Fuel Cost Heuristics for Starshade Slews and Station-Keeping in Exoplanet Imaging Mission Simulations

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1. Exoplanet Imaging Simulator
2. Model for Starshade
3. Impulsive Fuel Cost + Heuristics
4. Continuous Thrust Fuel Cost + Heuristics
5. Observation Scheduling
Savransky and Garrett (2016) “WFIRST-AFTA coronagraph science yield modeling with EXOSIMS.” JATIS
https://github.com/dsavransky/EXOSIMS
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Starshade Configuration

- In-band starlight is suppressed
  - Off-axis exoplanet light collected directly
- Maintains constant separation distance $s$ along target star line of sight (LOS)
- Tight tolerance in lateral direction
  - Starlight floods pupil plane if $>1m$ from LOS

Limited fuel on board

On a halo orbit
Telescope Orbit (Not Drawn to Scale)

Ecliptic ("Inertial") Frame
(Rotation only to show structure)
Telescope Orbit (Not Drawn to Scale)

Ecliptic ("Inertial") Frame
(Rotation only to show structure)

Rotating Frame
(Earth and Sun stationary)
Starshade in the CR3BP Frame

\[ \ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} + \mathbf{f}_{SRP} \cdot \hat{x} \]
\[ \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} + \mathbf{f}_{SRP} \cdot \hat{y} \]
\[ \ddot{z} = \frac{\partial \Omega}{\partial z} + \mathbf{f}_{SRP} \cdot \hat{z} \]

\[ \Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2}, \]

\[ r_1 = \sqrt{(\mu - x)^2 + y^2 + z^2}, \]
\[ r_2 = \sqrt{(1 - \mu - x)^2 + y^2 + z^2} \]
Flight Modes
Thruster Models

Impulsive Thrust Model

- Chemical Propulsion
- Instantaneous changes in velocity at $t_i$ and $t_j$
- Solved as boundary value problem (BVP) using collocation algorithm

$$\Delta m = m_0 \left(1 - e^{-\frac{\Delta v}{g_0 I_{sp}}} \right)$$
Thruster Models

Continuous Thrust Model

- Solar Electric Propulsion, Ion thruster, etc.
- Thrust can be throttled throughout trajectory
- Must add mass as state variable
Starshade Lateral Disturbance Forces
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Parameterizing Fuel Costs

\[ \Delta v = f(i, j, \Delta t, t_0, T_{halo}, s) \]
Parameterizing Fuel Costs

- Stars arranged by ecliptic longitude
- Constant slew time of 20 days
- 3D cost matrix for multiple slew times

Impulsive Fuel Costs

\[ \Delta v = f(i, j, \Delta t, t_0) \]

\[ \Delta v = f(\psi, \Delta t) \]

Parameterizing Fuel Costs - Errors
Impulsive Fuel Costs

- Assume constant halo and separation distance
- Before: 12 minutes to compute map at every decision step
  - 5 day time step
- Now: single map generated offline for any target list

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Continuous Thrust Fuel Costs

- Use optimal control!
  - Combine dynamics with optimization space
  - Augmented CR3BP equations of motion
  - Introduce co-states (7 more) for each state

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} \dot{r} \\ \dot{\mathbf{v}} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{g}(r) + \mathbf{h}(\mathbf{v}) + \frac{\mu T_{\text{max}}}{m} \hat{\alpha} \\ -\frac{\mu T_{\text{max}}}{v_e} \end{bmatrix} \\
\dot{\lambda} &= -\frac{\partial H}{\partial x} = \begin{bmatrix} \dot{\lambda}_r \\ \dot{\lambda}_v \\ \dot{\lambda}_m \end{bmatrix} = \begin{bmatrix} -G^T \lambda_v \\ -\lambda_r - H^T \lambda_v \\ -\frac{\lambda_v \mu T_{\text{max}}}{m^2} \end{bmatrix}
\end{align*}
\]

- Define cost function for Hamiltonian $H$
  - $\epsilon \in [0, 1]$ switches cost function between minimum fuel and minimum energy optimization

\[
J = T_{\text{max}} \frac{c}{u} \int_{t_0}^{t_f} [u - \epsilon u (1 - u)] dt
\]

Continuous Thrust Fuel Costs

- Thruster throttle values are a function of states and costates
  \[ u = f(\lambda_v, \lambda_m, m, \epsilon) = \begin{cases} 
    0 & \text{Thruster Off} \\
    (0, 1) & \text{Partially Throttled} \\
    1 & \text{Thruster Max} 
  \end{cases} \]

- Solve BVP with 14 boundary conditions instead of 6

- \( \epsilon \) used to vary control law
  - \( \epsilon = 1 \) is minimum energy
  - \( \epsilon = 0 \) is minimum fuel

Parameterizing Fuel Costs (6000 kg Starshade)

- Control law minimizes energy

- Fuel cost is directly a function of fuel mass used

\[ \Delta m \approx f(\psi, \Delta t, t_0, m_0) \]

- Dependence on initial mass at start of maneuver:

\[ \Delta m \approx f(\psi, \Delta t, t_0)A_0e^{-\frac{uT_{\text{max}}}{v_e}(1-m_0)} \]

\[ A_0 \approx \Delta m(\psi, \Delta t, t_0, m_0 = 1) \]
Parameterizing Fuel Costs – Time Dependence

- Time dependence has more structure
- Perhaps 3-d interpolant more appropriate
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Keepout Constraints
Keepout Constraints

Keepout Constraints
Cost Function

Minimize fuel use for all stars $j$

Maximize completeness for each star $j$

Prioritize stars that haven't been observed yet

Prioritize stars designated for a revisit

$$c = c_1 \Delta v_{min} + c_2 (1 - C_O) - c_3 f_{unv} + c_4 f_{rev}$$

$$J = \arg \min_j (c)$$

Observation Schedule

Mission Ensembles

Conclusions

- Fuel cost interpolant based on full solutions to CR3BP trajectories
- Effectively explores slew time tradespace
  - Mission time constraints applied as upper and lower bounds
- Interpolant used as heuristic within full end-to-end starshade mission simulations
- Realistically and accurately treating starshade fuel costs increase confidence in simulations
  - Increase number of scheduled observations + possible detections

EXOSIMS main page: github.com/dsavransky/EXOSIMS

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Backup Slides
Starshade Configuration

- No starlight enters telescope directly
  - Off-axis exoplanet light collected

- Maintains constant separation $s$ along target star line of sight (LOS)

- Tight tolerance in lateral direction
  - Starlight floods pupil plane if >1m from LOS
Solar Radiation Pressure

Glassman et al (2011) "Creating optimal observation schedules for a starshade planet-finding mission" IEEE
Fake Star Catalog
Impulsive Fuel Costs - Errors

\[ \Delta v_{INT} = f(\Delta t) \big|_{(\psi_0, t_0)} \]

\[ \Delta v_{BVP} = f(\Delta t, t_0) \big|_{\psi_0} \]
Parameterizing Fuel Costs - Errors
Retargeting Trajectories

Collocation:
- Cubic polynomial
- Equal at endpoints
- Creates mesh and minimizes residual error
Retargeting Trajectories
Error Analysis

Test 1, $\mu_{abs} = 2.53$, $\text{med}_{abs} = 1.36$
Test 2, $\mu_{abs} = 8.97$, $\text{med}_{abs} = 3.90$
Test 3, $\mu_{abs} = 6.83$, $\text{med}_{abs} = 3.61$
Parameterizing Fuel Costs - Errors

\[ \Delta v_{BVP} = f(\Delta t)|_{(\psi_0, t_0)} \]

\[ \Delta v_{INT} = f(\Delta t, t_0)|_{\psi_0} \]
Parameterizing Fuel Costs - Errors

\[ \Delta v_{BVP} = f(\psi, \Delta t, t_0) \]
\[ \Delta v_{INT} = f(\psi, \Delta t, t_0) \]
Scheduler

- Star Catalog
  - Planet Catalog
  - Planet Population
  - Planet Physical Model
  - Optical System

Target List

Generate star completeness values

Apply completeness, binary star, other filters

Keepout Binary Map

Mission Simulation Start

- Keepout Parameters
  - Mission Start and End Times

Fake Star Catalog

Fuel Cost Interpolant
Scheduler

- Old Star
- Target List
- Current Time

Find Next Best Target Star

- Calculate Observable Time Windows
- Estimate Fuel Costs
- Filter Target List
- Select Star with Lowest Total Cost

Update

While Fuel Left on Board

Determine Detections and Populate Schedule

- Stars visited more than a maximum amount of times
- Long integration times
- Retargeting fuel costs too expensive
- Observable times too far in the future
Completeness

- Joint Probability Density function
  - Star-planet brightness difference
  - Star-planet projected separation
- Based on instrument parameters, integrate over region
- Probability that a planet with assumed parameters is observable near a star