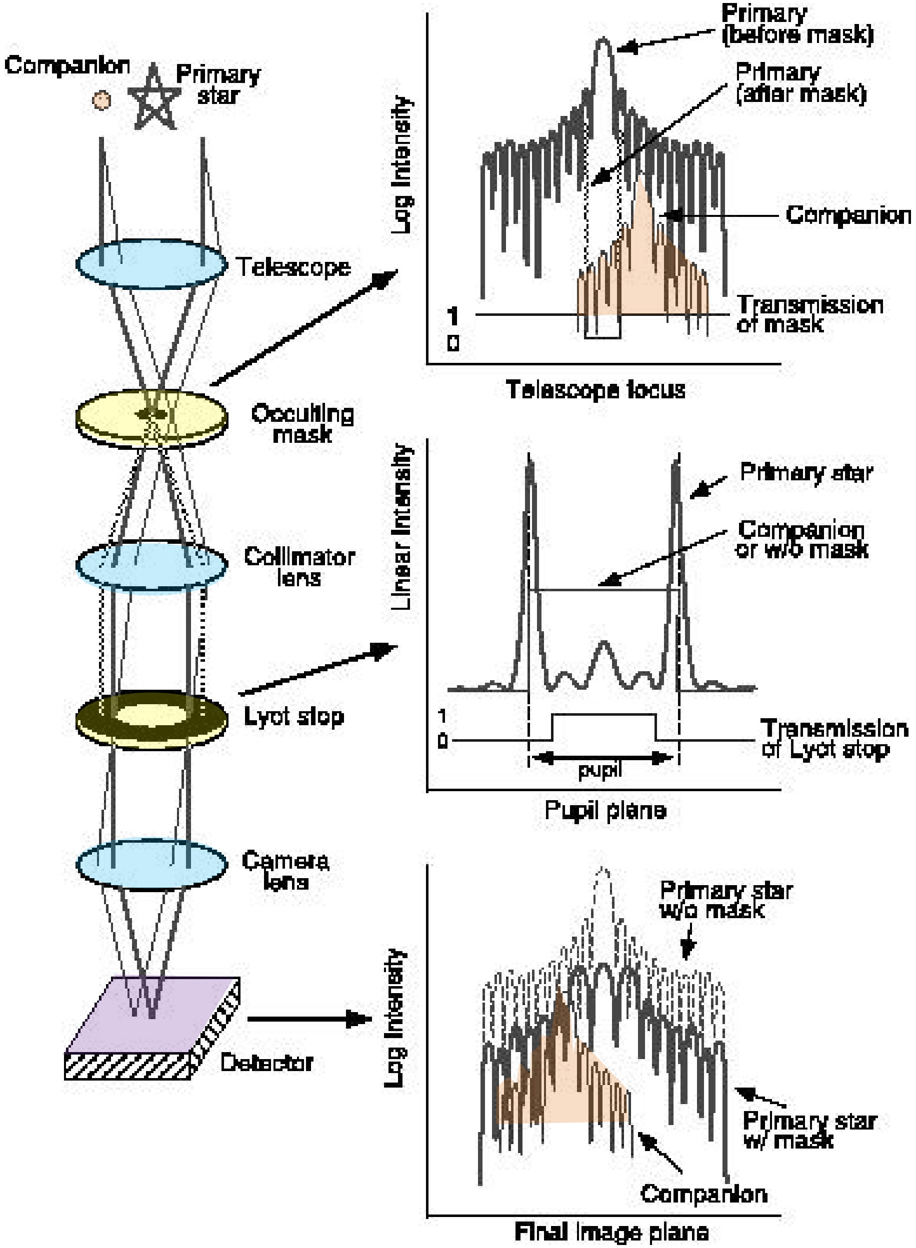






The Coronagraph

Bernard Lyot, 1939
French Astronomer
Inventor of the Coronagraph



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