Fuel Cost Heuristics for Starshade Slews and Station-Keeping in Exoplanet Imaging Mission Simulations

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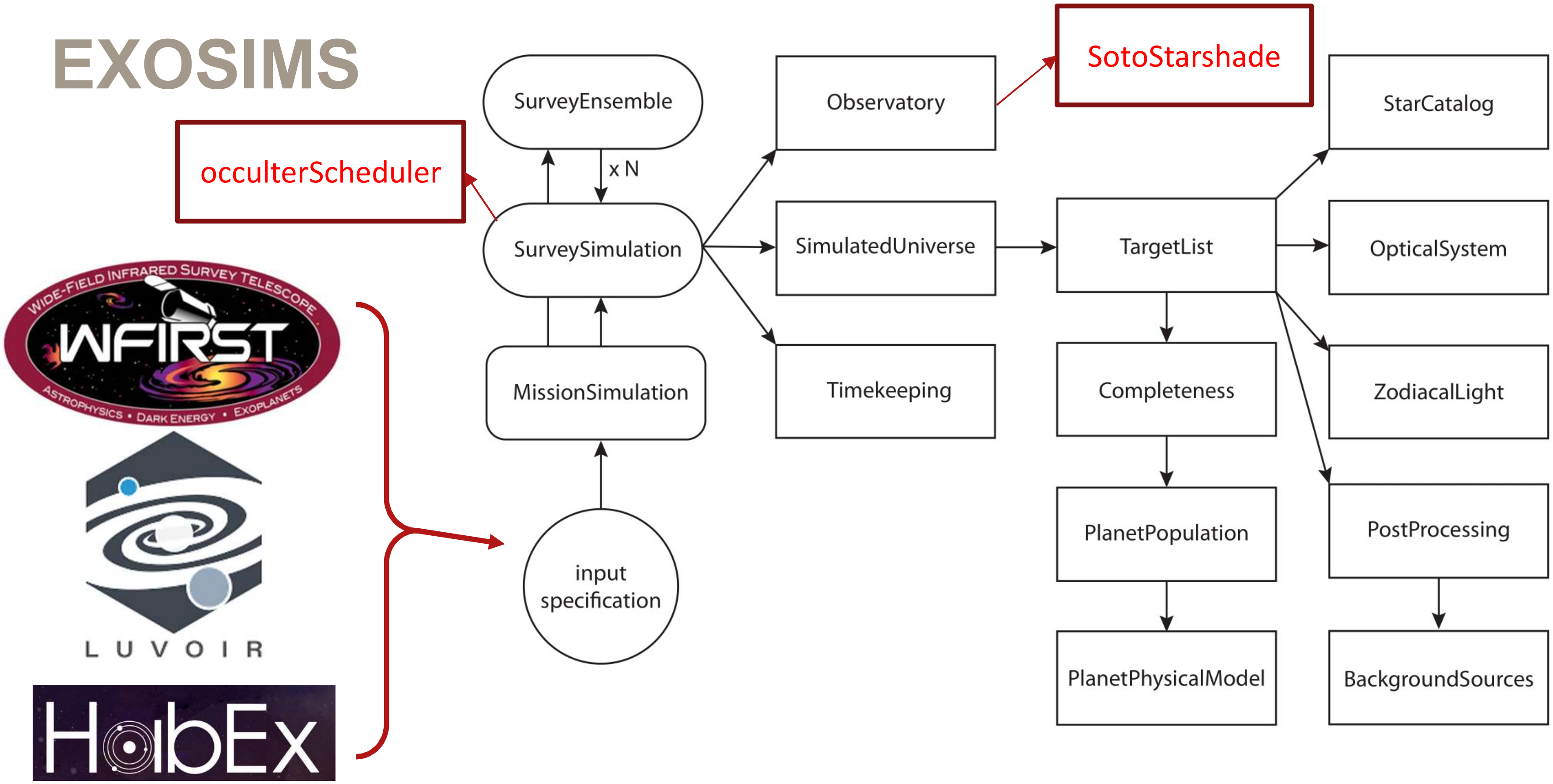
1. Exoplanet Imaging Simulator
2. Model for Starshade
3. Impulsive Fuel Cost + Heuristics
4. Continuous Thrust Fuel Cost + Heuristics
5. Observation Scheduling

EXOSIMS



Savransky and Garrett (2016) “WFIRST-AFTA coronagraph science yield modeling with EXOSIMS.” *JATIS*
https://github.com/dsavransky/EXOSIMS

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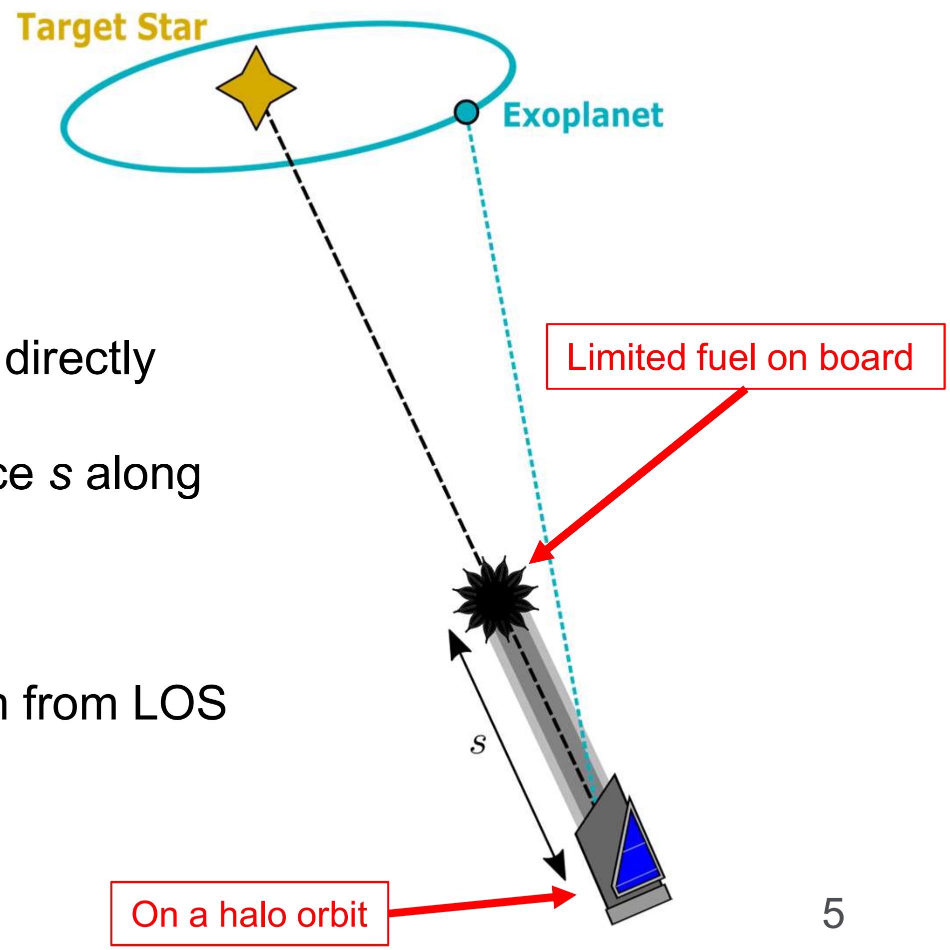
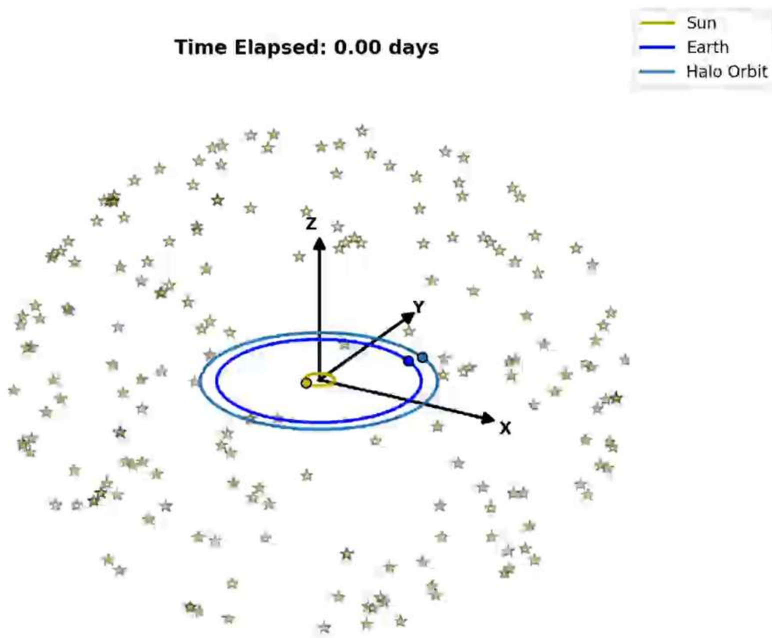
Starshade Configuration



- In-band starlight is suppressed
  - Off-axis exoplanet light collected directly
- Maintains constant separation distance *s* along target star line of sight (LOS)
- Tight tolerance in lateral direction
  - Starlight floods pupil plane if >1m from LOS

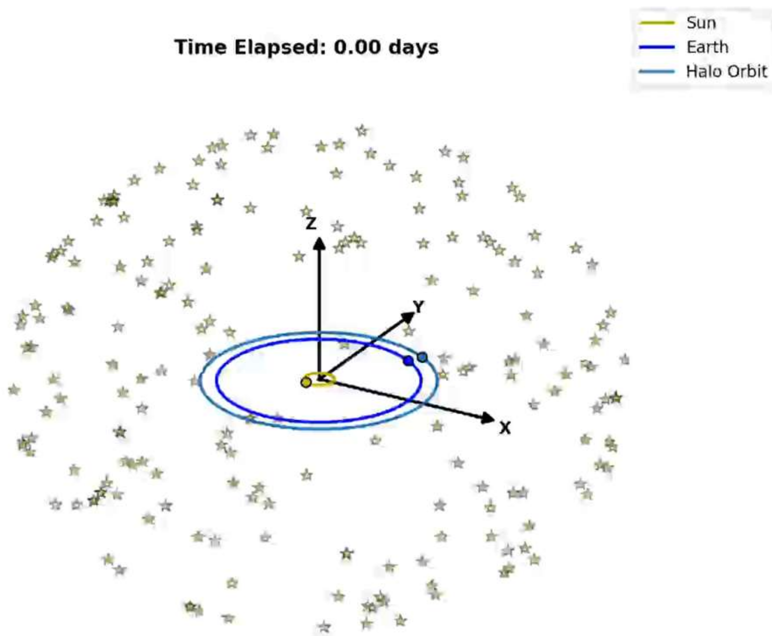
Telescope Orbit (Not Drawn to Scale)



Ecliptic (“Inertial”) Frame

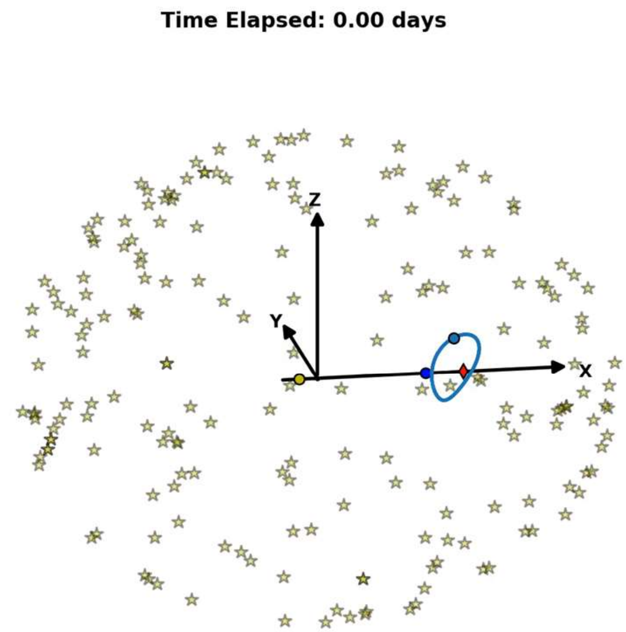
(Rotation only to show structure)

Telescope Orbit (Not Drawn to Scale)



Ecliptic (“Inertial”) Frame

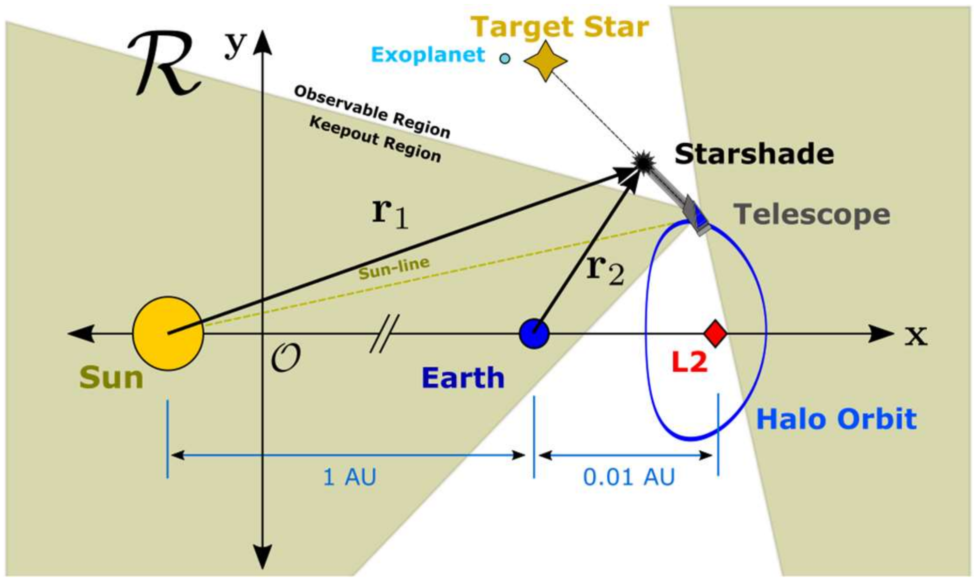
(Rotation only to show structure)



Rotating Frame

(Earth and Sun stationary)

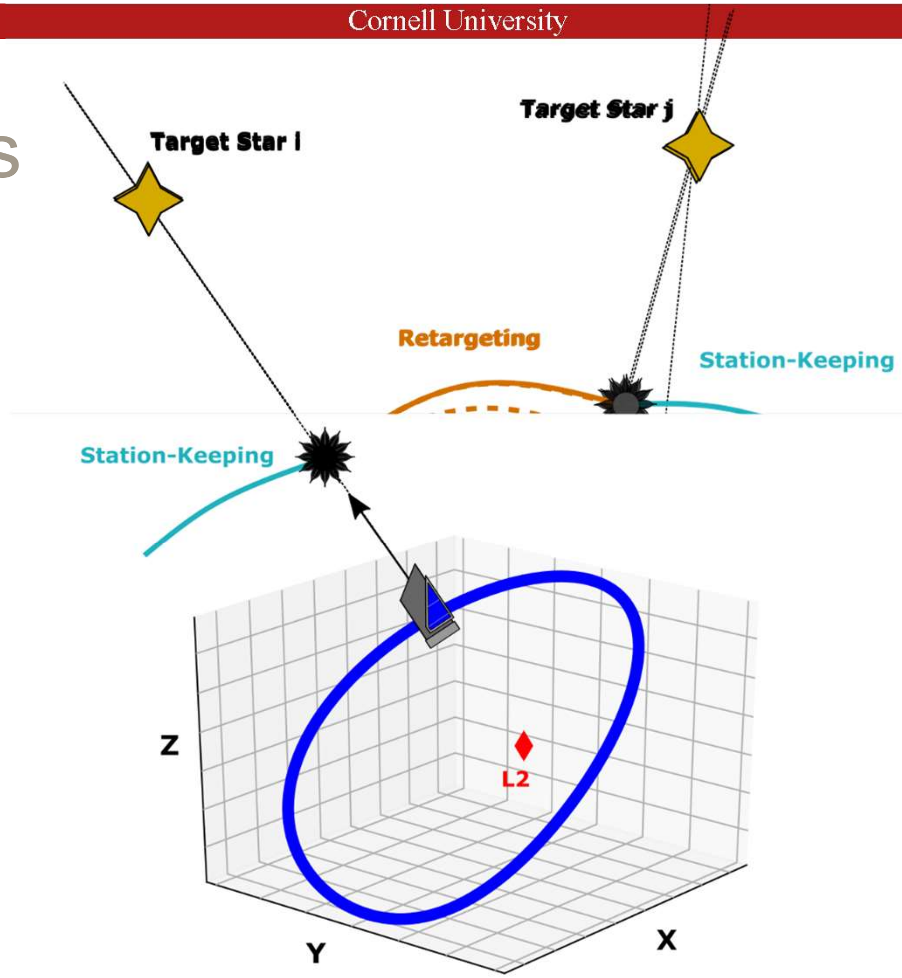
Starshade in the CR3BP Frame



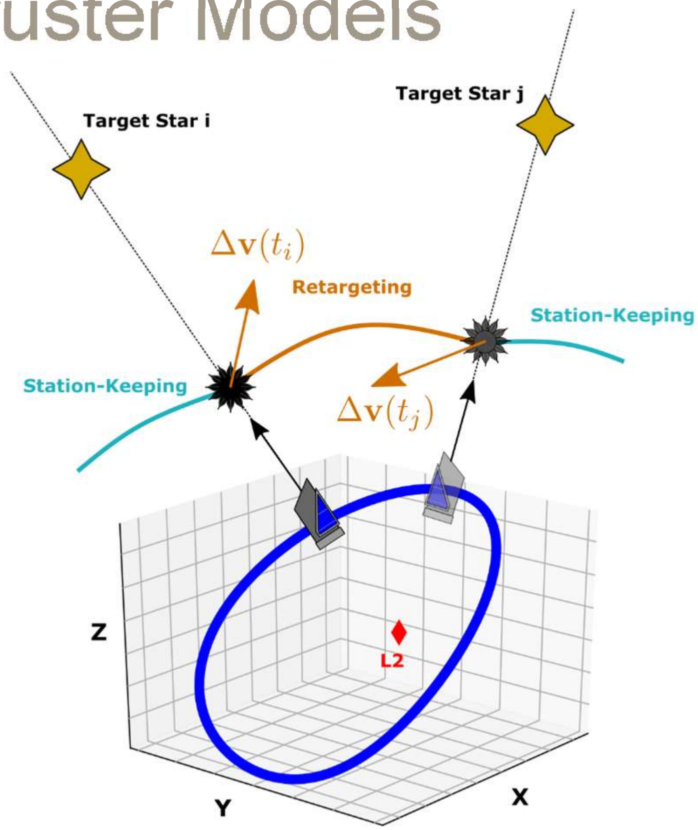
$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} + \mathbf{f}_{SRP} \cdot \hat{\mathbf{x}}$$
$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} + \mathbf{f}_{SRP} \cdot \hat{\mathbf{y}}$$
$$\ddot{z} = \frac{\partial \Omega}{\partial z} + \mathbf{f}_{SRP} \cdot \hat{\mathbf{z}}$$

$$\Omega(x,y,z) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2},$$
$$r_1 = \sqrt{(\mu - x)^2 + y^2 + z^2},$$
$$r_2 = \sqrt{(1 - \mu - x)^2 + y^2 + z^2}$$

Flight Modes



Thruster Models



Impulsive Thrust Model

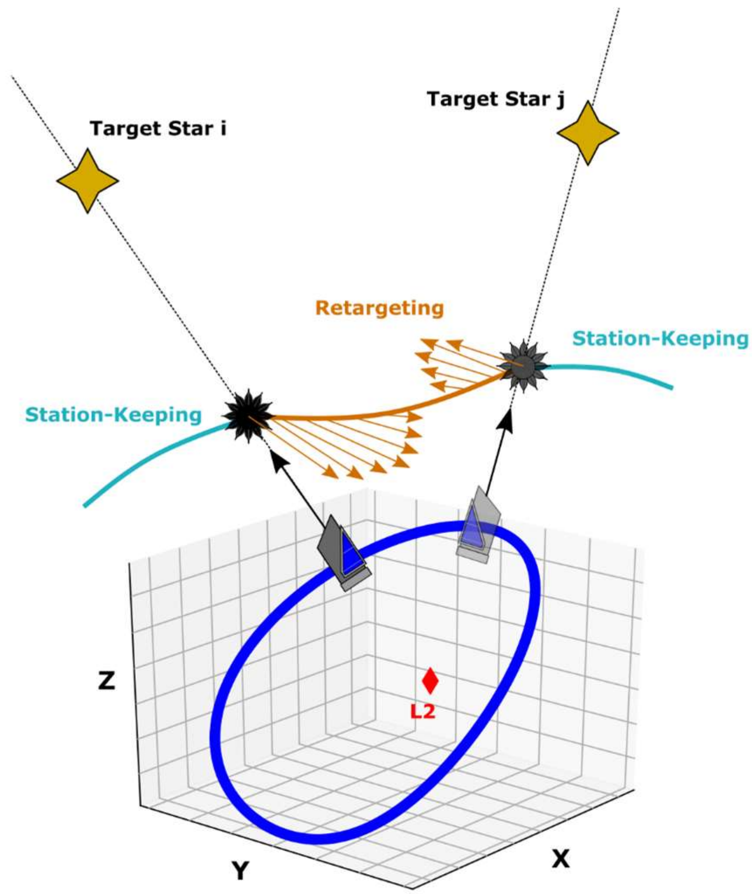
- Chemical Propulsion
- Instantaneous changes in velocity at t_i and t_j
- Solved as boundary value problem (BVP) using collocation algorithm

$$\Delta m = m_0(1 - e^{-\frac{\Delta v}{g_0 I_{sp}}})$$

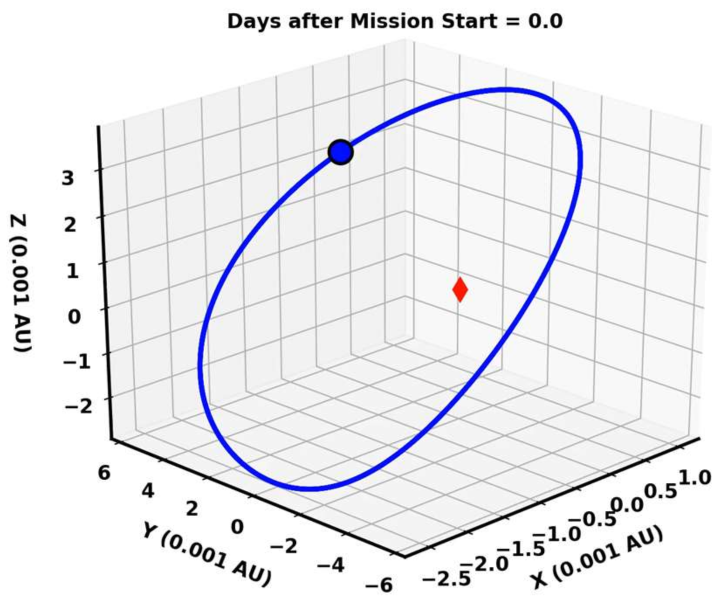
Thruster Models

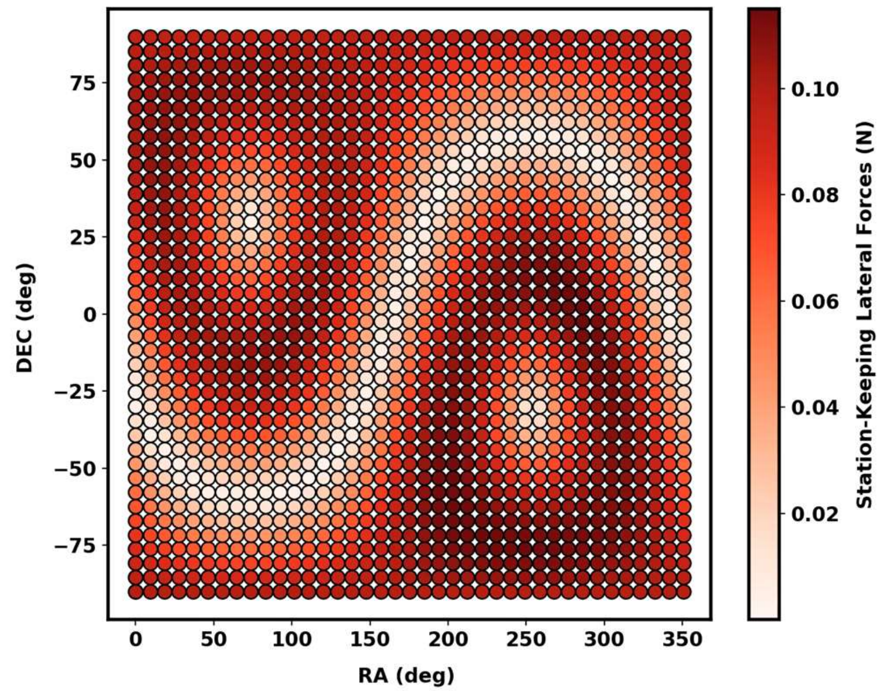
Continuous Thrust Model

- Solar Electric Propulsion, Ion thruster, etc.
- Thrust can be throttled throughout trajectory
- Must add mass as state variable



Starshade Lateral Disturbance Forces





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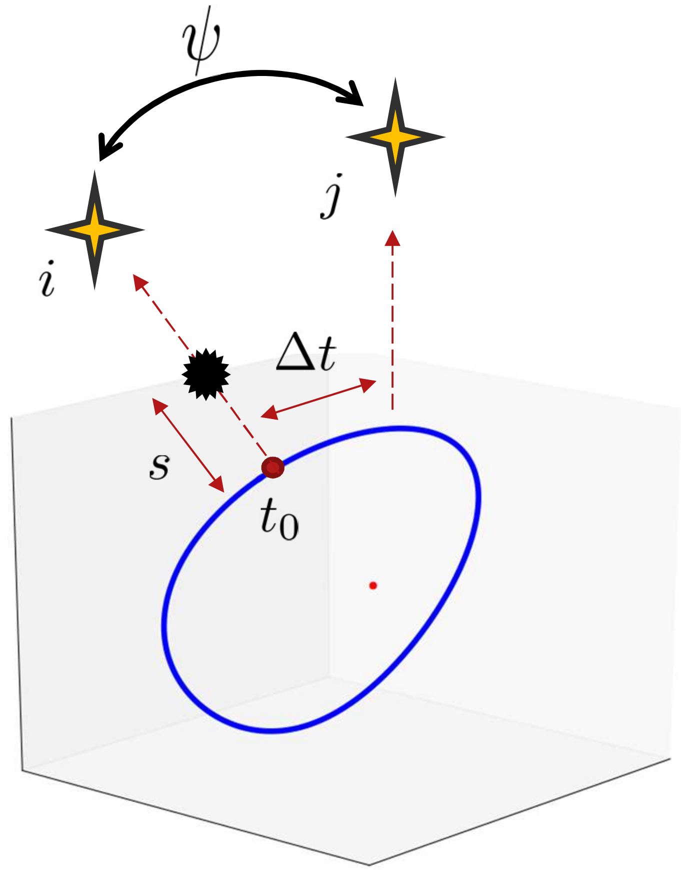
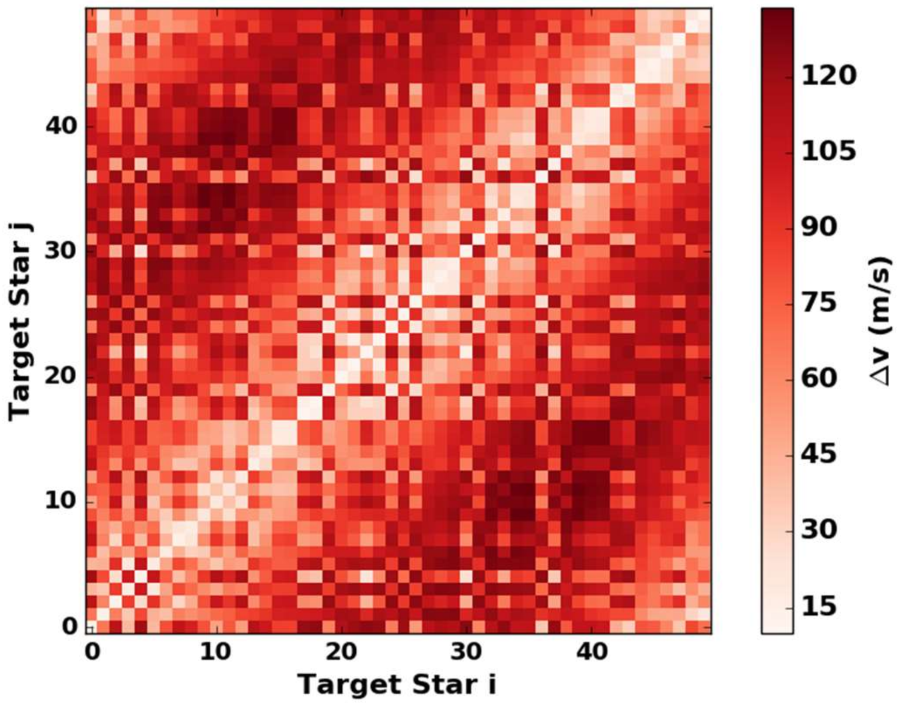
Parameterizing Fuel Costs

$$\Delta v = f(i, j, \Delta t, t_0, \underline{T_{halo}, s})$$



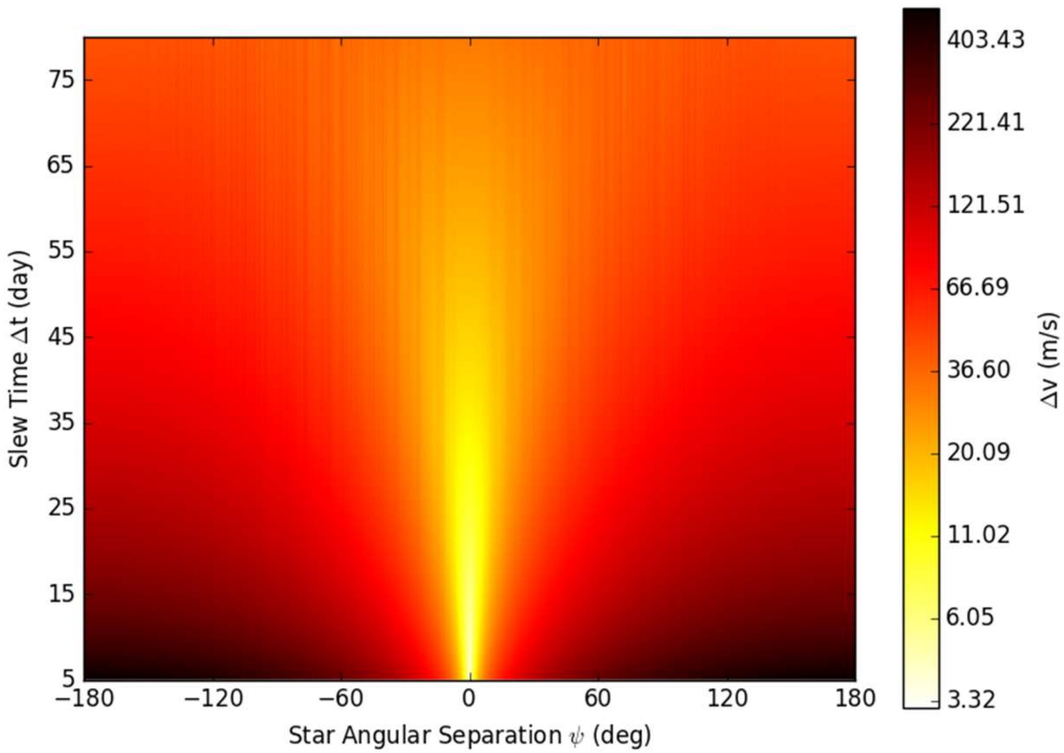
Parameterizing Fuel Costs



- Stars arranged by ecliptic longitude
- Constant slew time of 20 days
- 3D cost matrix for multiple slew times

Based on Kolemen and Kasdin (2012) "Optimization of an occulter-based extrasolar-planet-imaging mission" *JGCD*

Impulsive Fuel Costs



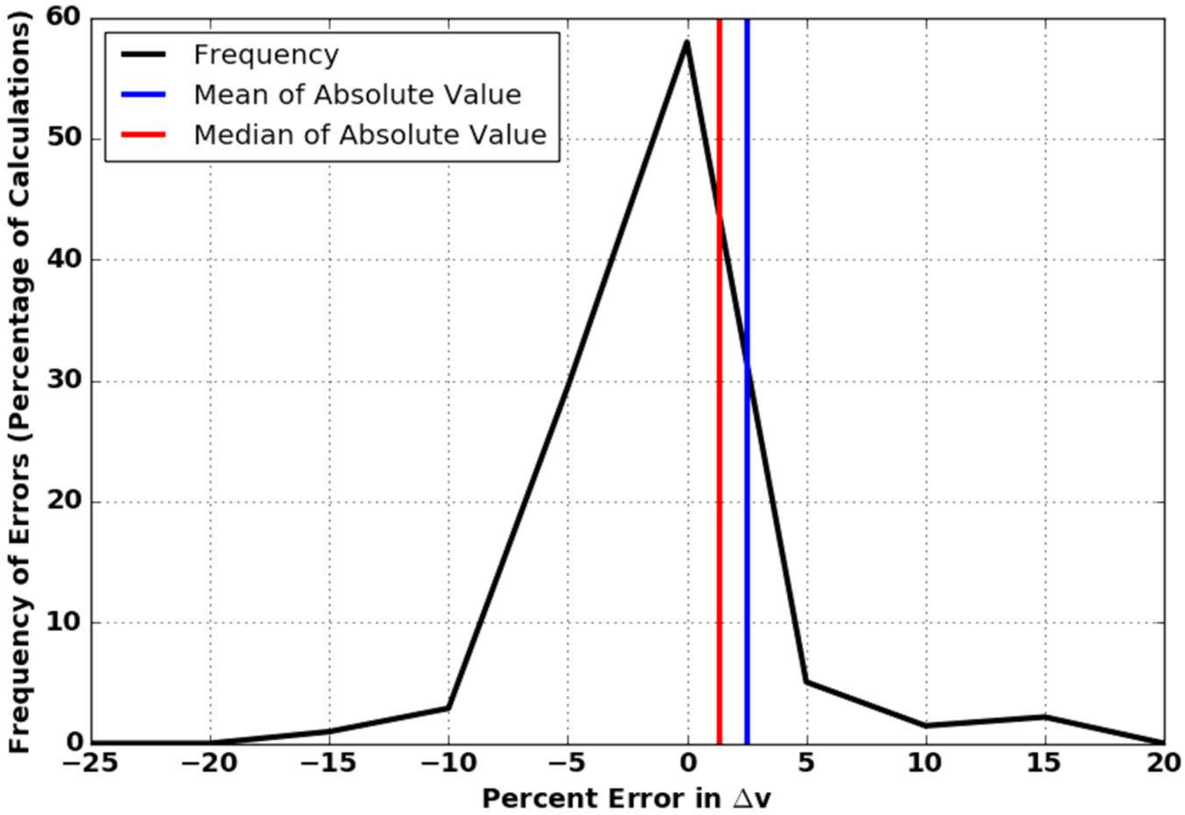
$$\Delta v = f(i, j, \Delta t, t_0)$$

↓

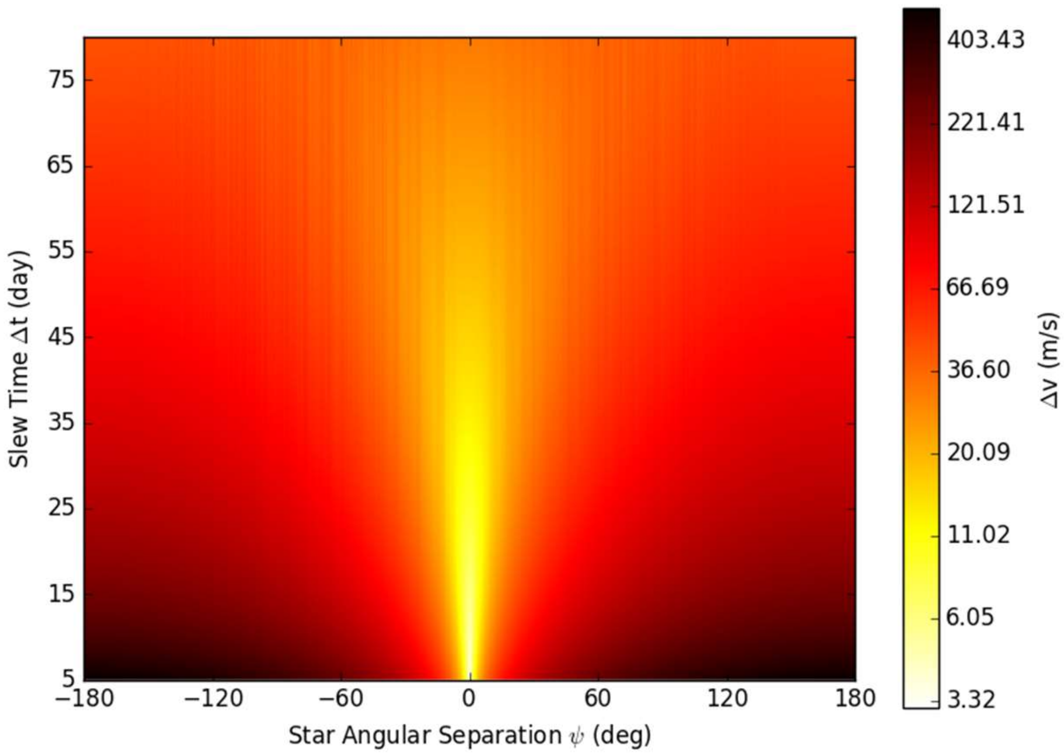
$$\Delta v = f(\psi, \Delta t)$$

Soto et al (2019) "Parameterizing the Search Space of Starshade Fuel Costs for Optimal Observation Schedules." *JGCD*

Parameterizing Fuel Costs - Errors



Impulsive Fuel Costs



- Assume constant halo and separation distance
- Before: 12 minutes to compute map at every decision step
  - 5 day time step
- Now: single map generated offline for any target list

Soto et al (2019) “Parameterizing the Search Space of Starshade Fuel Costs for Optimal Observation Schedules.” *JGCD*

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4. **Continuous Thrust Fuel Cost + Heuristics**
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Continuous Thrust Fuel Costs

- Use optimal control!
  - Combine dynamics with optimization space
  - Augmented CR3BP equations of motion
  - Introduce co-states (7 more) for each state

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{g}(\mathbf{r}) + \mathbf{h}(\mathbf{v}) + \frac{uT_{\max}}{m}\hat{\boldsymbol{\alpha}} \\ -\frac{uT_{\max}}{v_e} \end{bmatrix}$$

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \mathbf{x}} = \begin{bmatrix} \dot{\boldsymbol{\lambda}}_{\mathbf{r}} \\ \dot{\boldsymbol{\lambda}}_{\mathbf{v}} \\ \dot{\lambda}_m \end{bmatrix} = \begin{bmatrix} -\mathbf{G}^T\boldsymbol{\lambda}_{\mathbf{v}} \\ -\boldsymbol{\lambda}_{\mathbf{r}} - \mathbf{H}^T\boldsymbol{\lambda}_{\mathbf{v}} \\ \frac{-\lambda_v uT_{max}}{m^2} \end{bmatrix}$$

- Define cost function for Hamiltonian *H*
  - $\epsilon \in [0, 1]$ switches cost function between minimum fuel and minimum energy optimization

$$J = \frac{T_{\max}}{c} \int_{t_0}^{t_f} [u - \epsilon u(1-u)]dt$$

Zhang et al (2015) “Low-Thrust Minimum-Fuel Optimization in the Circular Restricted Three-Body Problem.” *JGCD*

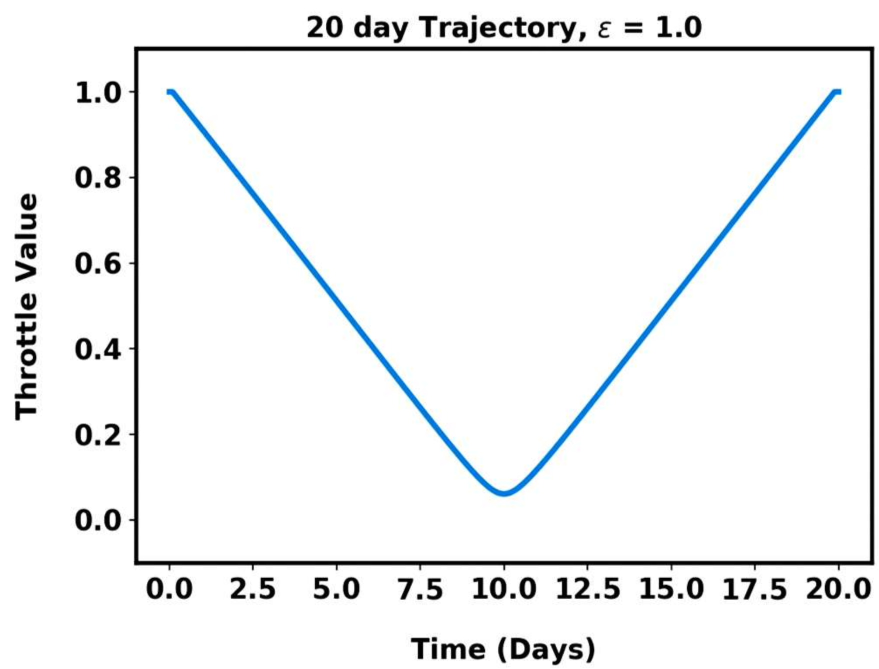
Continuous Thrust Fuel Costs

- Thruster throttle values are a function of states and costates

$$u = f(\boldsymbol{\lambda}_{\mathbf{v}}, \lambda_m, m, \epsilon) = \begin{cases} 0 & \text{Thruster Off} \\ (0,1) & \text{Partially Throttled} \\ 1 & \text{Thruster Max} \end{cases}$$

- Solve BVP with 14 boundary conditions instead of 6

- ε used to vary control law
  - ε=1 is minimum energy
  - ε=0 is minimum fuel



Zhang et al (2015) “Low-Thrust Minimum-Fuel Optimization in the Circular Restricted Three-Body Problem.” *JGCD*

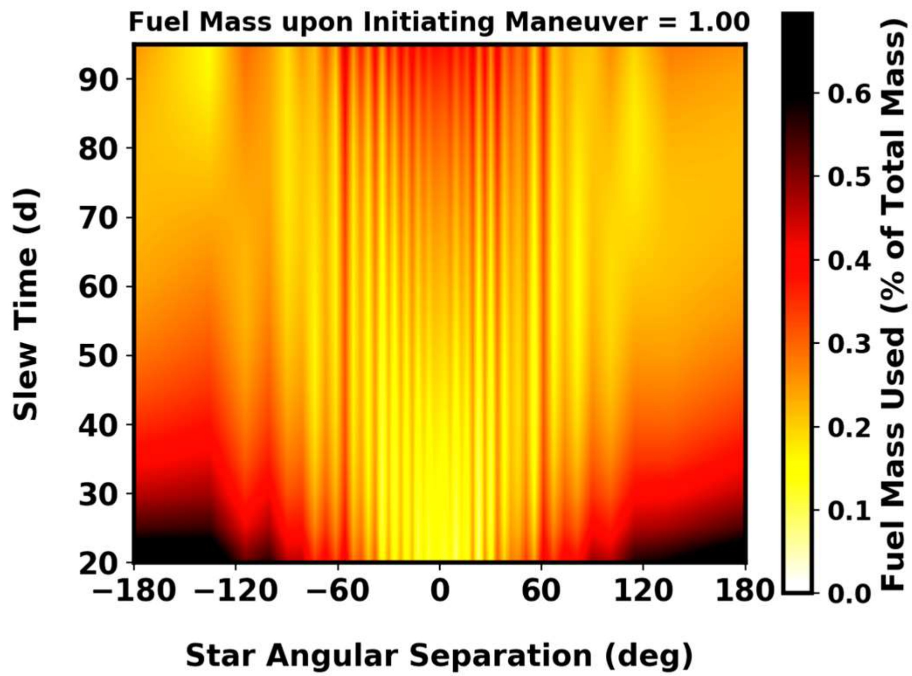
Parameterizing Fuel Costs (6000 kg Starshade)

- Control law minimizes energy
- Fuel cost is directly a function of fuel mass used

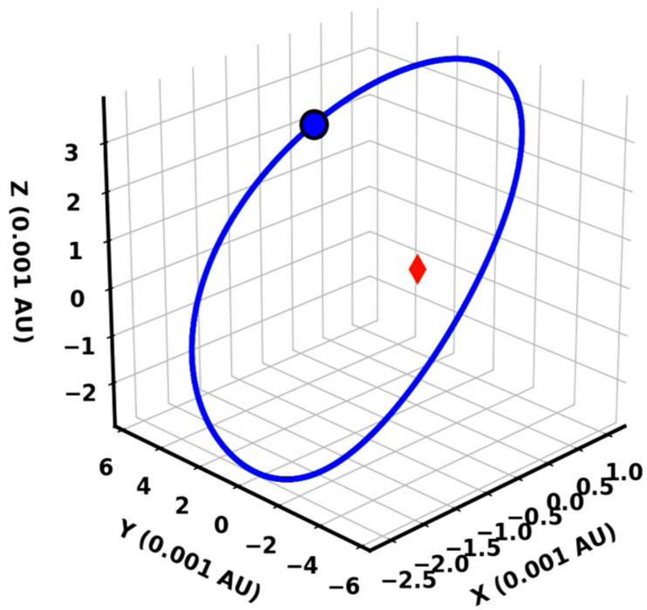
$$\Delta m \approx f(\psi, \Delta t, t_0, m_0)$$

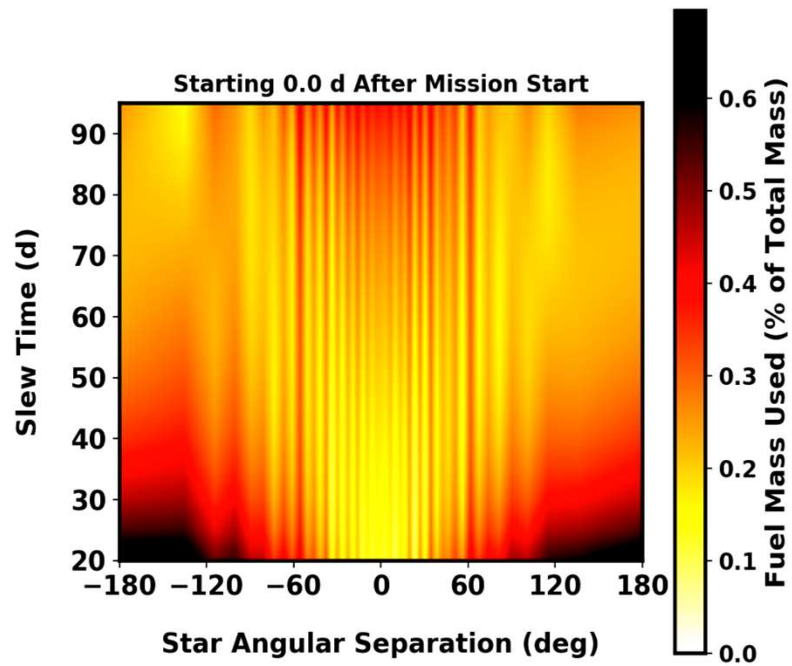
- Dependence on initial mass at start of maneuver:

$$\Delta m \approx f(\psi, \Delta t, t_0) A_0 e^{-\frac{u T_{\max}}{v_e}(1-m_0)}$$
$$A_0 \approx \Delta m(\psi, \Delta t, t_0, m_0 = 1)$$



Parameterizing Fuel Costs – Time Dependence





- Time dependence has more structure
- Perhaps 3-d interpolant more appropriate

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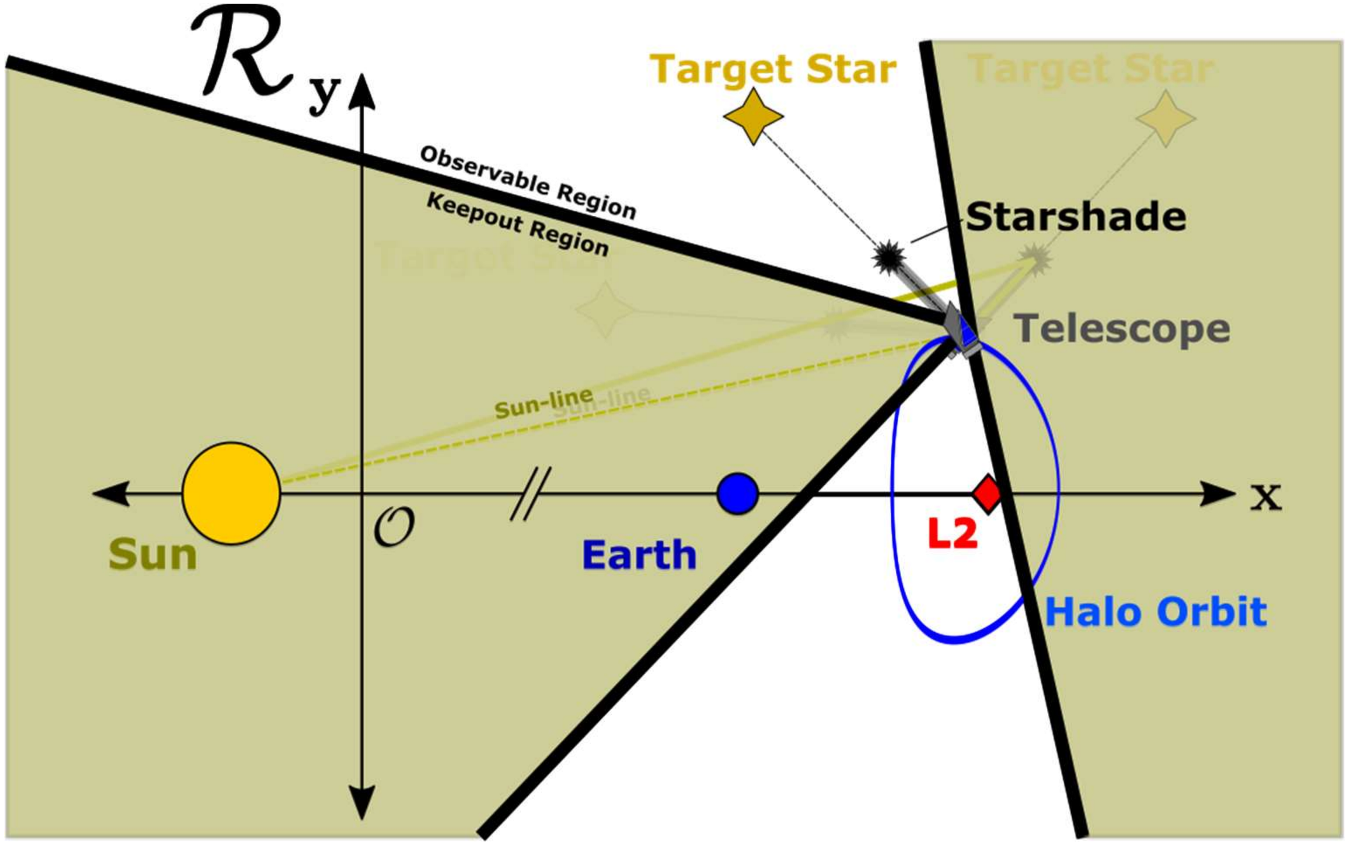
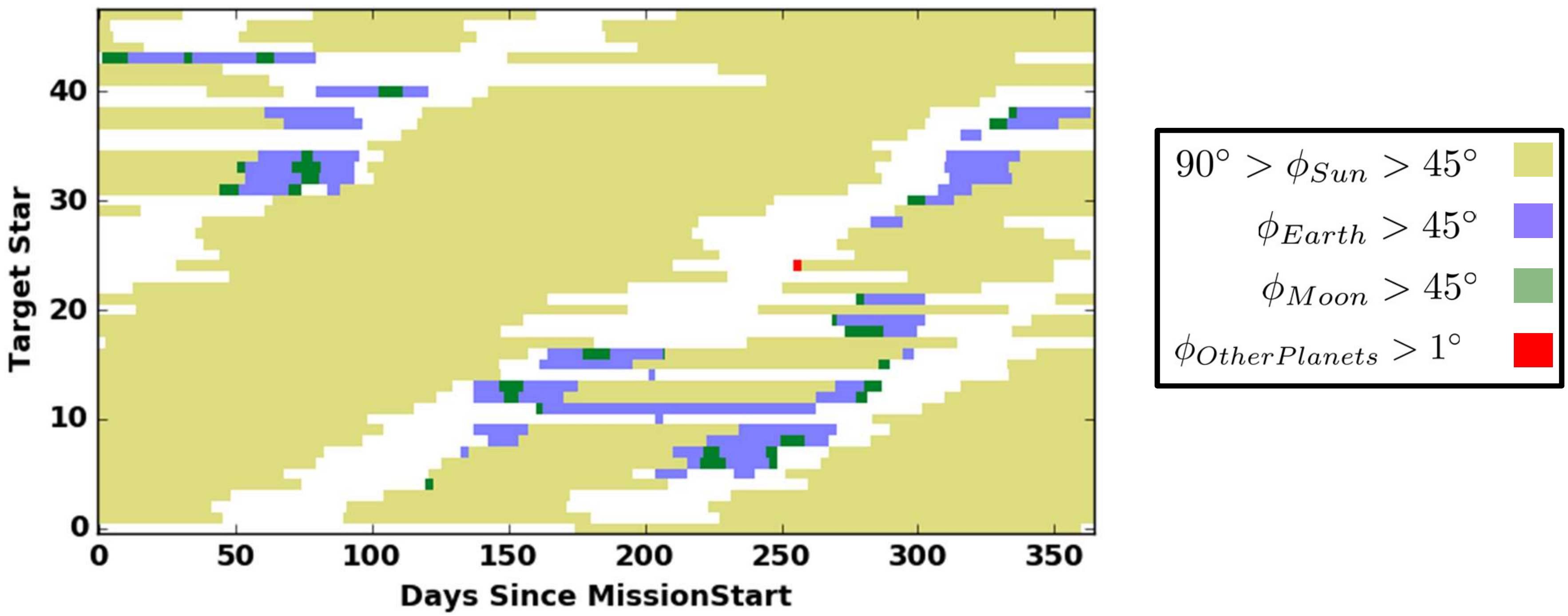
Keepout Constraints

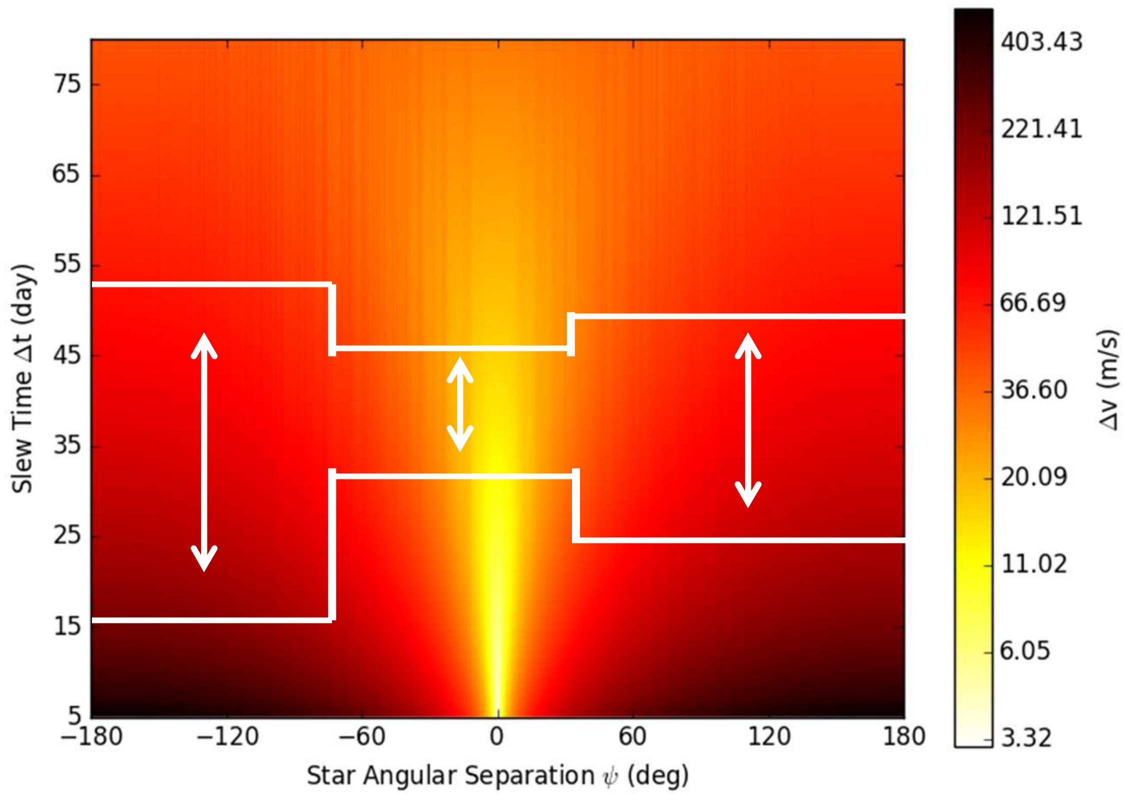


Keepout Constraints



Soto et al (2019) "Parameterizing the Search Space of Starshade Fuel Costs for Optimal Observation Schedules." *JGCD*

Keepout Constraints



Cost Function

Minimize **fuel use** for all stars *j*

Maximize **completeness** for each star *j*

Prioritize stars that haven't been observed yet

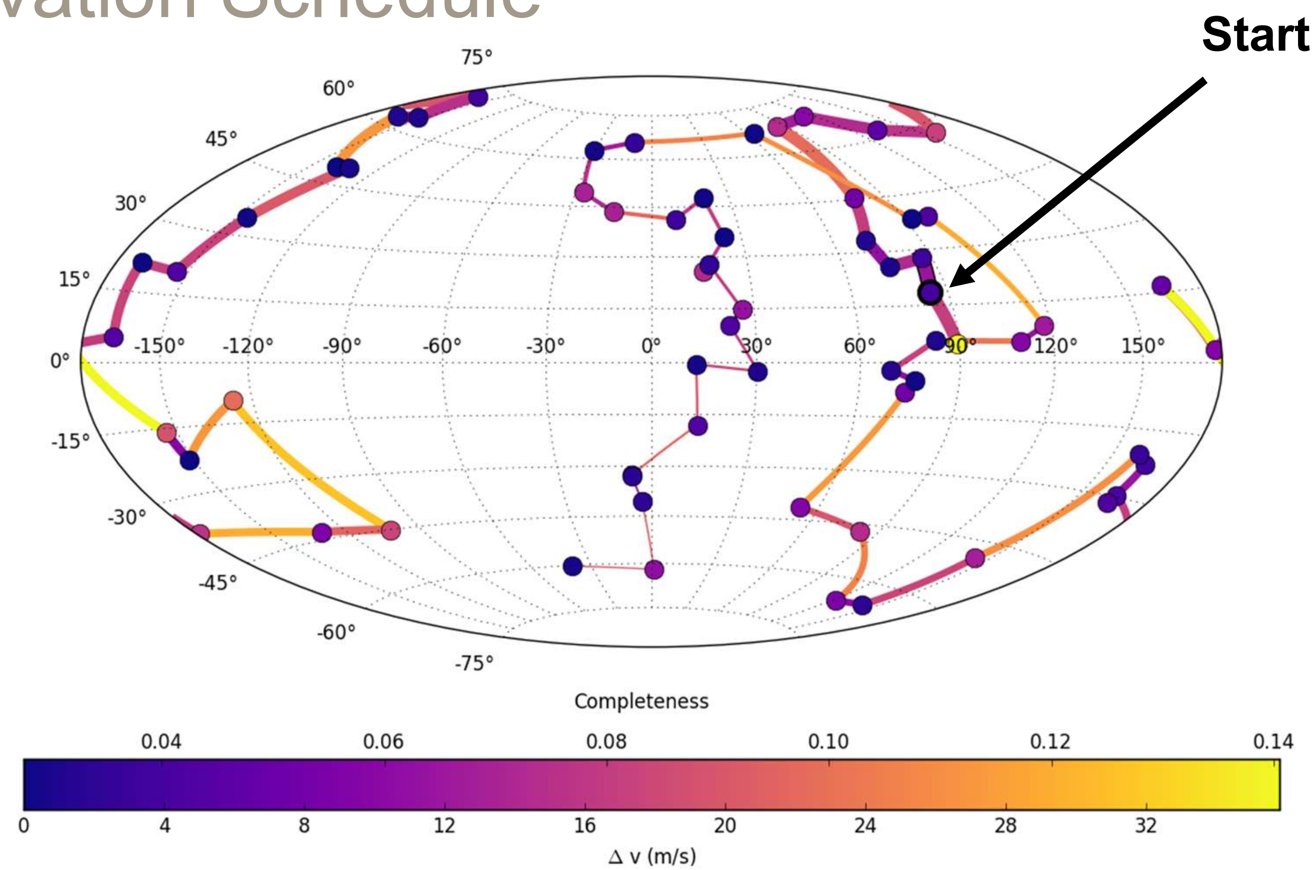
Prioritize stars designated for a revisit

$$\mathbf{c} = c_1 \Delta v_{min} + c_2(1 - C_O) - c_3 f_{unv} + c_4 f_{rev}$$

$$J = \arg\min_j(\mathbf{c})$$

Savransky et al (2010) "Analyzing the Designs of Planet-Finding Missions" *PASP*
Soto et al (2019) "Parameterizing the Search Space of Starshade Fuel Costs for Optimal Observation Schedules." *JGCD*

Observation Schedule



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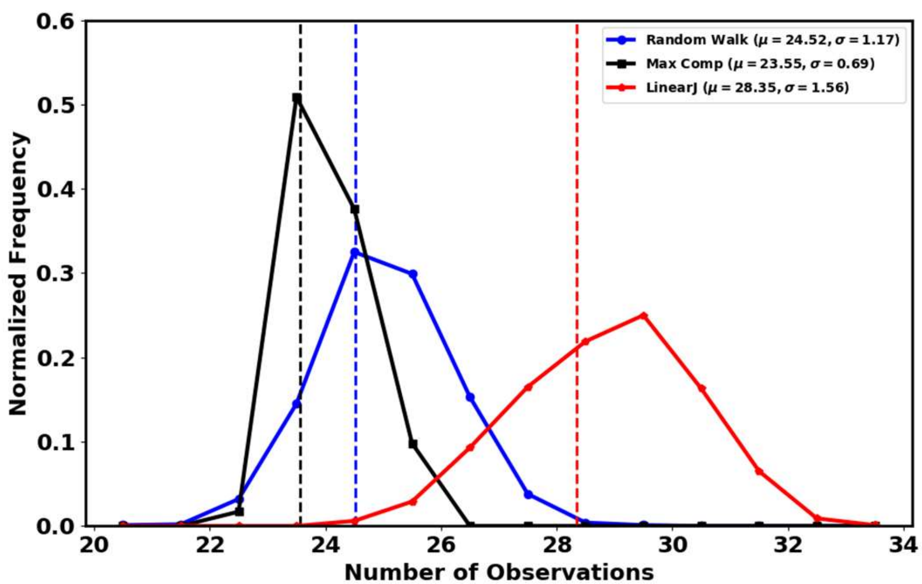
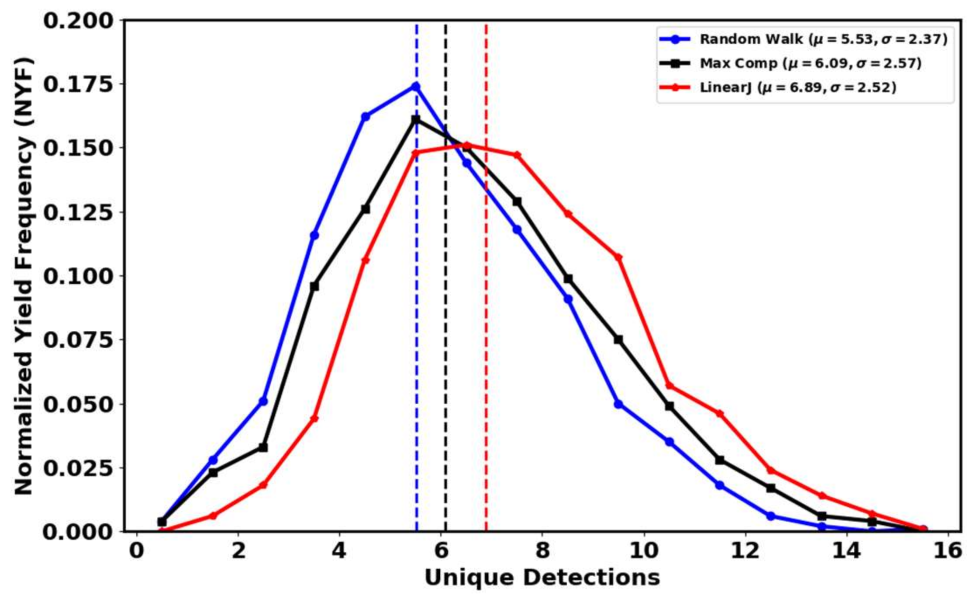
Mission Ensembles





Soto et al (2019) “Parameterizing the Search Space of Starshade Fuel Costs for Optimal Observation Schedules.” *JGCD*

Conclusions

- Fuel cost interpolant based on full solutions to CR3BP trajectories
- Effectively explores slew time tradespace
    - Mission time constraints applied as upper and lower bounds
- Interpolant used as heuristic within full end-to-end starshade mission simulations
- Realistically and accurately treating starshade fuel costs increase confidence in simulations
    - Increase number of scheduled observations + possible detections

EXOSIMS main page:
github.com/dsavransky/EXOSIMS

Work Funded by:
- NASA Grant No. NNG16PJ24C (SIT)
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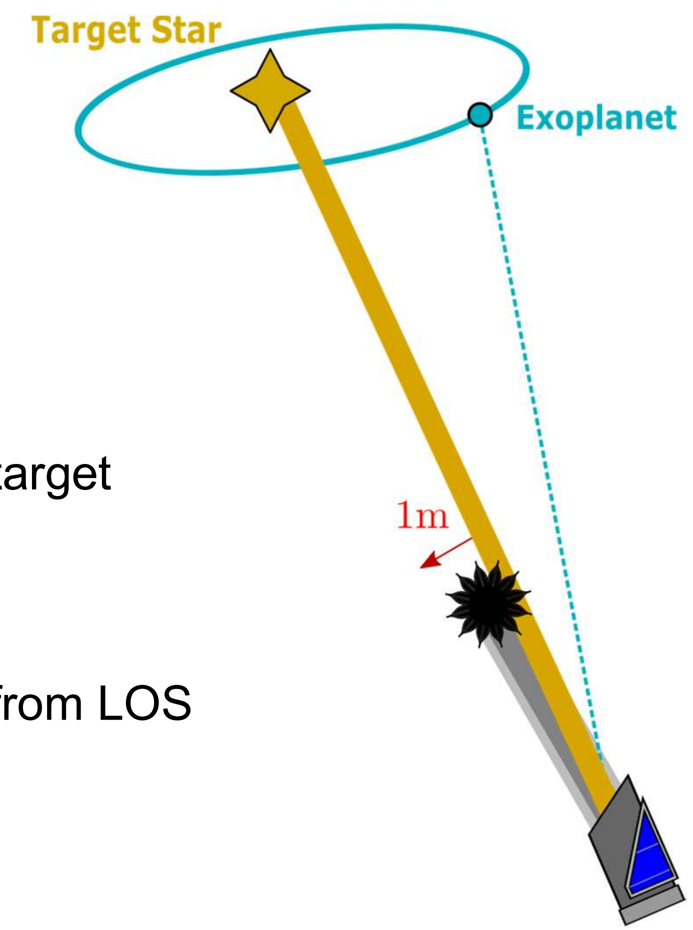
Backup Slides

Starshade Configuration

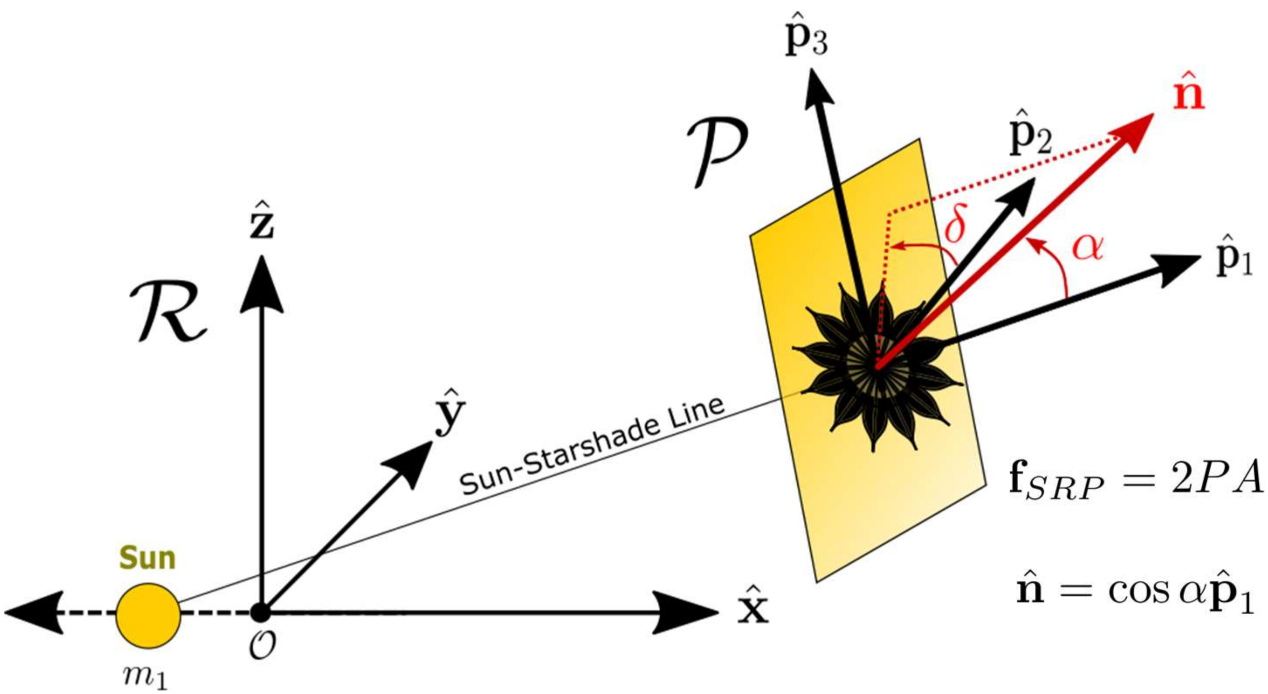


- No starlight enters telescope directly
  - Off-axis exoplanet light collected

- Maintains constant separation *s* along target star line of sight (LOS)

- Tight tolerance in lateral direction
  - Starlight floods pupil plane if >1m from LOS

Solar Radiation Pressure

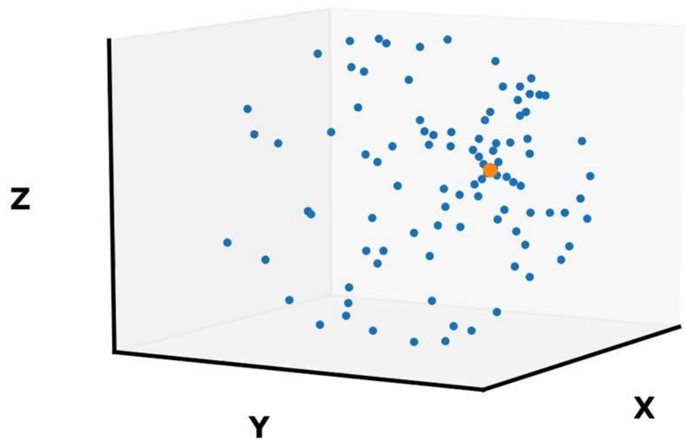


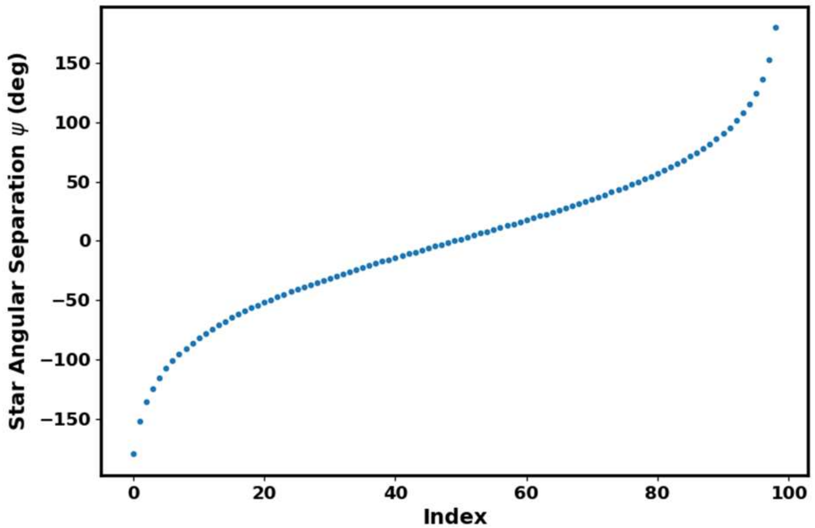
$$\mathbf{f}_{SRP} = 2PA\cos\alpha\left[b_1\hat{\mathbf{p}}_1 + (b_2\cos\alpha + b_3)\hat{\mathbf{n}}\right]$$

$$\hat{\mathbf{n}} = \cos\alpha\hat{\mathbf{p}}_1 + \sin\alpha\cos\delta\hat{\mathbf{p}}_2 + \sin\alpha\sin\delta\hat{\mathbf{p}}_3$$

Glassman et al (2011) “Creating optimal observation schedules for a starshade planet-finding mission” *IEEE*
McIness (1999) *Solar Sailing: Technology, Dynamics, and Mission Applications*
Soto et al (2019) “Parameterizing the Search Space of Starshade Fuel Costs for Optimal Observation Schedules.” *JGCD*

Fake Star Catalog





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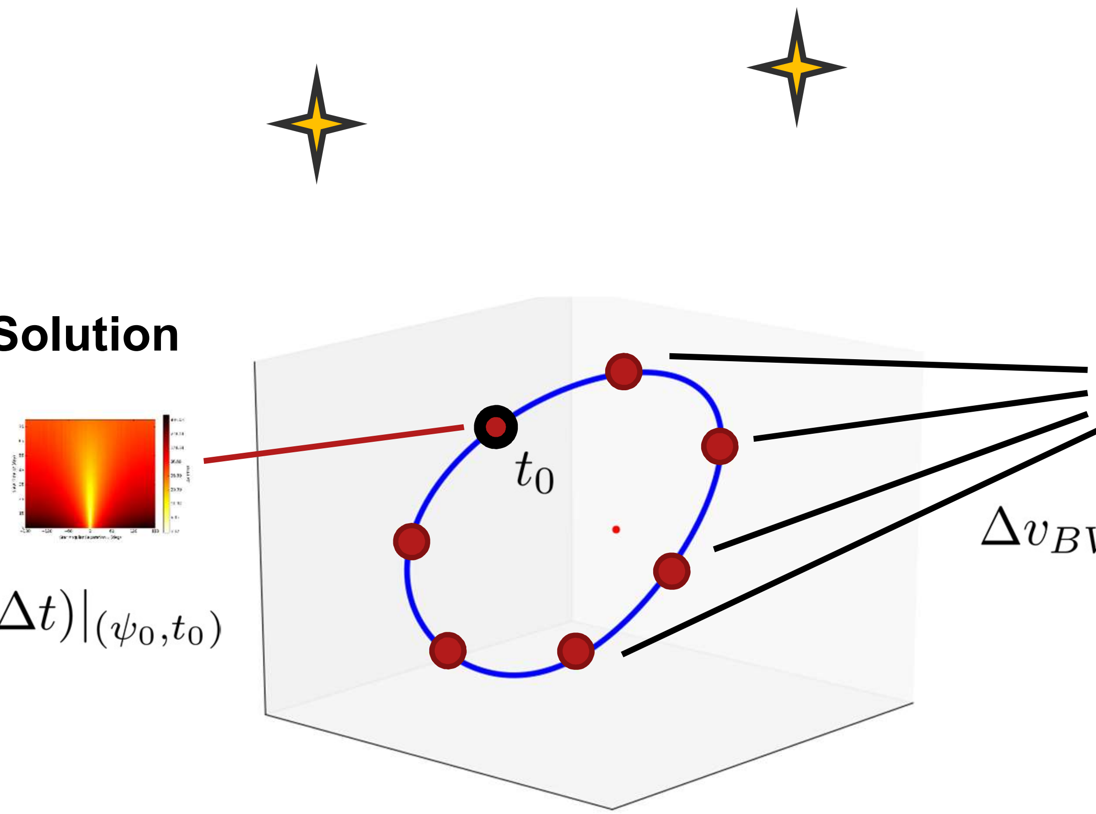
Impulsive Fuel Costs - Errors



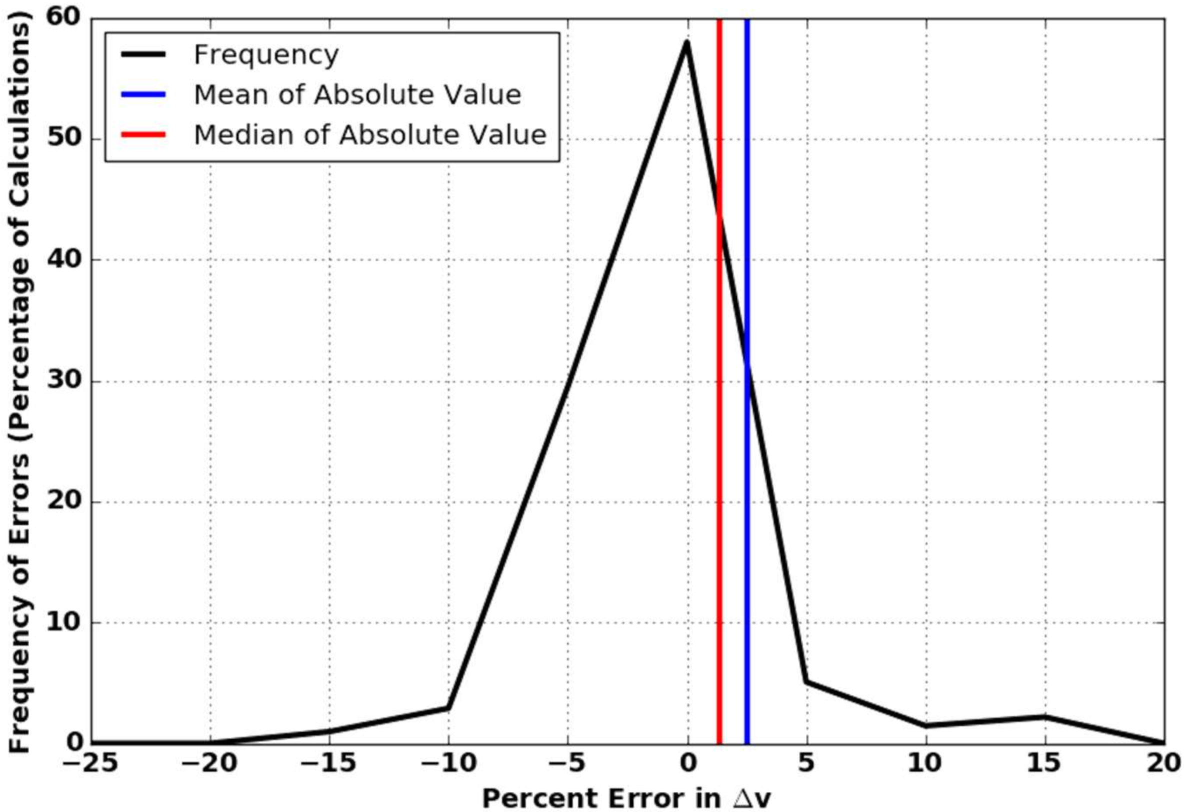
Interpolated Solution

$$\Delta v_{INT} = f(\Delta t)|_{(\psi_0, t_0)}$$

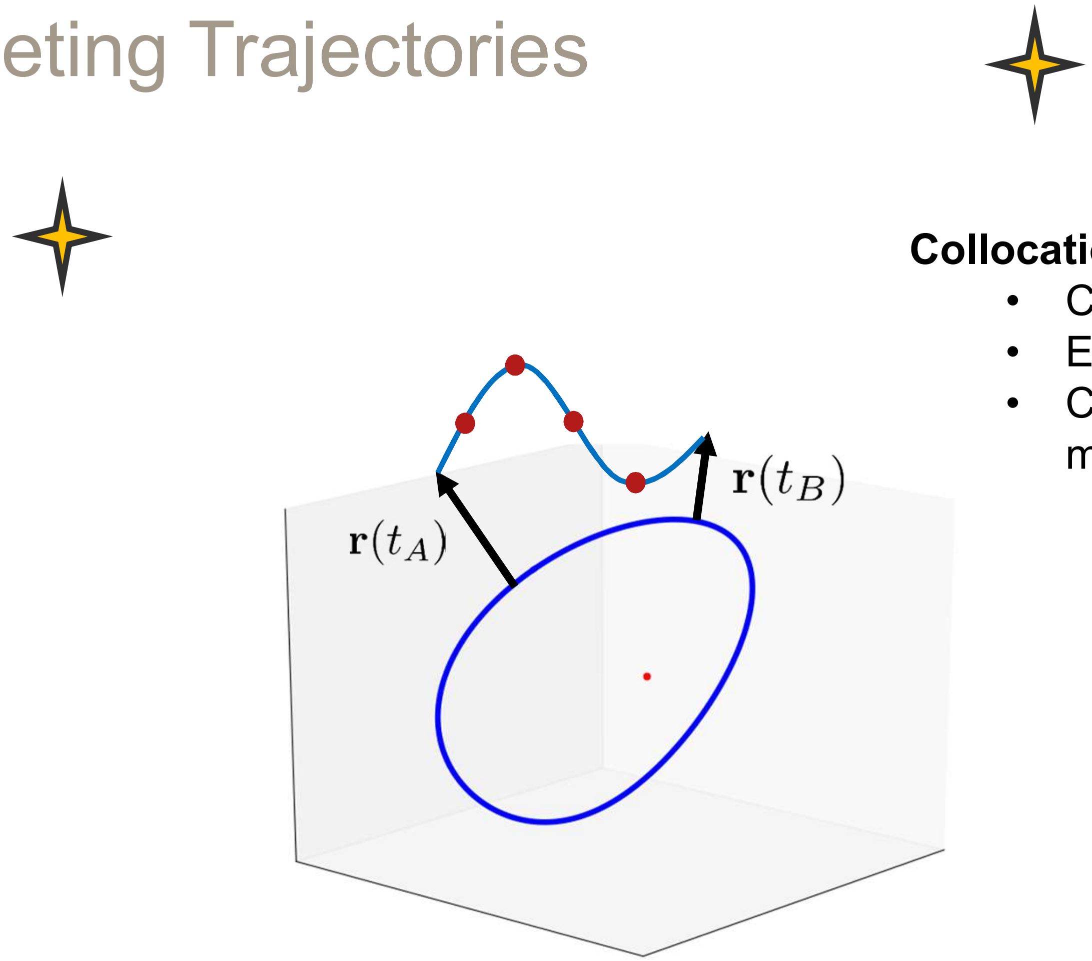
BVP Solution

$$\Delta v_{BVP} = f(\Delta t, t0)|_{\psi_0}$$

Parameterizing Fuel Costs - Errors



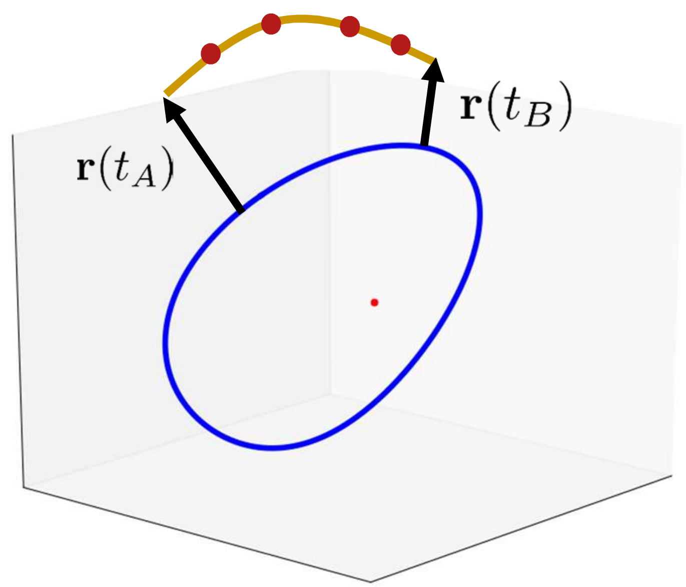
Retargeting Trajectories



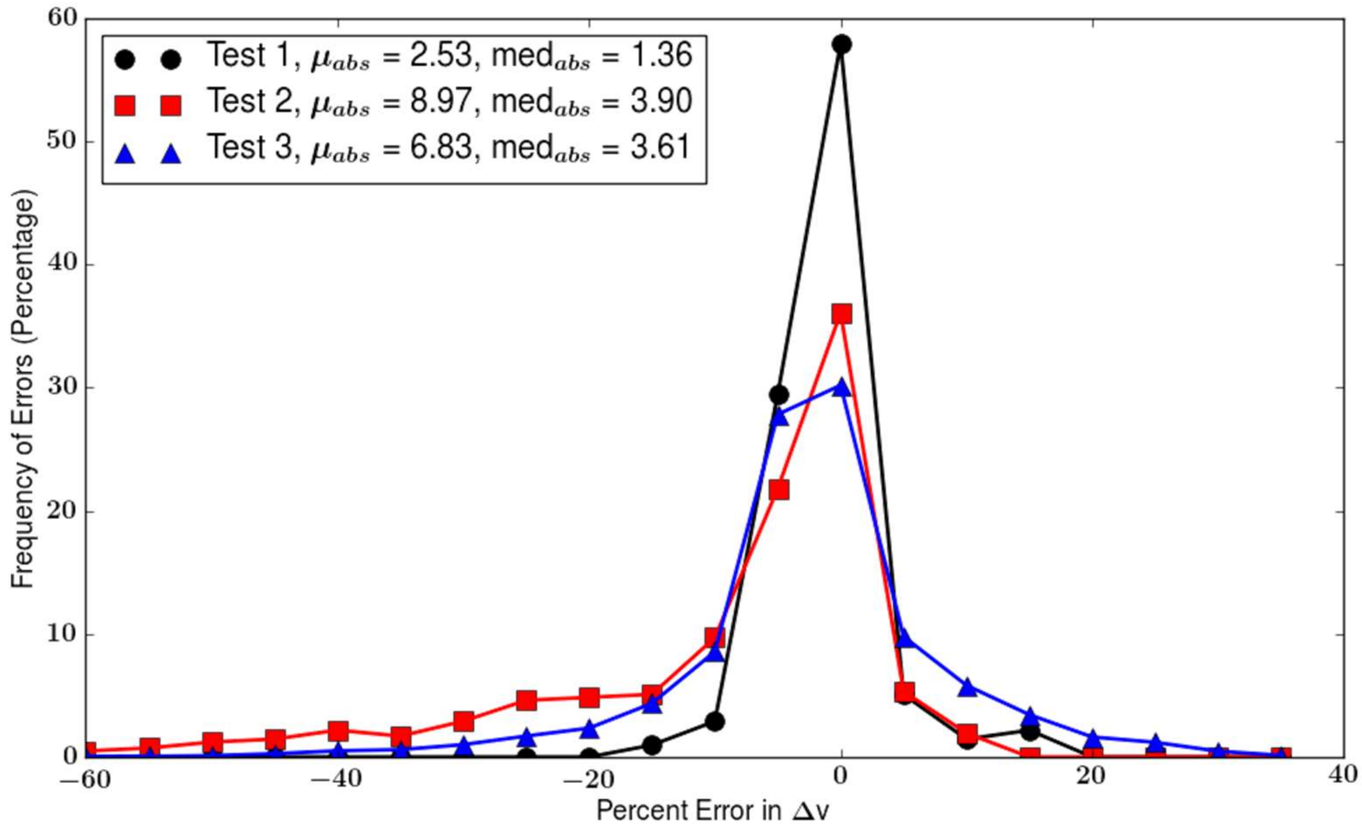
Collocation:

- Cubic polynomial
- Equal at endpoints
- Creates mesh and minimizes residual error

Retargeting Trajectories

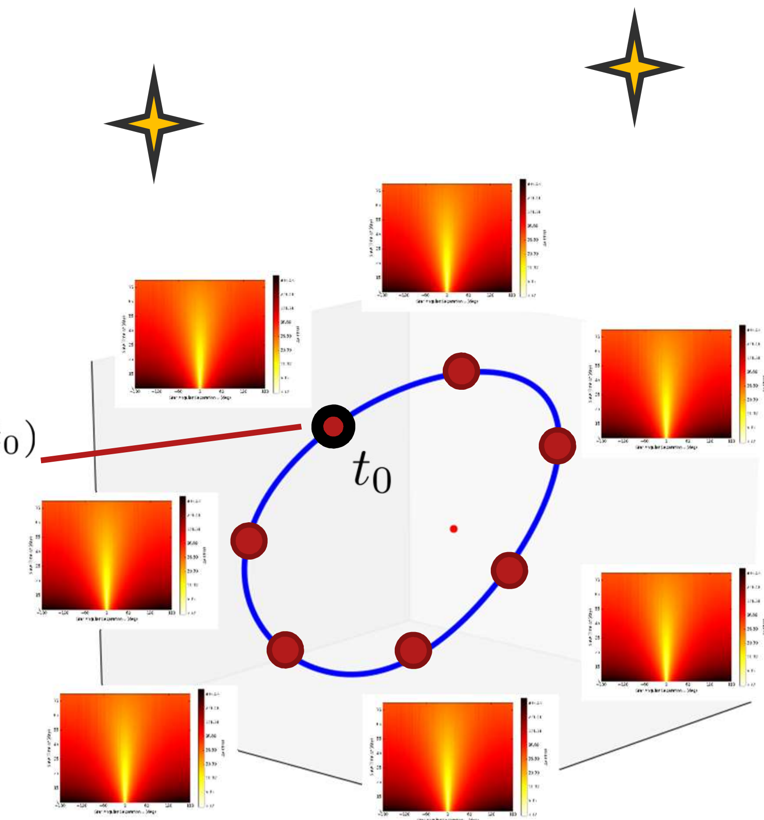


Error Analysis



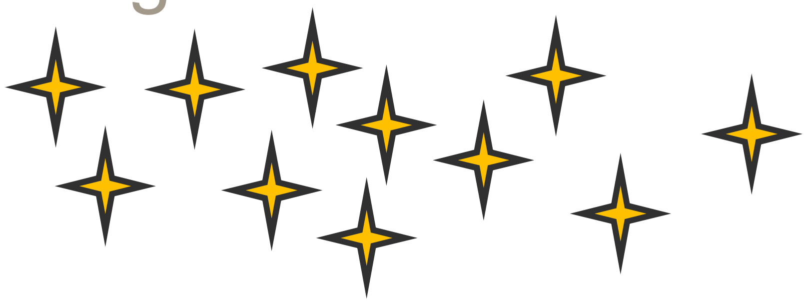
Parameterizing Fuel Costs - Errors

$$\Delta v_{BVP} = f(\Delta t)|_{(\psi_0, t_0)}$$

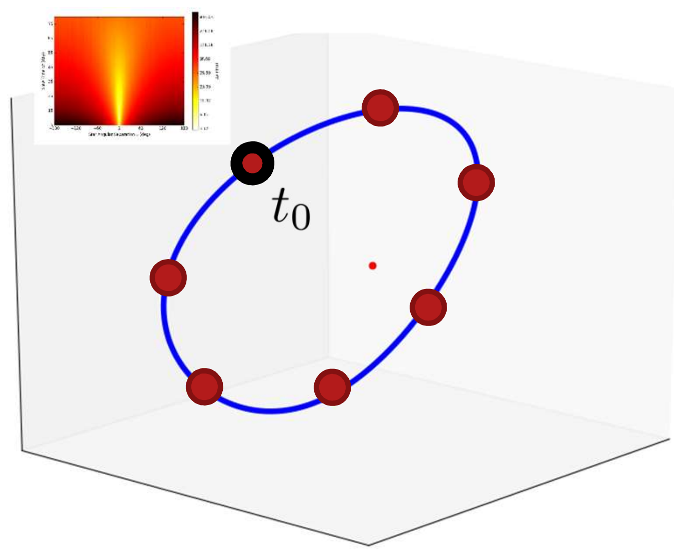


$$\Delta v_{INT} = f(\Delta t, t_0)|_{\psi_0}$$

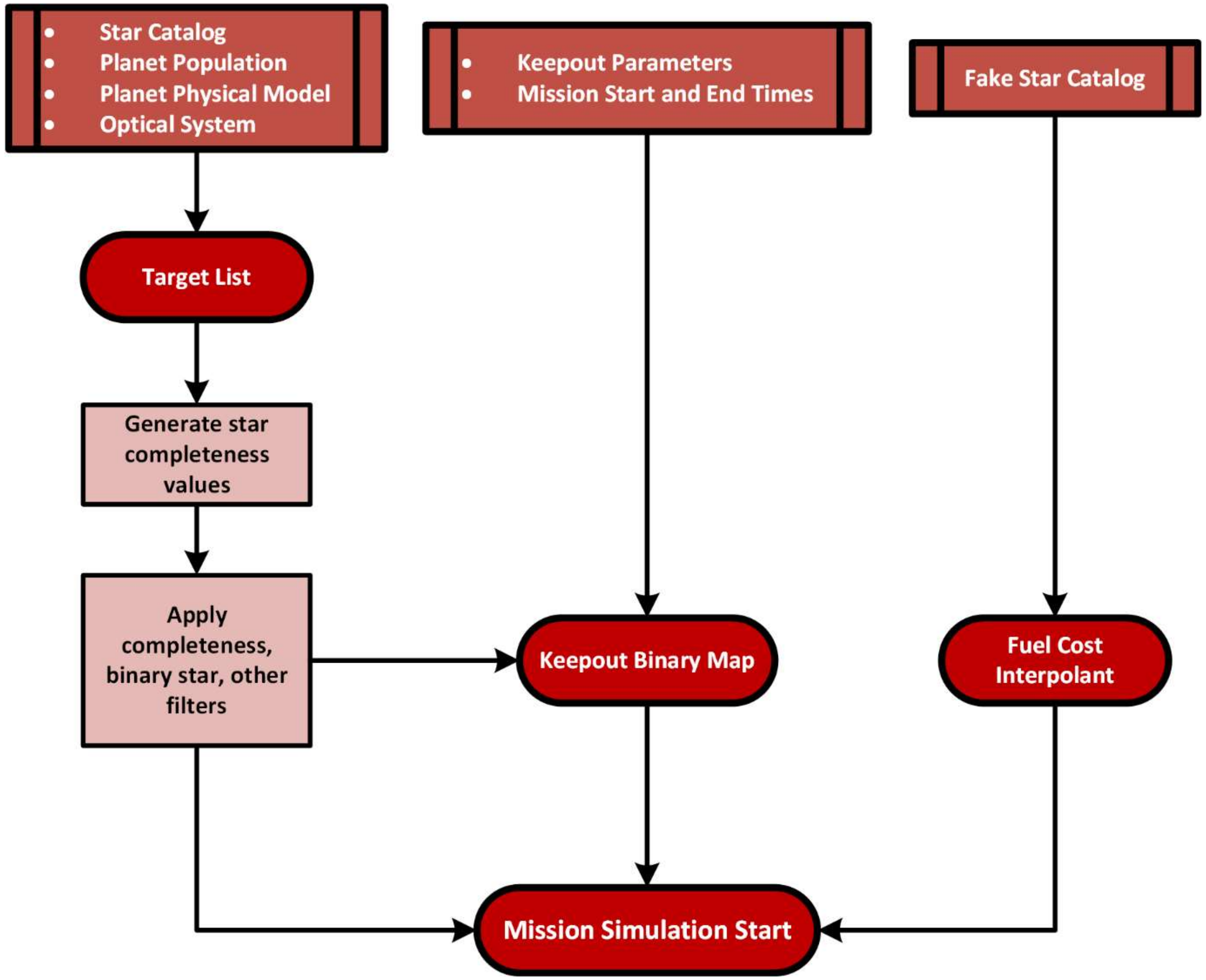
Parameterizing Fuel Costs - Errors



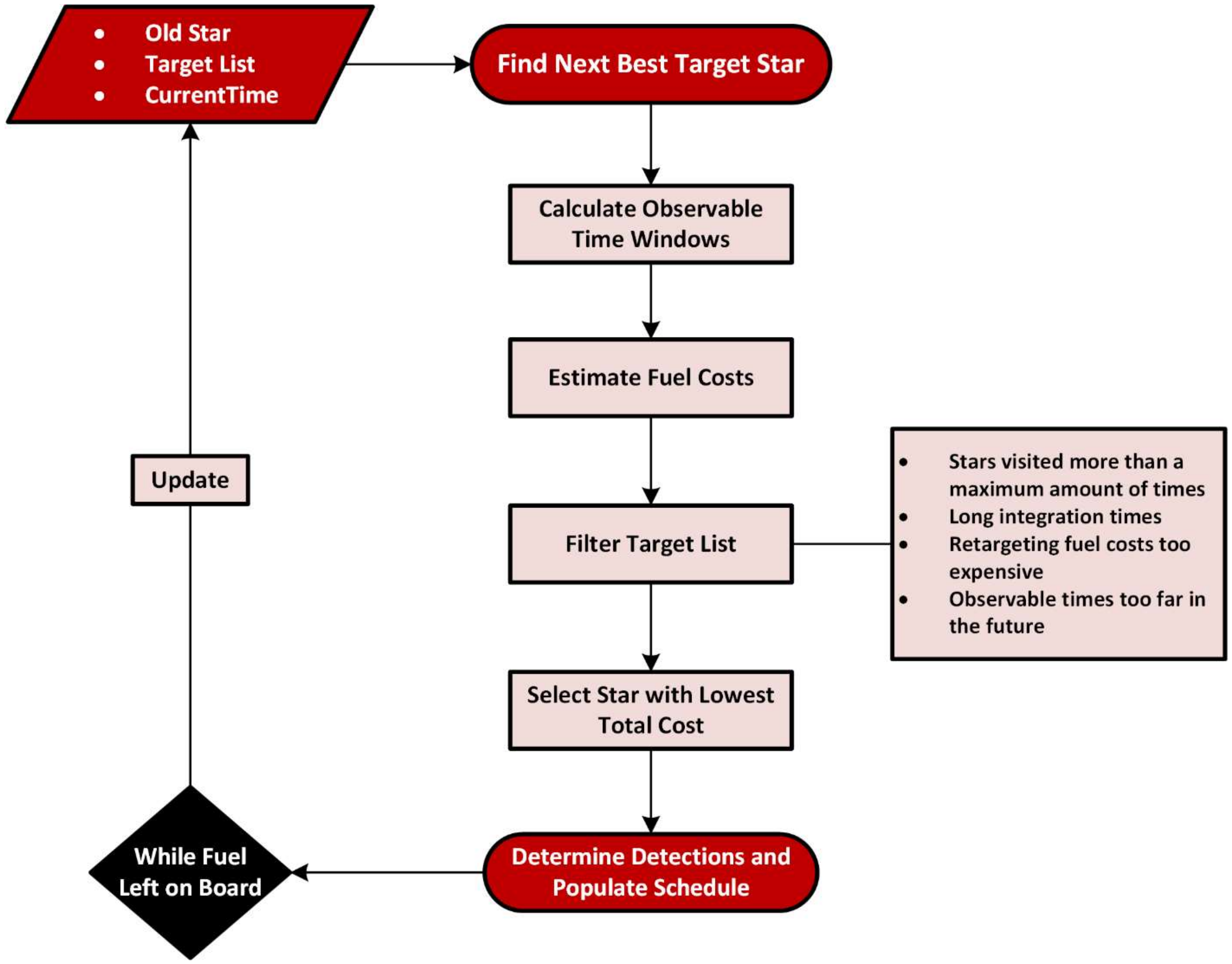
$$\Delta v_{BVP} = f(\psi, \Delta t, t_0)$$
$$\Delta v_{INT} = f(\psi, \Delta t, t_0)$$



Scheduler


Star Catalog
Planet Population
Planet Physical Model
Optical System
Keepout Parameters
Mission Start and End Times
Fake Star Catalog
Target List
Generate star completeness values
Apply completeness, binary star, other filters
Keepout Binary Map
Fuel Cost Interpolant
Mission Simulation Start

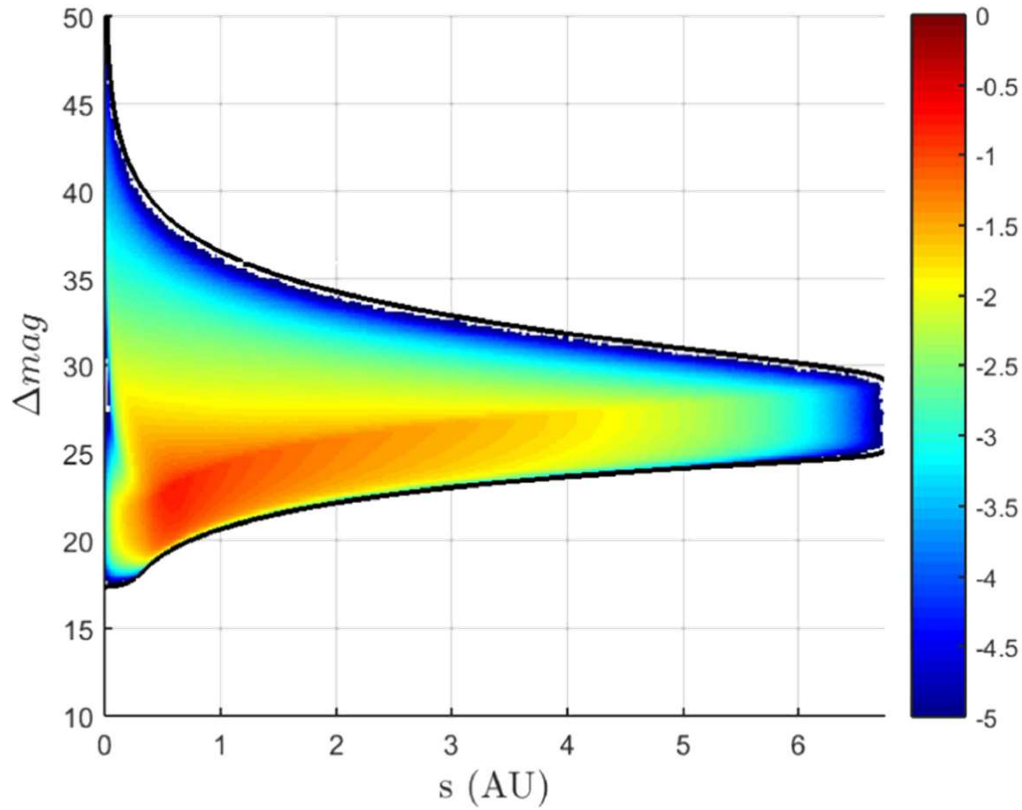
Scheduler



• Old Star
• Target List
• CurrentTime
Find Next Best Target Star
Calculate Observable Time Windows
Estimate Fuel Costs
Filter Target List
• Stars visited more than a maximum amount of times
• Long integration times
• Retargeting fuel costs too expensive
• Observable times too far in the future
Select Star with Lowest Total Cost
Determine Detections and Populate Schedule
While Fuel Left on Board
Update

Completeness

- Joint Probability Density function
  - Star-planet brightness difference
  - Star-planet projected separation
- Based on instrument parameters, integrate over region
- Probability that a planet with assumed parameters is observable near a star



Garrett, D. and Savransky, D. (2016) “Analytical Formulation of the Single-Visit Completeness Joint Probability Density Function”