## Hardware Demonstration of Starshade Formation Flying

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## Hardware Demonstration of Starshade Formation Flying

## Objectives:

© To experimentally validate formation sensing and control algorithms while maintaining high contrast with a flight-like starshade

Our approach:
© Princeton starshade testbed
© Pupil image sensor of diffraction pattern
© Discrete-time control and estimation

## Introduction

## Formation flying in starshade technology?



In science mode:
© The width of the deep of starshade's shadow depends on:

- size of the starshade
- separation distance
© The width can be resized but at a cost

Thus, better formation-keeping performance helps to maximize the scientific yield.

## Position Sensing

© Technology development needed in position sensing

- Need position accuracy $<10 \mathrm{~cm}$
- Centimeters over 10,000's kilometers separation is sub-mas angular measurement
© We can exploit nature of diffraction to avoid angular measurement
© Starshade's diffraction pattern moves one-to-one with starshade position over large distance
© Out-of-band light is bright and generates distinct Poisson spot - provides good guiding signal
© Pupil image is direct measurement of diffraction pattern


## S5 Work

© Lateral Sensing technology gap at TRL 5 (as of Nov 2018)
© SLATE testbed at JPL demonstrated position sensing accuracy $<30 \mathrm{~cm}$ and control accuracy $<1 \mathrm{~m}$ (flight equivalent)
© See Milestone \#4 Report by Flinois, Bottom, et al. (2018)
© Excellent work validating concept and showing robustness of sensing + control scheme
© Optical performance limited by small starshade and collimating optics

## Opportunity with Princeton testbed

© Existing testbed with flight-representative starshade
© Optical performance validated to $10^{-10}$ contrast level
© Can validate formation sensing and high contrast simultaneously
© Realistic optical performance allows exploration of new sensing techniques
© Starshade alignment with out-of-band light already used in high-contrast experiments
© Can be used for TRL 6+ work

## Princeton Starshade Testbed



Starshade diameter: 26 mm
Science channel: $\lambda=641 \mathrm{~nm}$
Guiding channel: $\lambda=405 \mathrm{~nm}$


## Formation Flying

## The formation flying problem


(a) Initial misalignment

(b) Final alignment

## Control and Estimation

## Control and estimation diagram

## Feedback control loop



## Equations of Motion

## The N-body problem for relative motion

The leader spacecraft:

$$
\begin{equation*}
\ddot{r}_{L}=-\sum_{i=1}^{N} \mu_{i}\left(\frac{r_{L i}}{\left\|r_{L i}\right\|^{3}}\right)+u_{L} \quad \text { for } \quad i=1,2, \ldots N \tag{1}
\end{equation*}
$$

The follower spacecraft:

$$
\begin{equation*}
\ddot{\boldsymbol{r}}_{F}=-\sum_{i=1}^{N} \mu_{i}\left(\frac{r_{F i}}{\left\|r_{F i}\right\|^{3}}\right)+u_{F} \quad \text { for } \quad i=1,2, \ldots N \tag{2}
\end{equation*}
$$

The term $\boldsymbol{u}_{X}$ correspond to control inputs.

## Equations of Motion

The relative motion is $x=r_{F}-r_{L}$
$\ddot{x}=-\sum_{i=1}^{N} \mu_{i}\left(\frac{r_{F i}}{\left\|r_{F i}\right\|^{3}}-\frac{r_{L i}}{\left\|r_{L i}\right\|^{3}}\right)+\Delta u+w \quad$ for $\quad i=1,2, \ldots N$
© $\boldsymbol{w} \sim \mathcal{N}\left(\mu_{S R P}, \sigma\right)$ adds uncertainty (normal random noise)
© $\mu_{S R P}$ is the average of the differential acceleration caused by the solar radiation pressure
© According to the configuration of our laboratory, we assume:

- The starshade is the leader in free motion
- The telescope is the follower in controlled motion
- Therefore $\Delta \boldsymbol{u}=\boldsymbol{u}_{F}-\boldsymbol{u}_{L^{*}}=\frac{0}{=} \boldsymbol{u}_{F}$


## Control Strategy

## Linear quadratic regulator with integral action



## Estimation Strategy

## Unscented Kalman filter



Other relevant features:
© Approximates the actual motion as:
$\ddot{x}=-\sum_{i=1}^{N} \mu_{i}\left(\frac{r_{F i}}{\left\|r_{F i}\right\|^{3}}-\frac{r_{L i}}{\left\|r_{L i}\right\|^{3}}\right)+\Delta \boldsymbol{u}+\boldsymbol{z}_{\boldsymbol{\psi}^{-0}}$
© Fed by the sensors and the controller

## Pupil Sensor

## Some background


© Deep shadow $\sim 1 \mathrm{~m}$ larger than telescope (in radius)
© Formation-keeping needs lateral position error $\lesssim 10 \mathrm{~cm}$
© For 1000's km of separation it means milliarcsec measurement
© Direct sampling the one-to-one diffraction pattern of the starshade

## Pupil Sensor



© Starshade's diffraction pattern approximated by a Bessel function:

$$
\begin{equation*}
I(x, y) \approx J_{0}^{2}\left(\frac{2 \pi R \sqrt{\left(x-x_{s}\right)^{2}+\left(y-y_{s}\right)^{2}}}{\lambda z}\right) \tag{4}
\end{equation*}
$$

© $x_{s}$ and $y_{s}$ solved via non-linear least squares

## Hardware-in-the-loop simulation

## Hardware-in-the-loop simulation features

© Intersatellite separation $26,000 \mathrm{~km}$
© For in-plane readings $<1$ m use pupil sensor
© For intersatellite separation and in-plane readings $>1 \mathrm{~m}$ :

- Star-tracking sensor
- Normal distributed random signal with a $\pm 5 \mathrm{~m}$ error
© Target star is Epsilon Eridani
© Maximum thrust firing of 1 N
© Simulation time 2 hour
© 10 celestial bodies from JPL's ephemeris DE432


## Experiment Procedure

© Start camera with initial misalignment
© Begin control loop...

- Switch to pupil plane camera
- Switch to out-of-band wavelength channel
- Take pupil image and extract position
- Or get position from noisy star tracker if at large offset
- Generate control signal + move camera
- Switch to focal plane camera
- Switch to science-band wavelength channel
- Take contrast image with focal plane camera


## Results



Focal Plane


## Results

## Position error in the line-of-sight frame

© Starting offset: 50 m
© Median pupil sensor error: 16 cm



## Results

## Control signals

(0 Total $\Delta v=0.89 \mathrm{~m} / \mathrm{s}$



## Conclusions

© Combination of an LQR+i and UKF
© High precision position sensing using a pupil image sensor
© Favorable performance includes:

- Position error convergence within tolerances
- Robust steady-state
- Low values of total $\Delta v$
© Initial results suggest the validity of formation flying approach in starshade technology


## Future Work

© Implement control scheme that maximizes observing time (minimizes thruster firings)
© Add beacon on starshade to do focal plane "retargeting" control
© Add control in "putting" region and implement proper sensor handoff
© Investigate position sensing in "putting" region
© Investigate other position extraction schemes (e.g., neural net)

## THANK YOU

## FOR YOUR ATTENTION

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