The search for another Earth using space telescopes with starshades: realistic image simulation and signal detection

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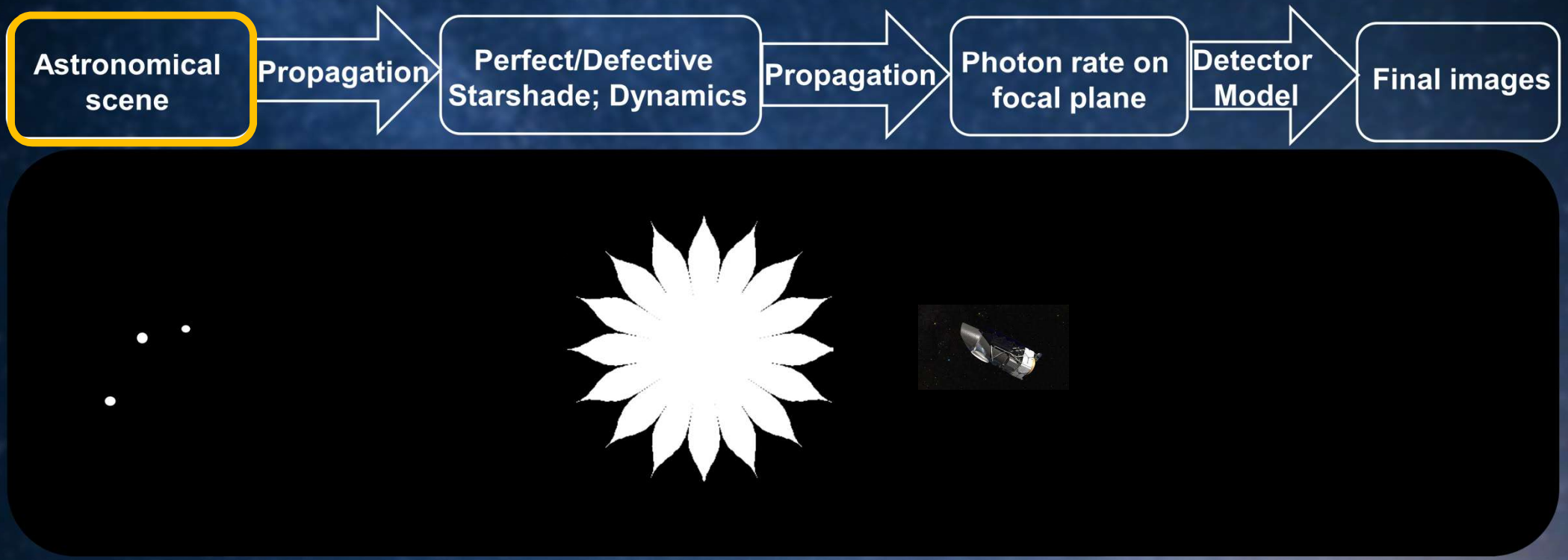
OUTLINE

- Image Simulation
  - Simulation Process
  - Photon Counting Mode
- Image Processing
  - Generalized Likelihood Ratio Test (GLRT)
  - Sequential GLRT (SGLRT) for Photon Counting Images

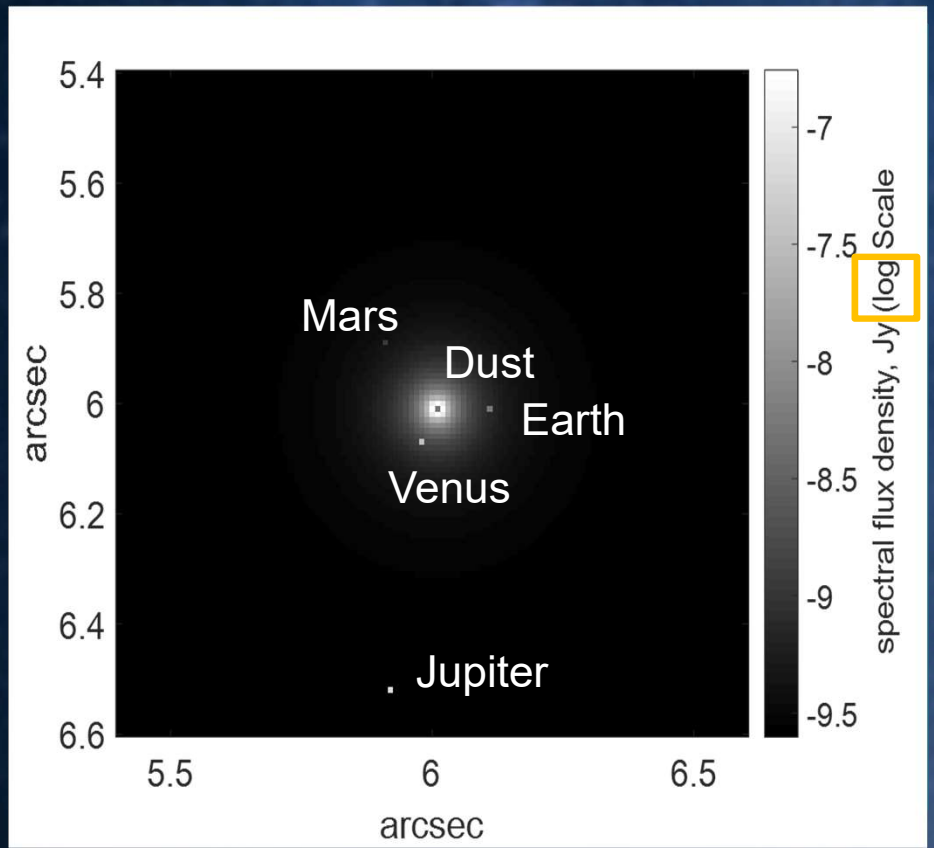
Image Simulation Process



Ref. Hu, M. M., Harness, A. D., Kim, Y., Kasdin, N. J., Vanderbei, R., Rizzo, M. J., & Roberge, A. (2017, September). Simulation of realistic images for Starshade missions. In *Techniques and Instrumentation for Detection of Exoplanets VIII* (Vol. 10400, p. 104001S). International Society for Optics and Photonics.

Simulation Input: Astronomical Scene

Original astronomical scene (Solar system) with the star 10^{10} times dimmer at 600nm wavelength (zoom-in for the scene)



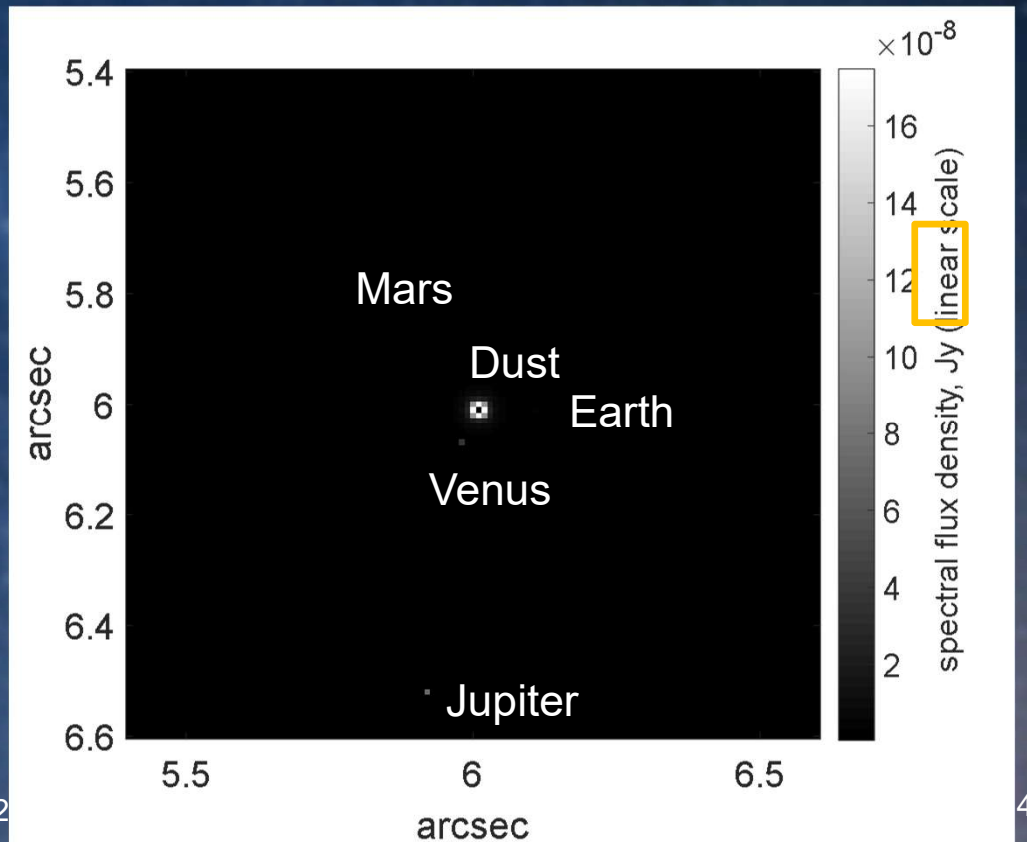
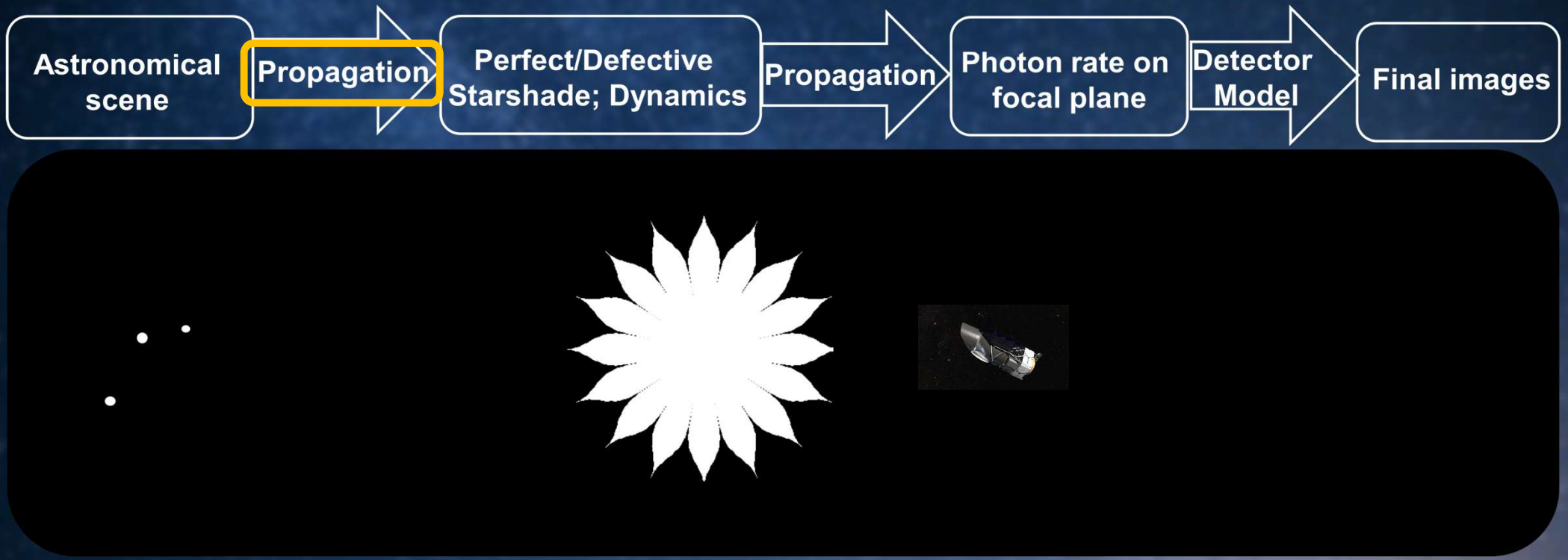
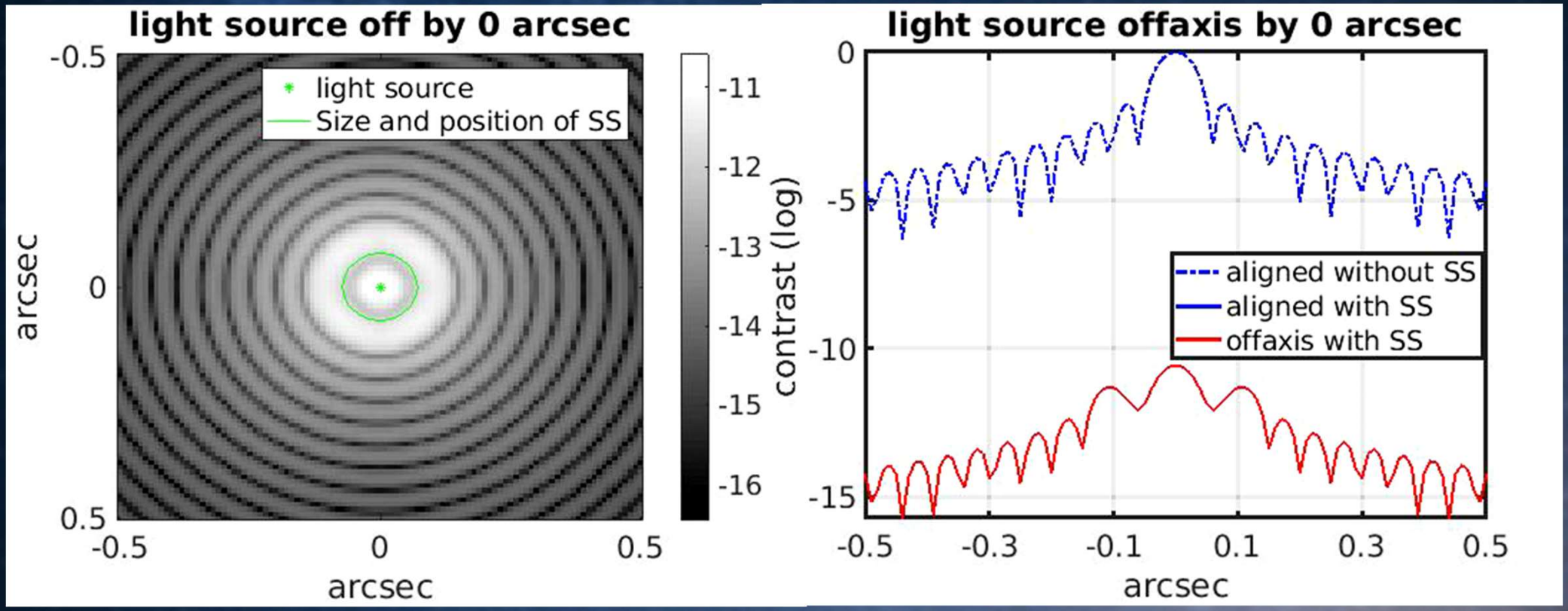


Image Simulation Process

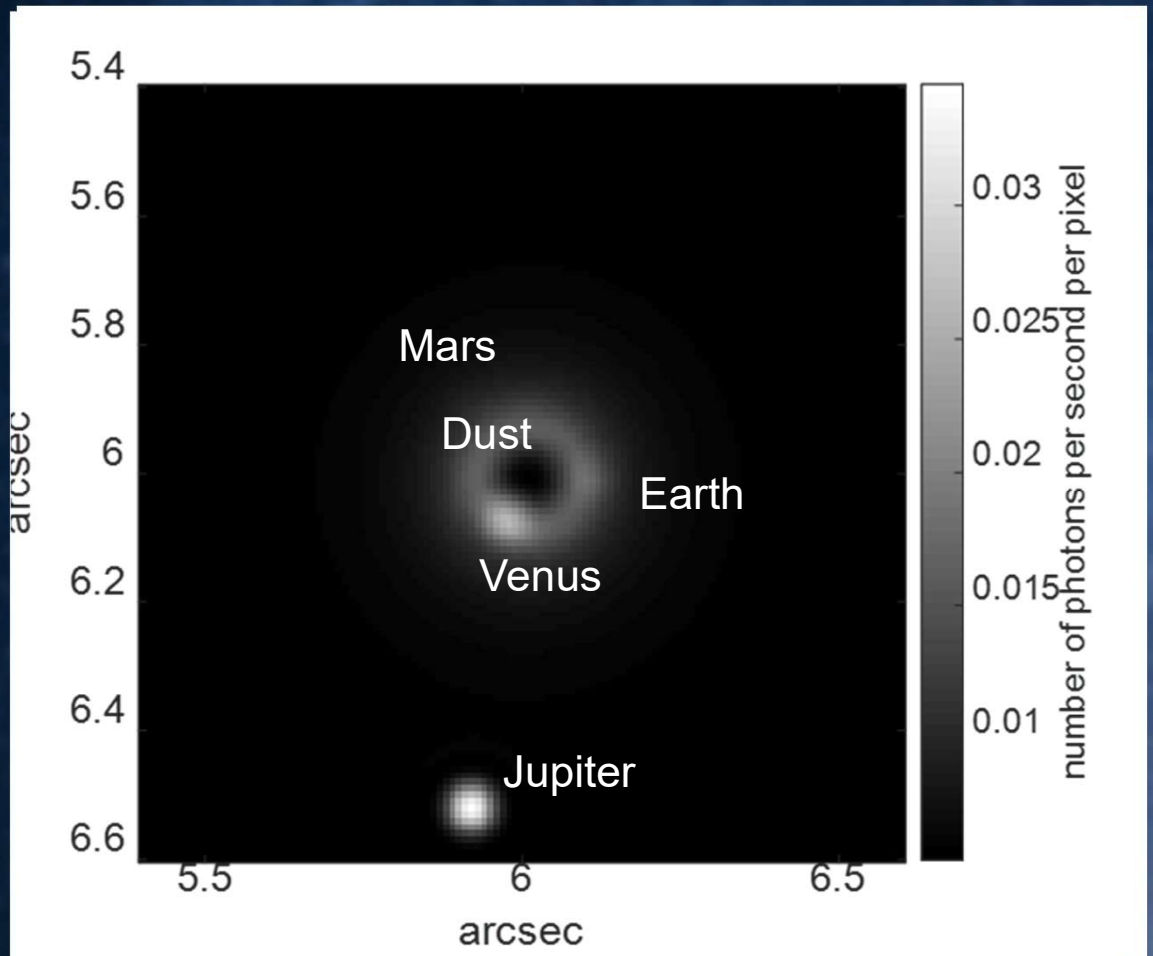


Ref. Hu, M. M., Harness, A. D., Kim, Y., Kasdin, N. J., Vanderbei, R., Rizzo, M. J., & Roberge, A. (2017, September). Simulation of realistic images for Starshade missions. In *Techniques and Instrumentation for Detection of Exoplanets VIII* (Vol. 10400, p. 104001S). International Society for Optics and Photonics.

PSF with a Starshade

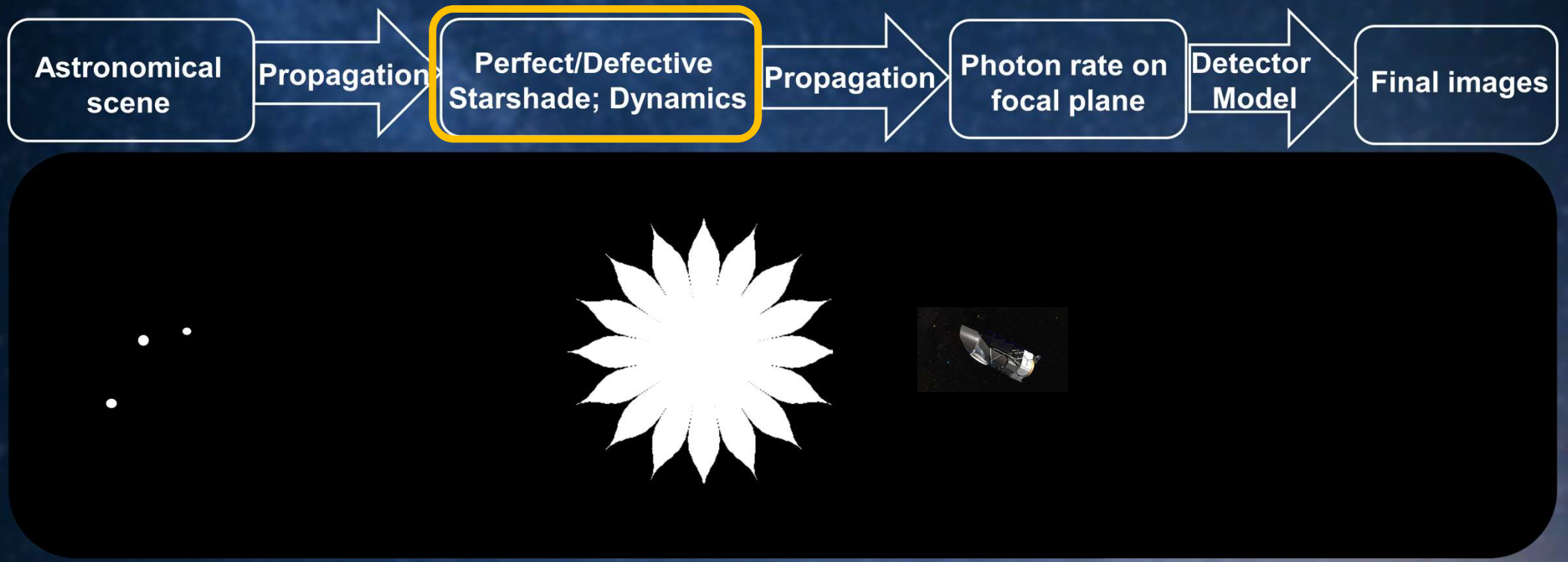


Simulation Simplification



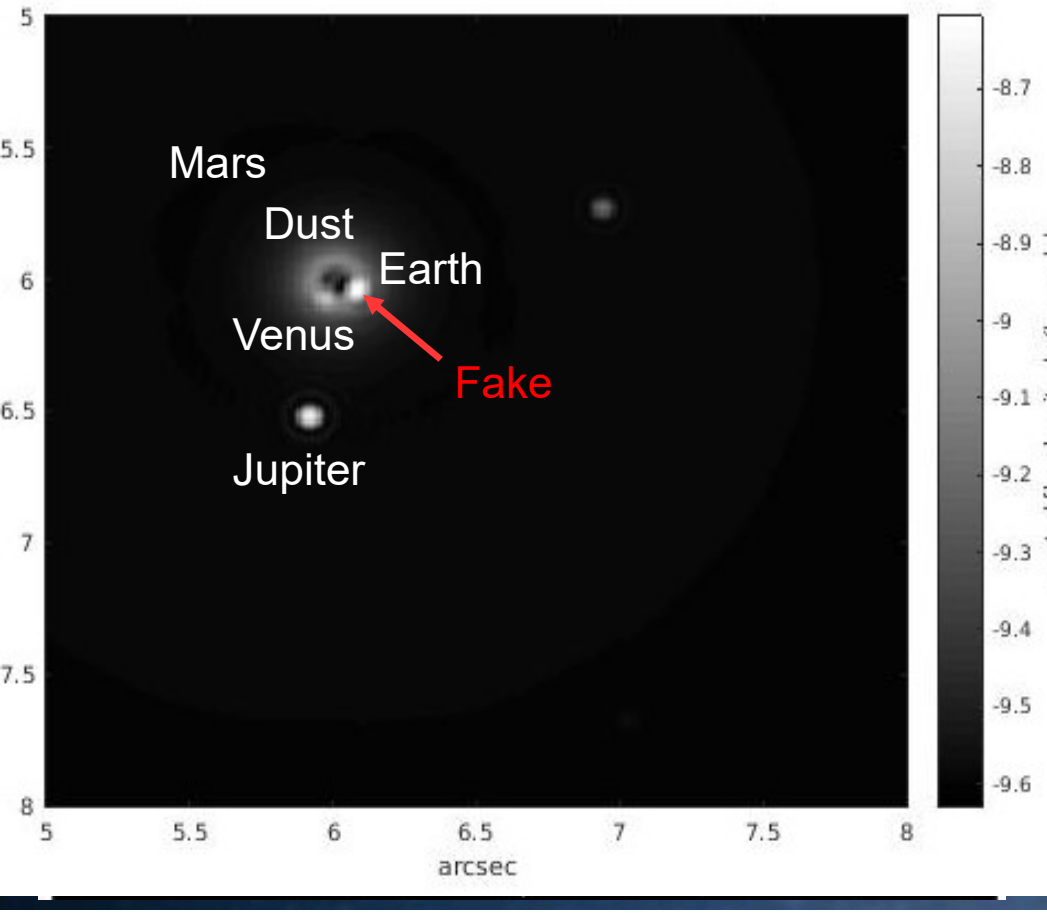
Simulated image for the perfect case *with all wavelengths (600nm~1100nm)*

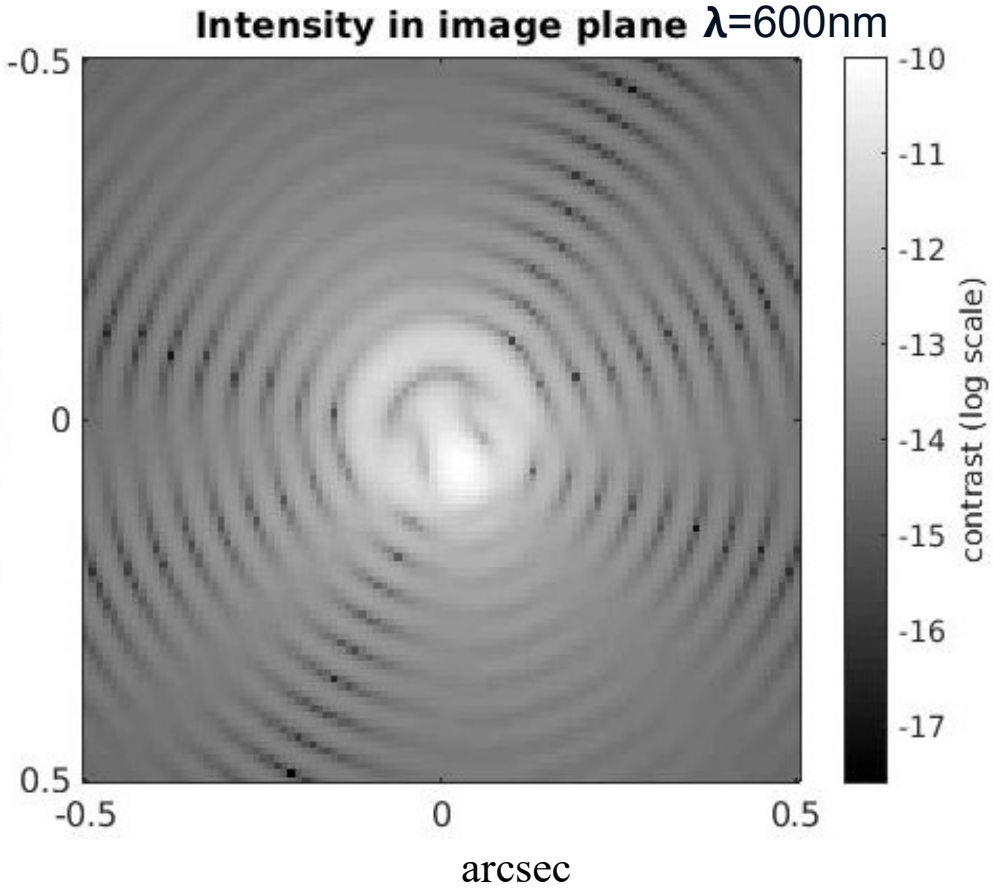
Image Simulation Process



Ref. Hu, M. M., Harness, A. D., Kim, Y., Kasdin, N. J., Vanderbei, R., Rizzo, M. J., & Roberge, A. (2017, September). Simulation of realistic images for Starshade missions. In *Techniques and Instrumentation for Detection of Exoplanets VIII* (Vol. 10400, p. 104001S). International Society for Optics and Photonics.

Errors On Starshade: Truncated Tip





Starshade Dynamics (Misalignment)

λ=600nm

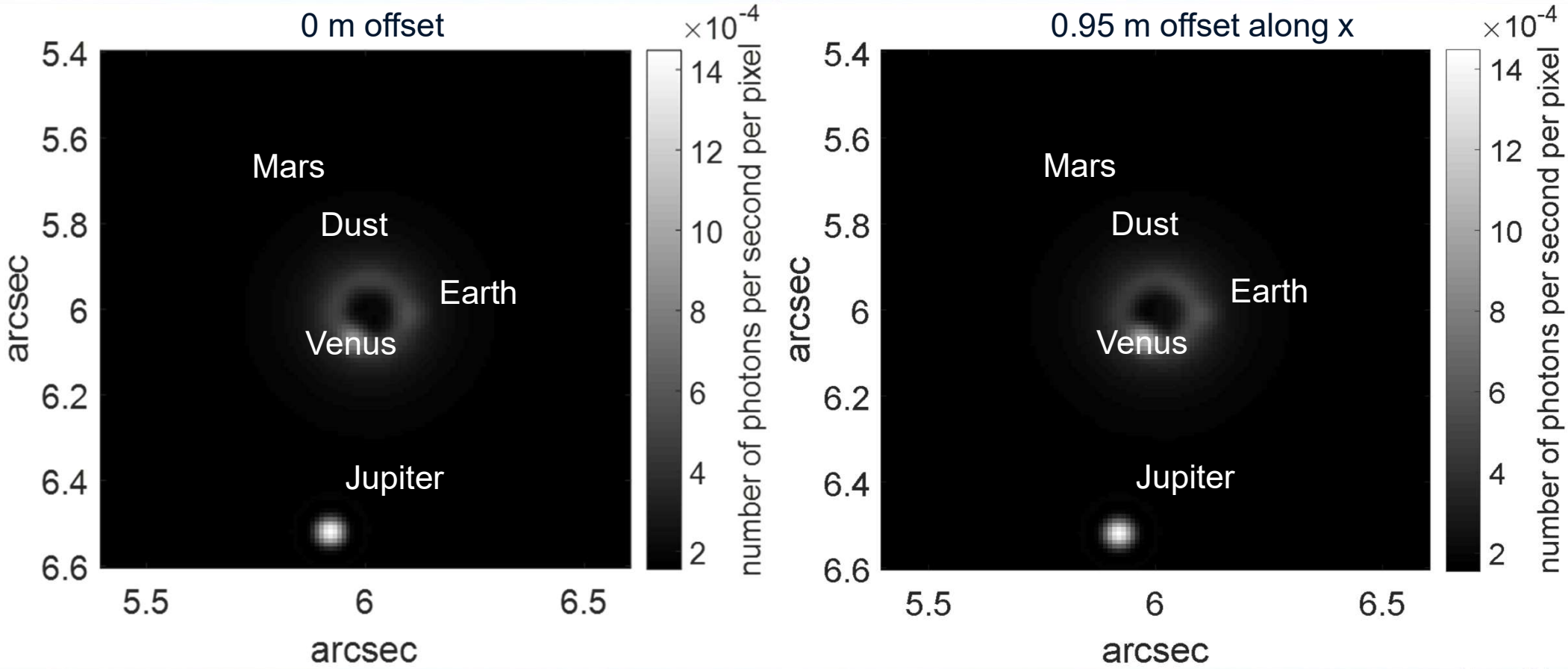
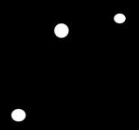
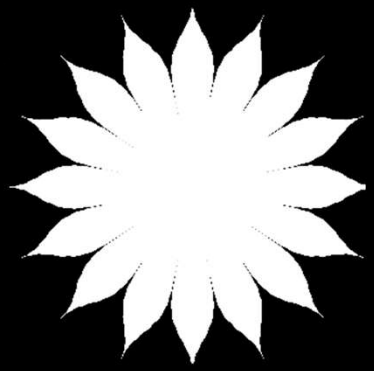
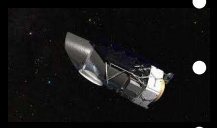


Image Simulation Process



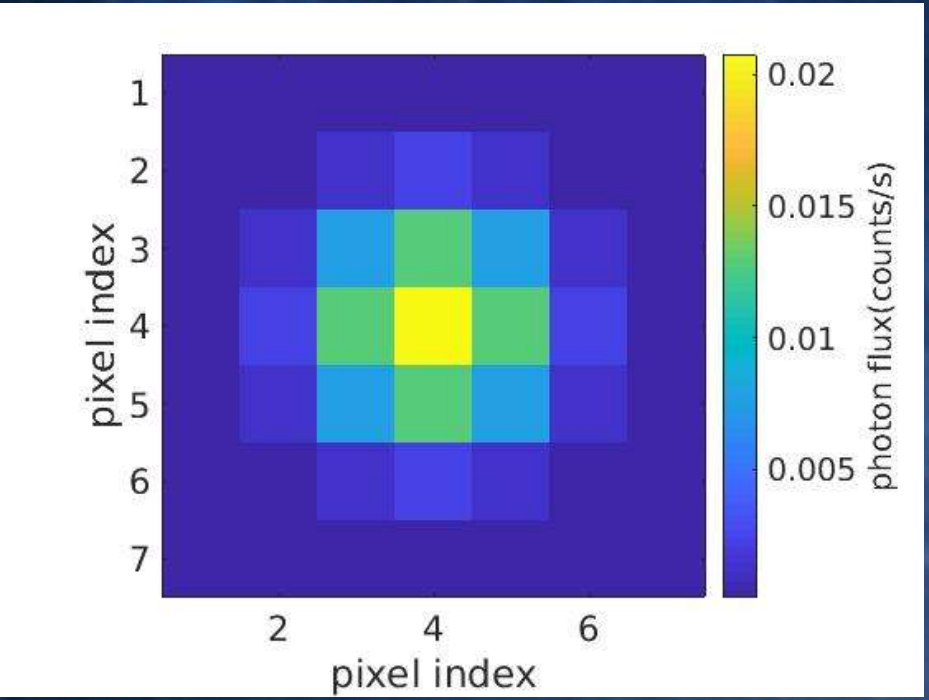




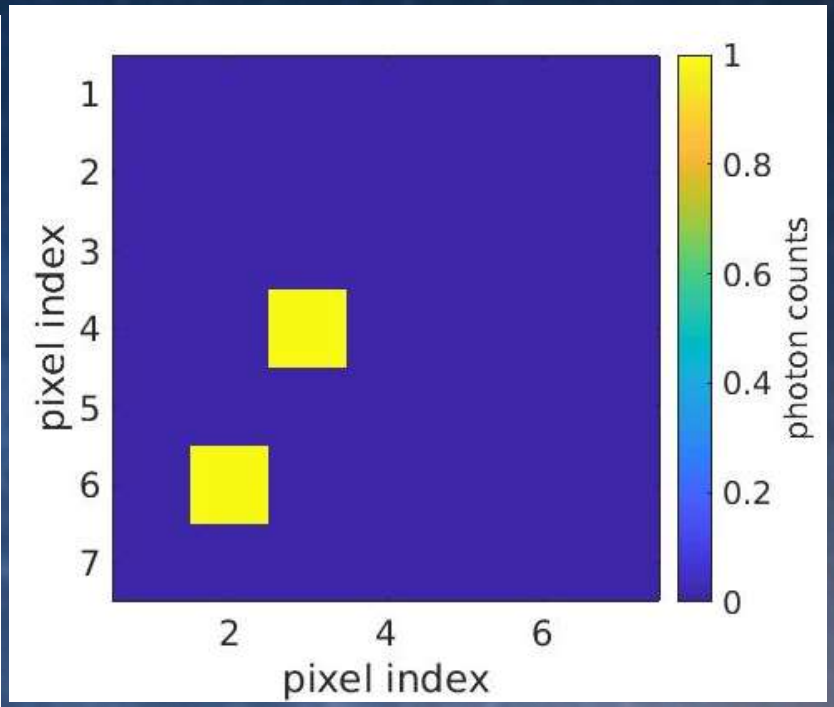
- Quantum Efficiency
- Photon counting mode
- Dark current
- Clock-induced charge
- Read noise
- Photon counting bias

Ref. Hu, M. M., Harness, A. D., Kim, Y., Kasdin, N. J., Vanderbei, R., Rizzo, M. J., & Roberge, A. (2017, September). Simulation of realistic images for Starshade missions. In *Techniques and Instrumentation for Detection of Exoplanets VIII* (Vol. 10400, p. 104001S). International Society for Optics and Photonics.

Photon Counting Mode

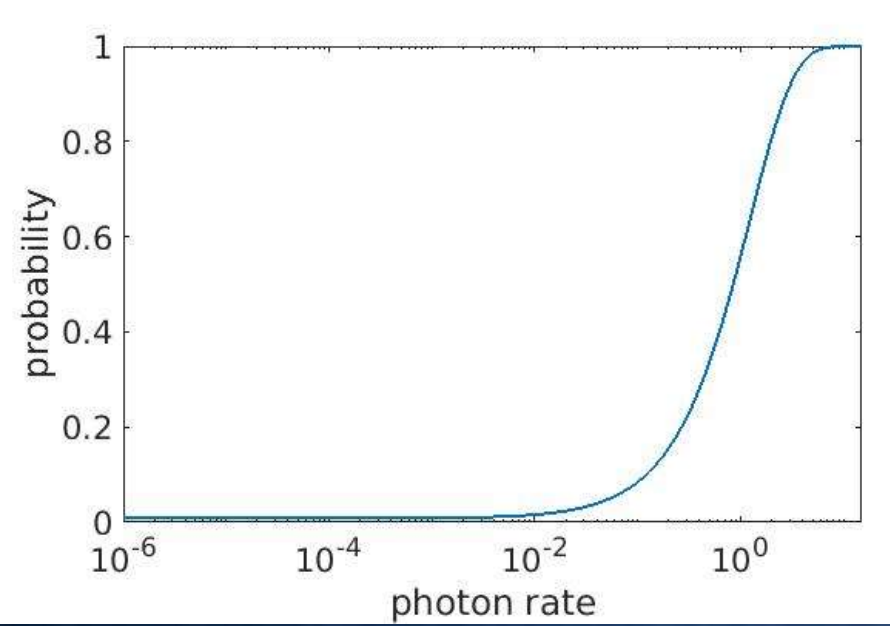


Simulated WFIRST's PSF for 1e-8 Jy light source. 0.021 arcsec/pixel.

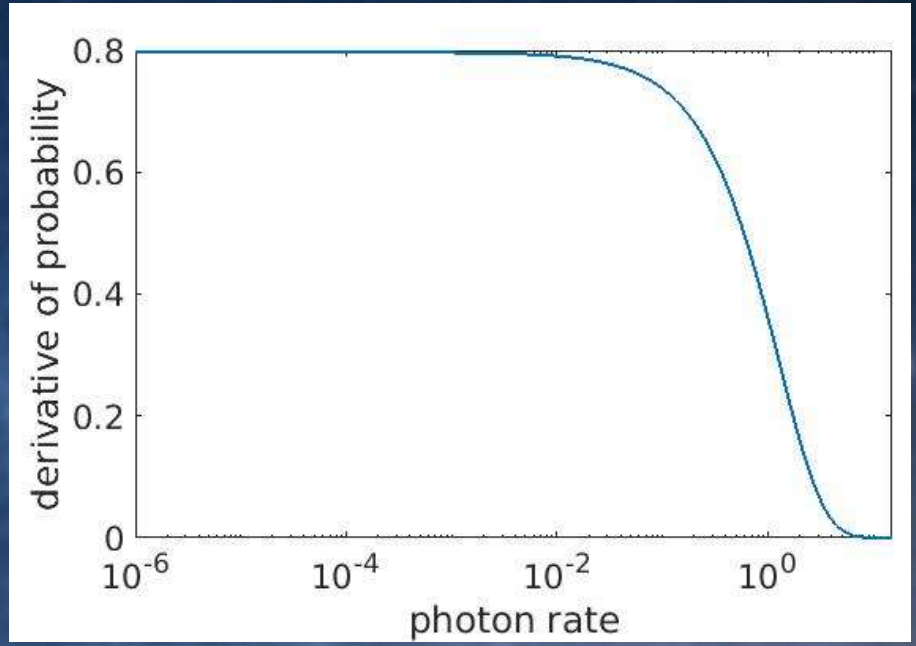


One photon counting (PC) image with 1s integration time

Photon Counting Mode

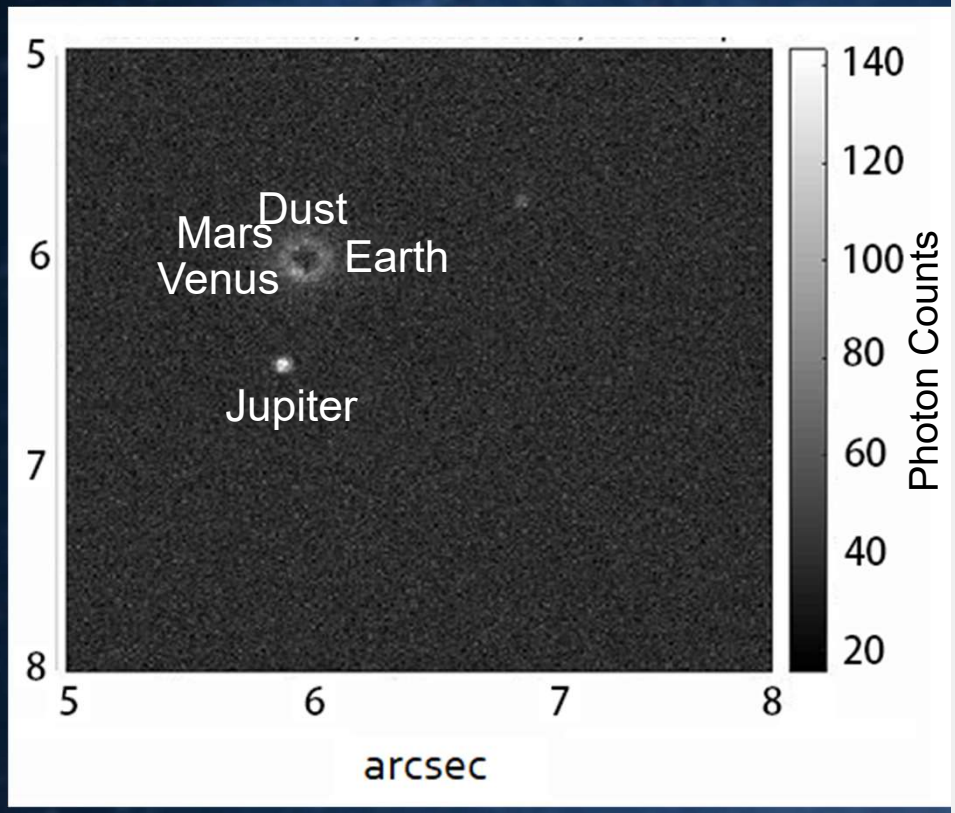


The probability of getting value one on the pixel for the PC image vs. the photon rate (counts/s) for WFIRST's detector.

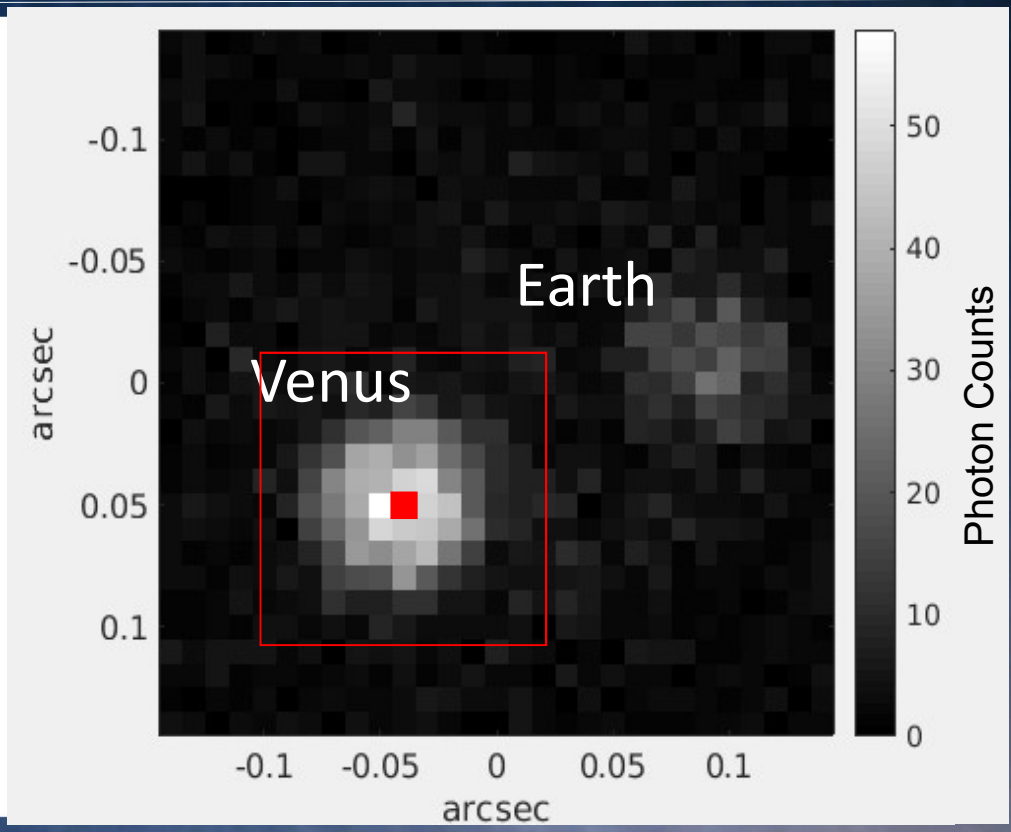


The derivative of probability of getting value one on the pixel for the PC image vs. the photon rate (counts/s) for WFIRST's detector.

Simulated Images



Simulated image for WFIRST with a perfect starshade



Zoom in & without exozodiacal dust

Generalized Likelihood Ratio Test

Maximum Likelihood Estimation

- Model: Image x = Intensity · PSF + Background + Noise
- Testing:

H_0 : Intensity = 0

H_1 : Intensity > 0

Decided by False Alarm Rate

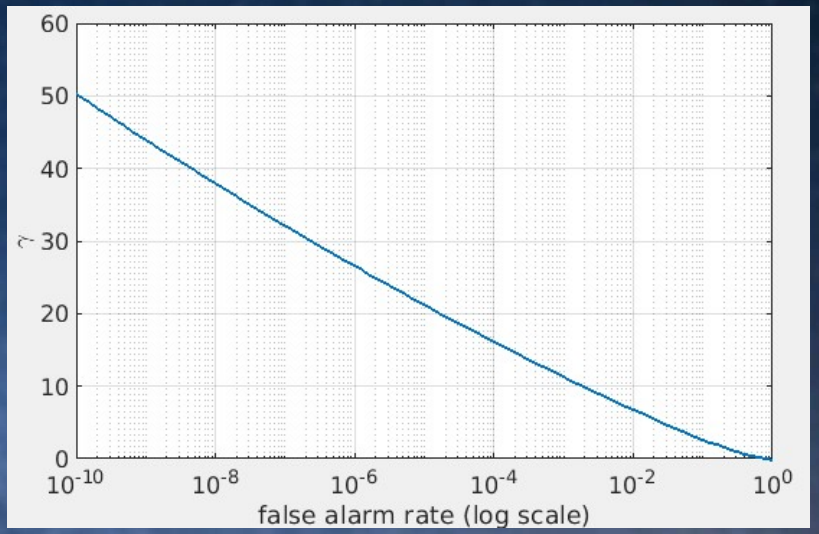
Reject H_0 , if $L_G(\boldsymbol{x}) = \frac{max_{\boldsymbol{\theta}} P(x\,|H_1)}{max_{\boldsymbol{\theta}} P(x|H_0)}$ > Threshold

Ref. Hu, M. M., Harness, A., & Kasdin, N. J. (2018, July). Image processing methods for exoplanets detection and characterization in starshade observations. In Space Telescopes and Instrumentation 2018: Optical, Infrared, and Millimeter Wave (Vol. 10698, p. 106985K). International Society for Optics and Photonics.

Kay, S. M. Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory. 1998.

Generalized Likelihood Ratio Test

- Assumption: Constant background;
  Gaussian noise
- Model: $\boldsymbol{x} = \underbrace{\begin{bmatrix} \boldsymbol{PSF}_1 & 1 \\ \vdots & \vdots \\ \boldsymbol{PSF}_N & 1 \end{bmatrix}}_{\boldsymbol{H}} \underbrace{\begin{bmatrix} I \\ b \end{bmatrix}}_{\boldsymbol{\theta}} + \boldsymbol{w}$
- Testing: $H_0: I = \underbrace{[1 \quad 0]}_{\boldsymbol{A}} \boldsymbol{\theta} = 0\,, \sigma^2 > 0;$
  $H_1: \boldsymbol{A\theta} > 0, \sigma^2 > 0$



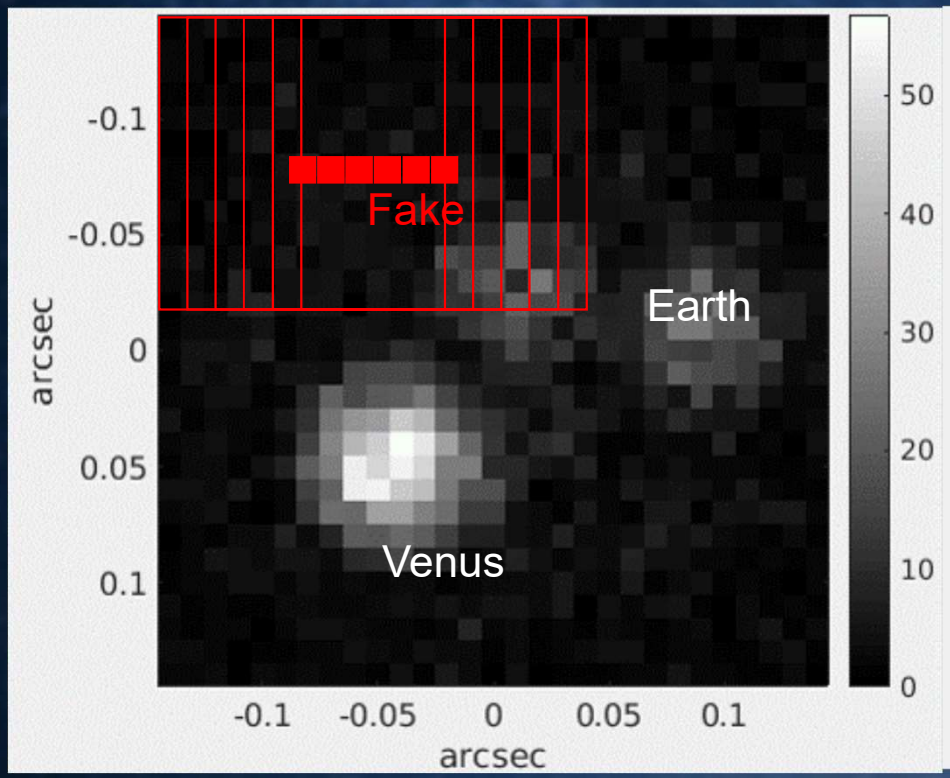
- Reject H_0, if $T(\boldsymbol{x}) = (N-2)\left(L_G(\boldsymbol{x})^{\frac{2}{N}} - 1\right) = \dfrac{\widehat{\boldsymbol{\theta}_1}^T \boldsymbol{A}^T [\boldsymbol{A}(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{A}^T]^{-1} \boldsymbol{A}\widehat{\boldsymbol{\theta}_1}}{\boldsymbol{x}^T(\boldsymbol{I}_N - \boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T)\boldsymbol{x}} > \gamma$

where $L_G(\boldsymbol{x}) = \dfrac{max_{\boldsymbol{\theta}}\, P(x\,|H_1)}{max_{\boldsymbol{\theta}}\, P(x|H_0)}$ and the ***false positive rate*** $P_{FA} = Q_{F_{1,N-2}}(\gamma)$

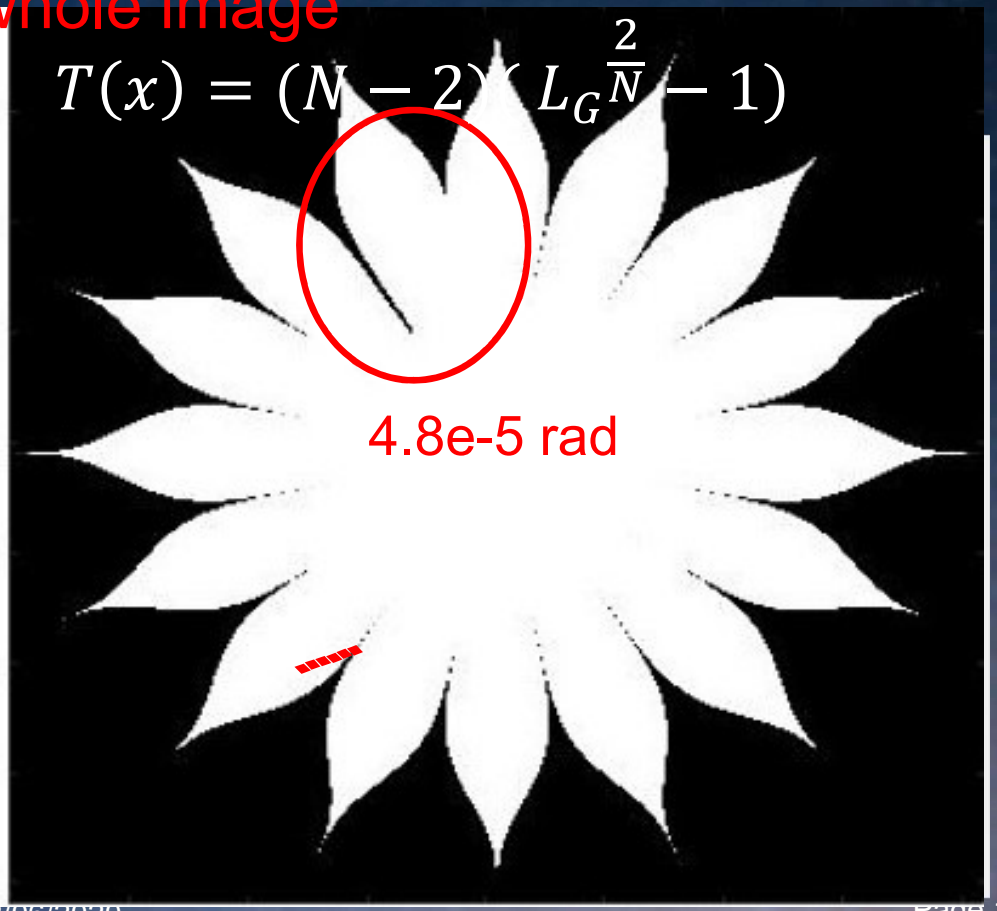
GLRT Results

Traverse the whole image

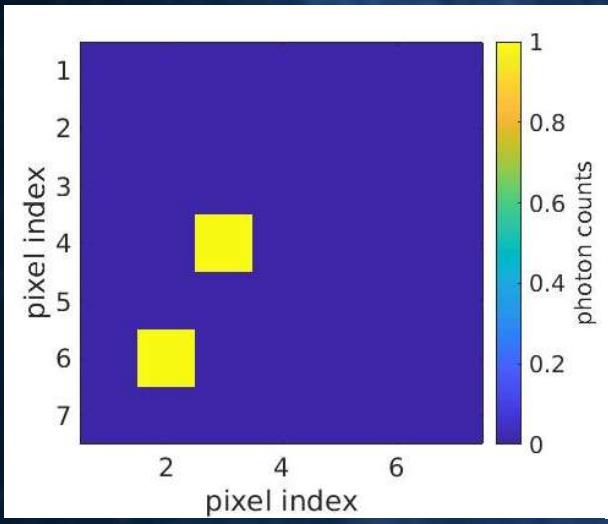
Image with a clocking petal [



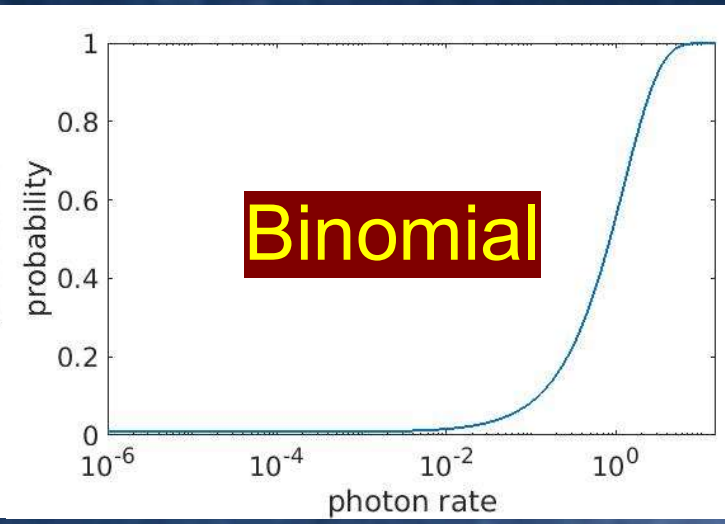
$$T(x) = (N-2)^{\left(L_G^{\frac{2}{N}}-1\right)}$$



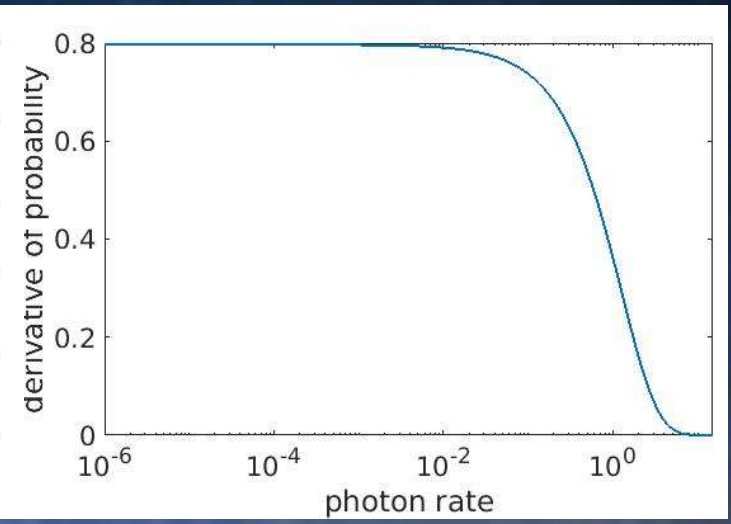
Photon Counting Mode



One photon counting (PC) image with 1s integration time



The probability of getting value one on the pixel for the PC image vs. the photon rate



The derivative of probability of getting value one on the pixel for the PC image vs. the photon rate

Sequential Generalized Likelihood Ratio Test

- Assumption: Constant background; Gaussian noise

Not Enough or Too

Long Integration Time

Binomial Each Pixel

Gaussian

- Model: $P(\text{Images } \{x_i\} \mid \text{Intensity}) = \prod_{\text{all images}} P(x_i \mid \text{Intensity} + \text{Background PSF} + \text{Background})$
- Testing:

H_0 : Intensity = 0

H_1 : Intensity > 0

- Reject H_0, if $L_G(\boldsymbol{x}) = \frac{\max_{\theta} P(\{x_i\} \mid H_1)}{\max_{\theta} P(\{x_i\} \mid H_0)} >$ Threshold

Ref. Hu, M. M., Sun, H., & Kasdin, N. J. (2019, September). Sequential generalized likelihood ratio test for planet detection with photon-counting mode. In Techniques and Instrumentation for Detection of Exoplanets IX (Vol. 11117, p. 111171K). International Society for Optics and Photonics.

Sequential Generalized Likelihood Ratio Test

- Model: $\boldsymbol{x} = \underbrace{\begin{bmatrix} \boldsymbol{PSF}_1 & 1 \\ \vdots & \vdots \\ \boldsymbol{PSF}_N & 1 \end{bmatrix}}_{\boldsymbol{H}} \underbrace{\begin{bmatrix} I \\ b \end{bmatrix}}_{\boldsymbol{\theta}}$, $k_{1,j} = \sum_{i=1}^{k} y_{i,j}$, $where\ i \in [1,k], j \in [1,N]$

$$p(\{\boldsymbol{y}_i\}|I,b) = \prod_{j=1}^{N}\prod_{i=1}^{k} \{ f(x_j\ )^{k_{1,j}} [1 - f(x_j\ )]^{k-k_{1,j}} \}$$

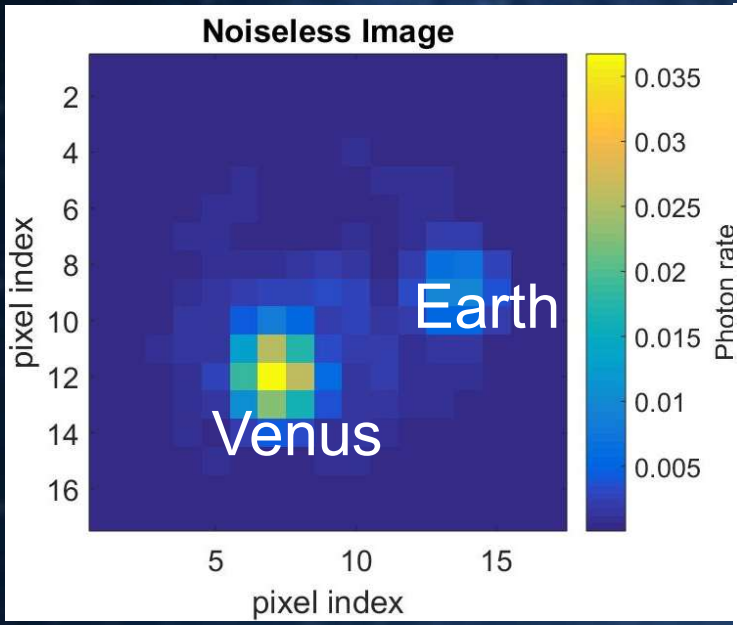
$$= \prod_{j=1}^{N} \{ f(I \cdot \boldsymbol{PSF}_j + b)^{k_{1,j}} [1 - f(I \cdot \boldsymbol{PSF}_j + b)]^{k-k_{1,j}} \}$$

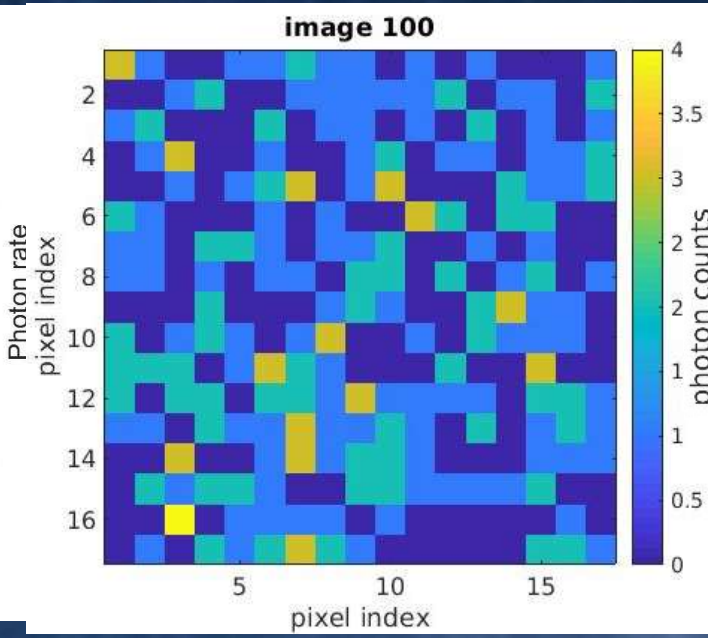
- Testing: $H_0: I = \underbrace{[1 \quad 0]}_{\boldsymbol{A}} \boldsymbol{\theta} = 0\ , \sigma^2 > 0;$

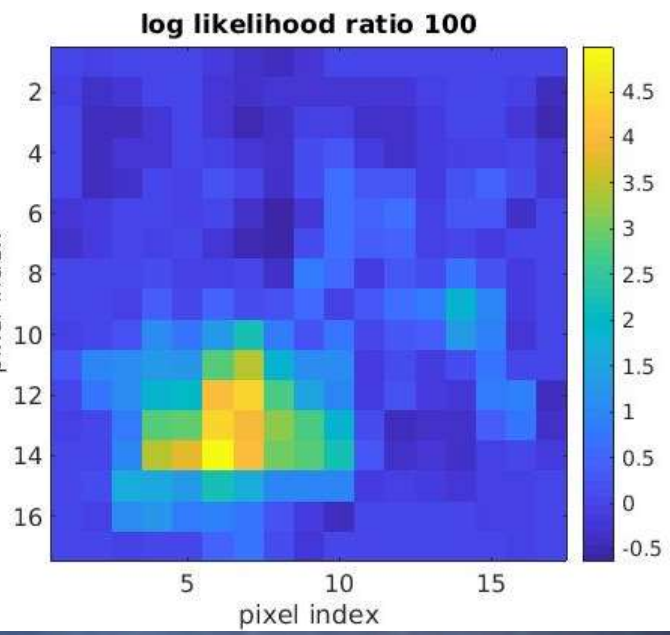
$H_1: \boldsymbol{A\theta} > 0, \sigma^2 > 0$

- Reject H_0, if $L_G(\boldsymbol{x}) = \frac{max_{\boldsymbol{\theta}}\ P(\{x_i\}\ |\ H_1)}{max_{\boldsymbol{\theta}}\ P(\{x_i\}|\ H_0)}$ > Threshold

Sequential Generalized Likelihood Ratio Test





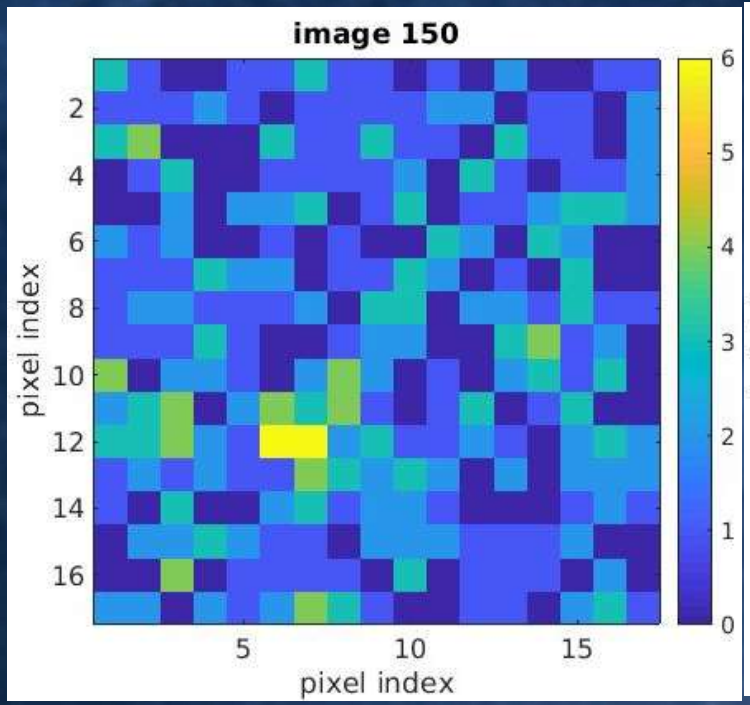


Simulated Noiseless image for Starshade-WFIRST system. This is the Ground-truth for the detection problem.

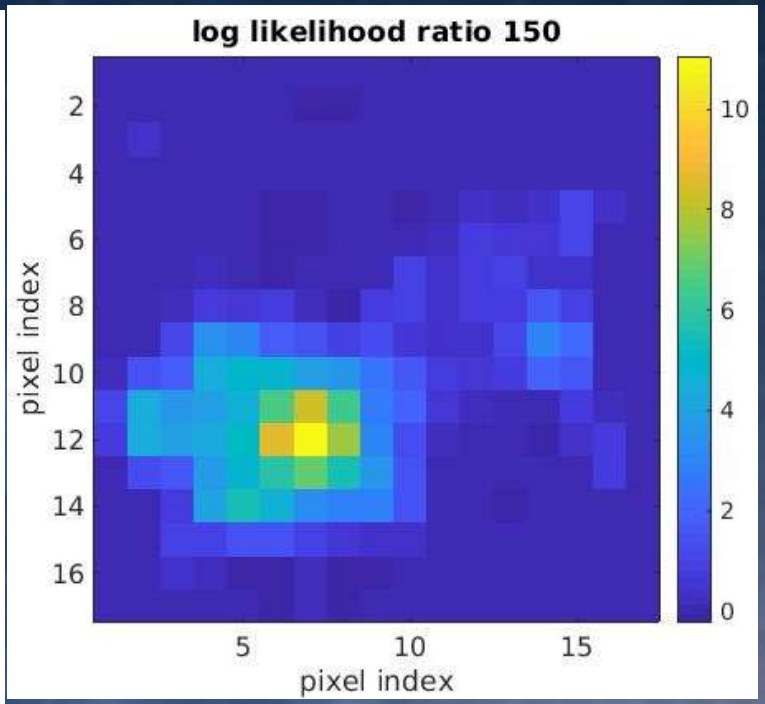
coadded image with 100 sequential PC images

False alarm rate of each pixel

Sequential Generalized Likelihood Ratio Test

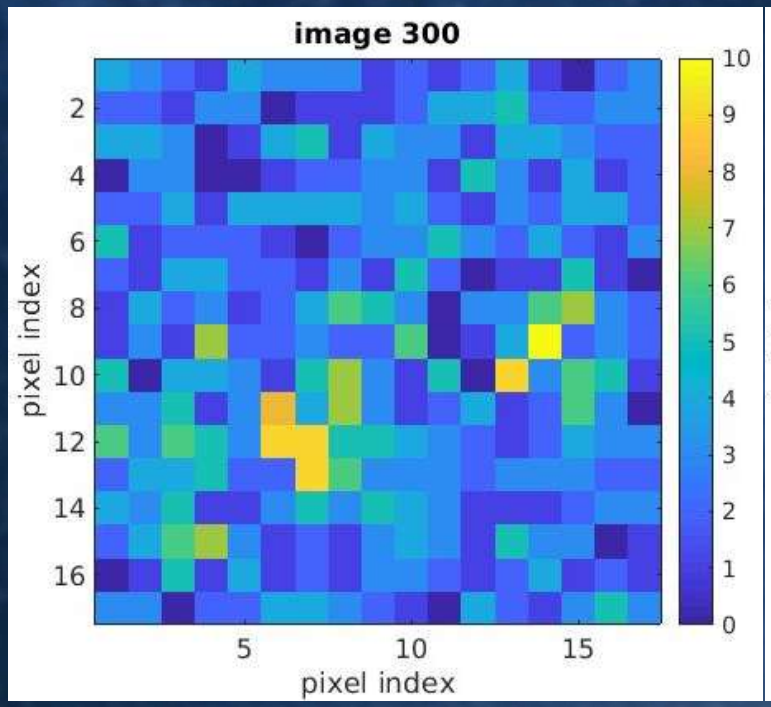


coadded image with 150 sequential PC images

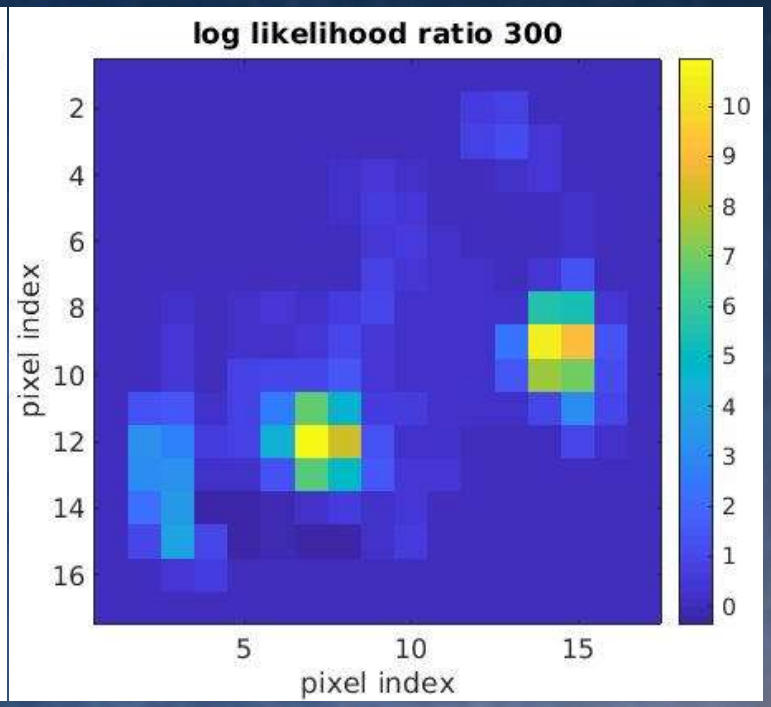


False alarm rate of each pixel

Sequential Generalized Likelihood Ratio Test

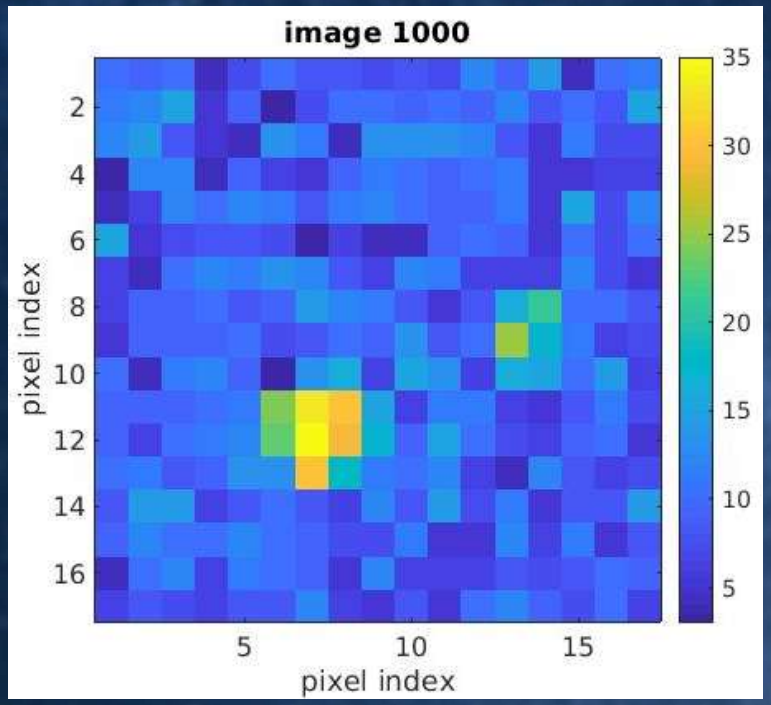


coadded image with 300 sequential PC images

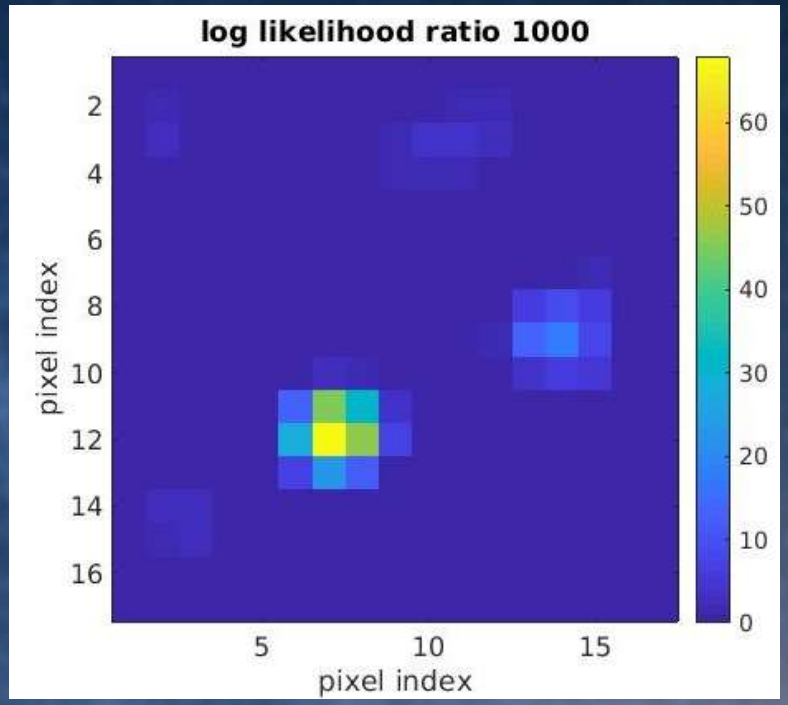


False alarm rate of each pixel

Sequential Generalized Likelihood Ratio Test

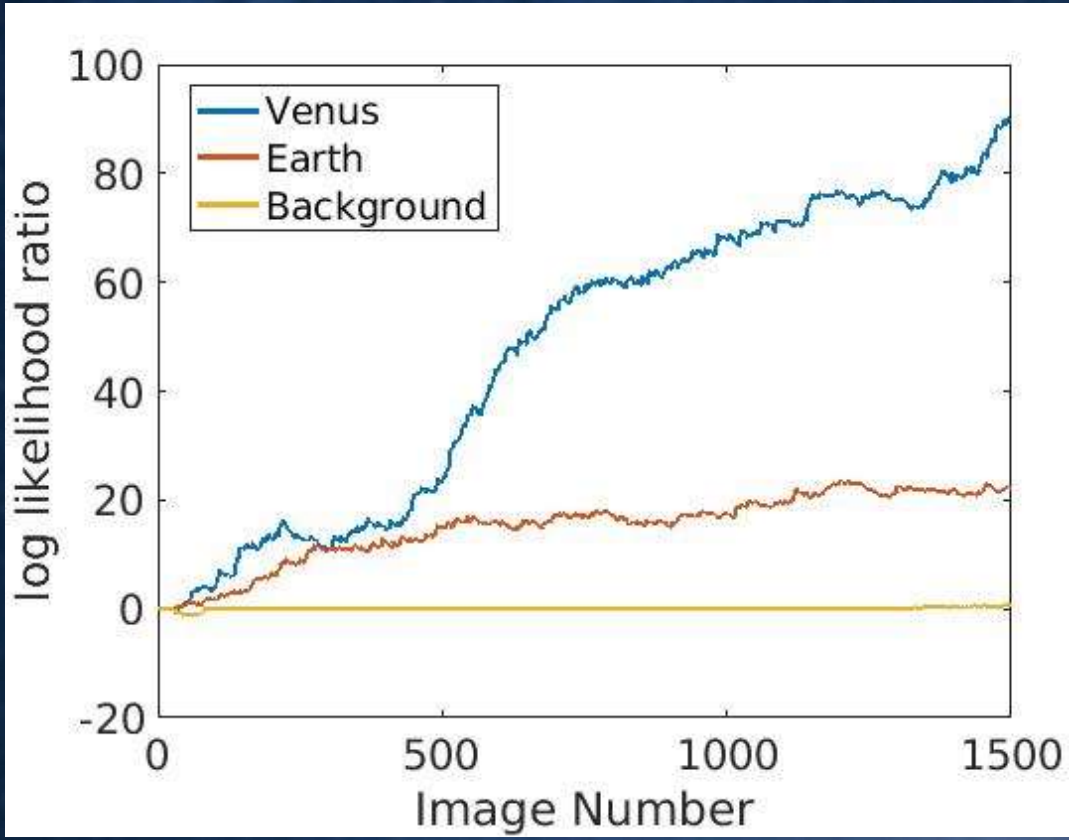


coadded image with 1000 sequential PC images



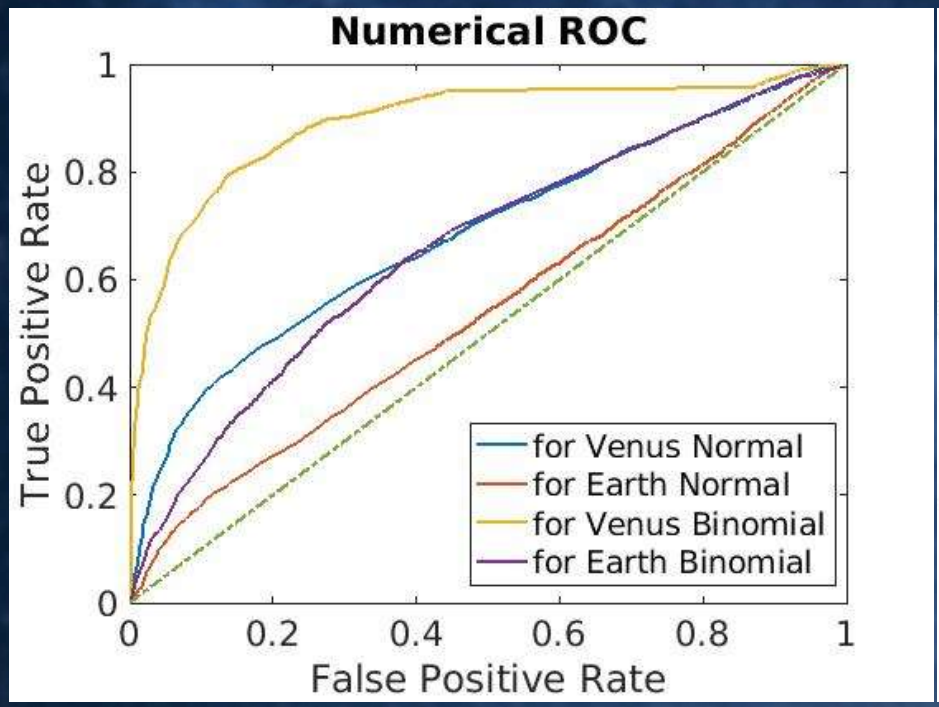
False alarm rate of each pixel

Sequential Generalized Likelihood Ratio Test

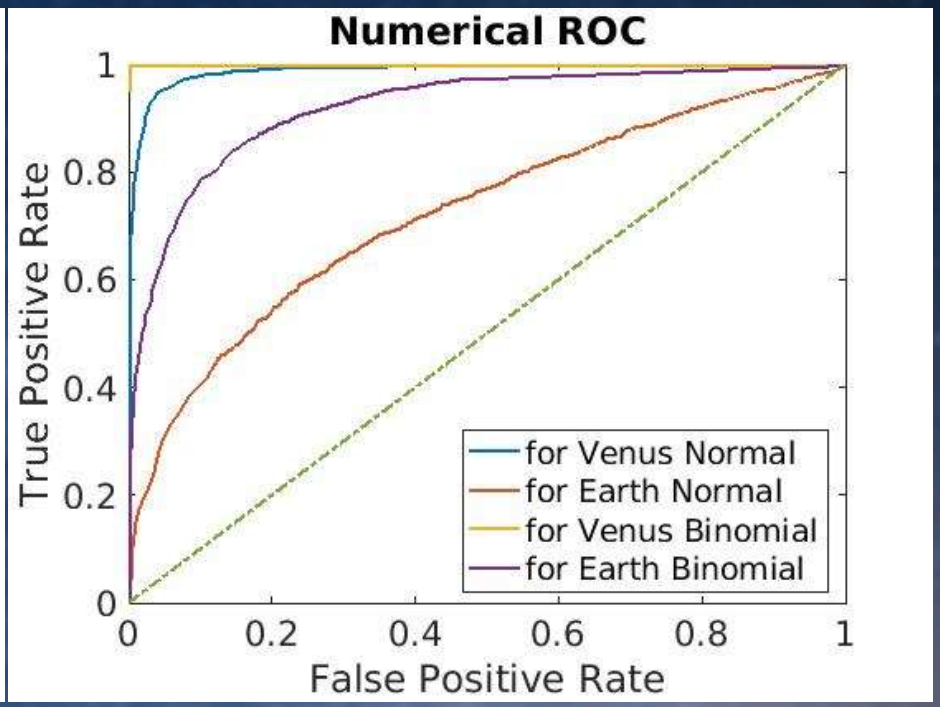


log likelihood ratio of Venus, Earth and a background pixel (5th row, 5th column) with increasing number of observations

Sequential Generalized Likelihood Ratio Test



ROC for 500 PC images



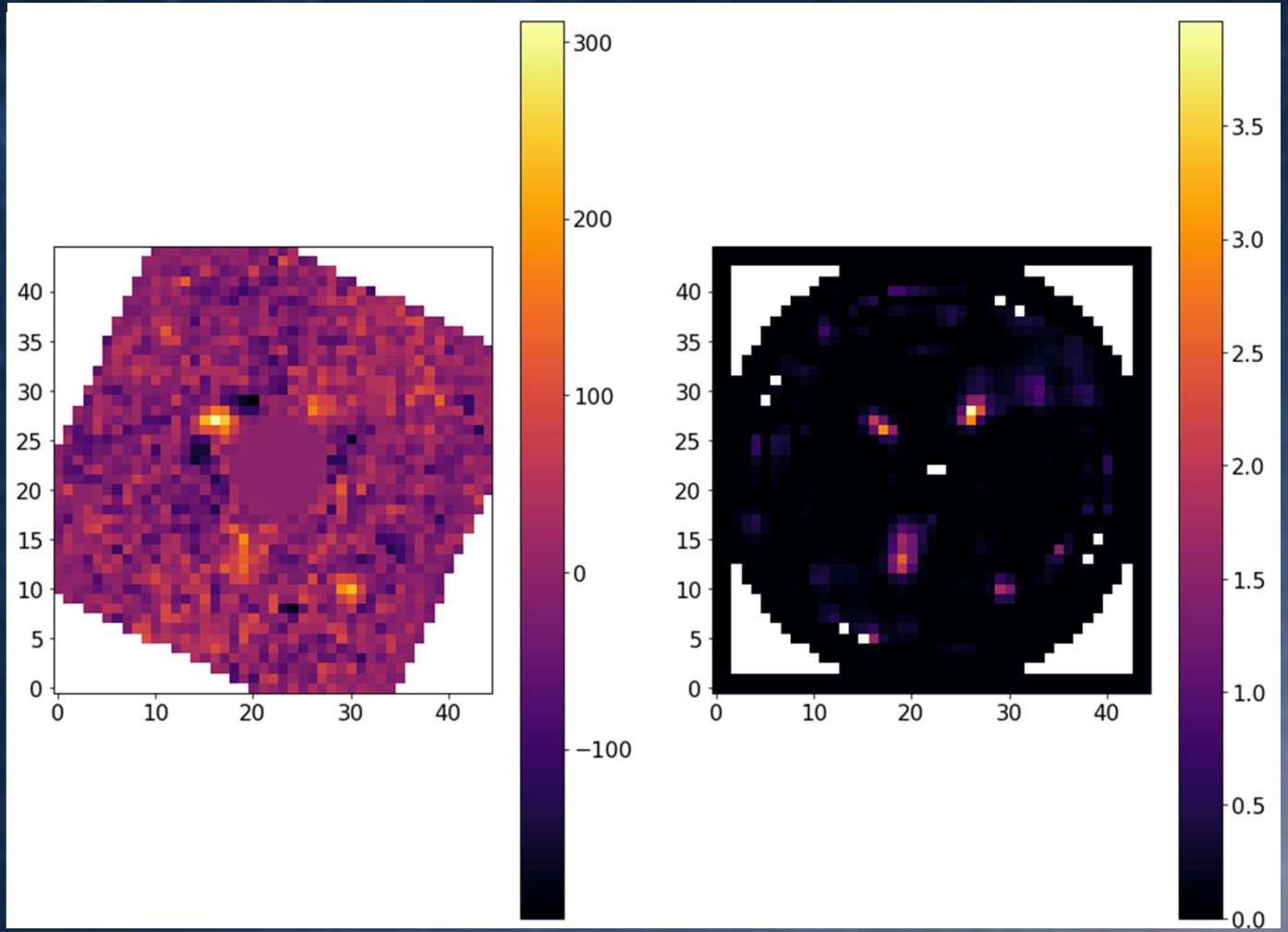
ROC for 2500 PC images

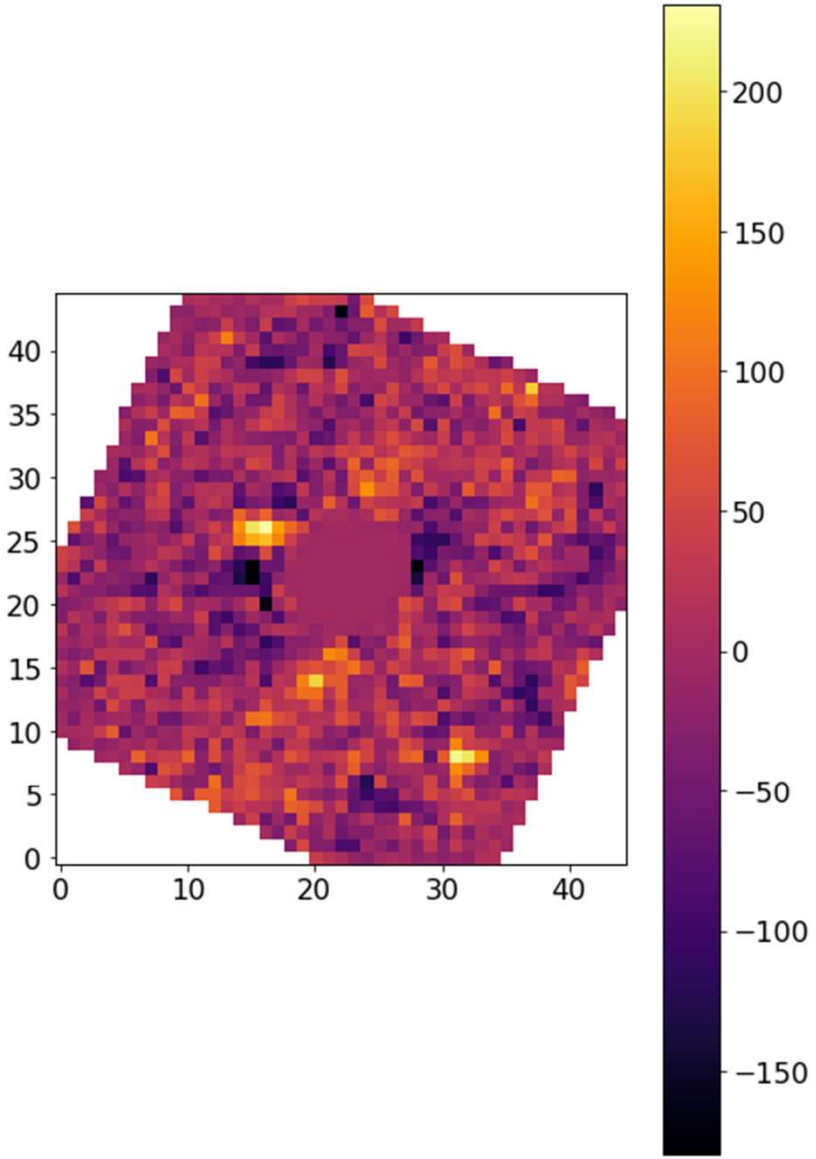
SUMMARY

Generalized Likelihood Ratio Test (GLRT)	Sequential GLRT (SGLRT) for Photon Counting Images
• Assumptions: Constant background; Gaussian noise • Simple & easy • May miss a detection or waste observation for inappropriate integration time	• Assumptions: Constant background; Binominal distribution for each pixel • Can process sequential images on-line • Maximizes the utilization of information in each observation

Thanks

Data challenge

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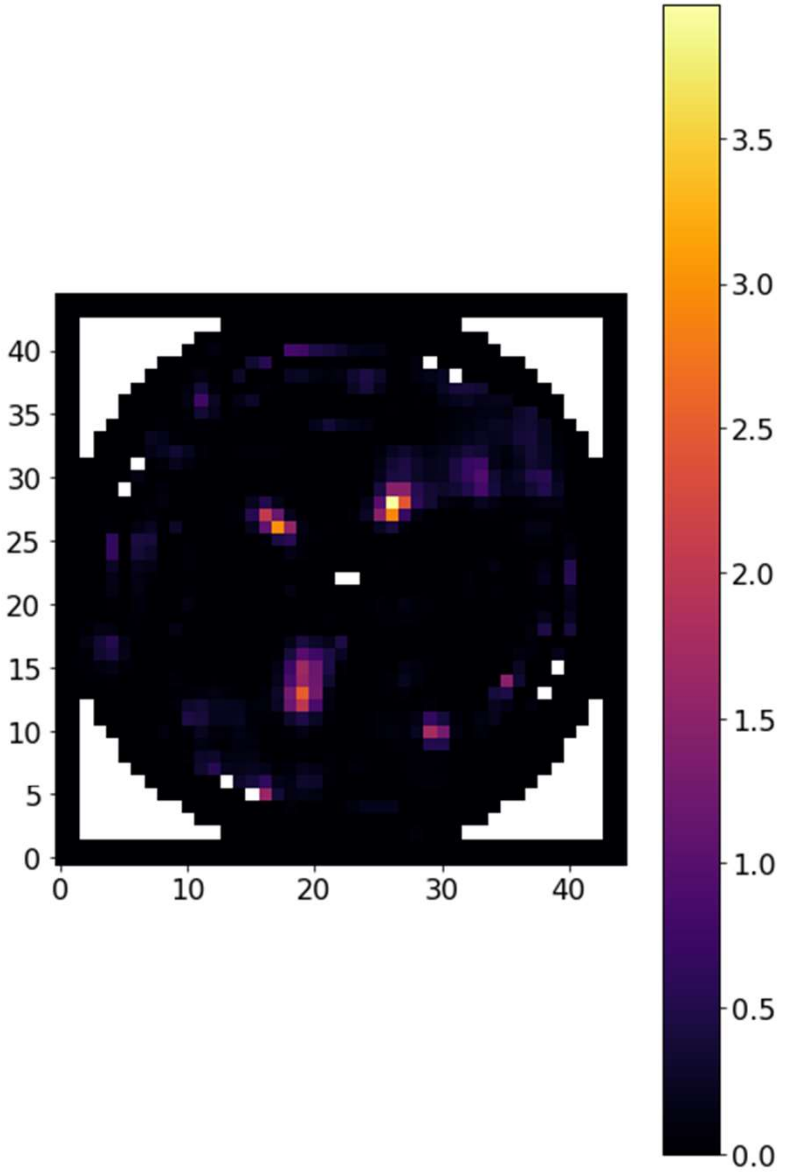
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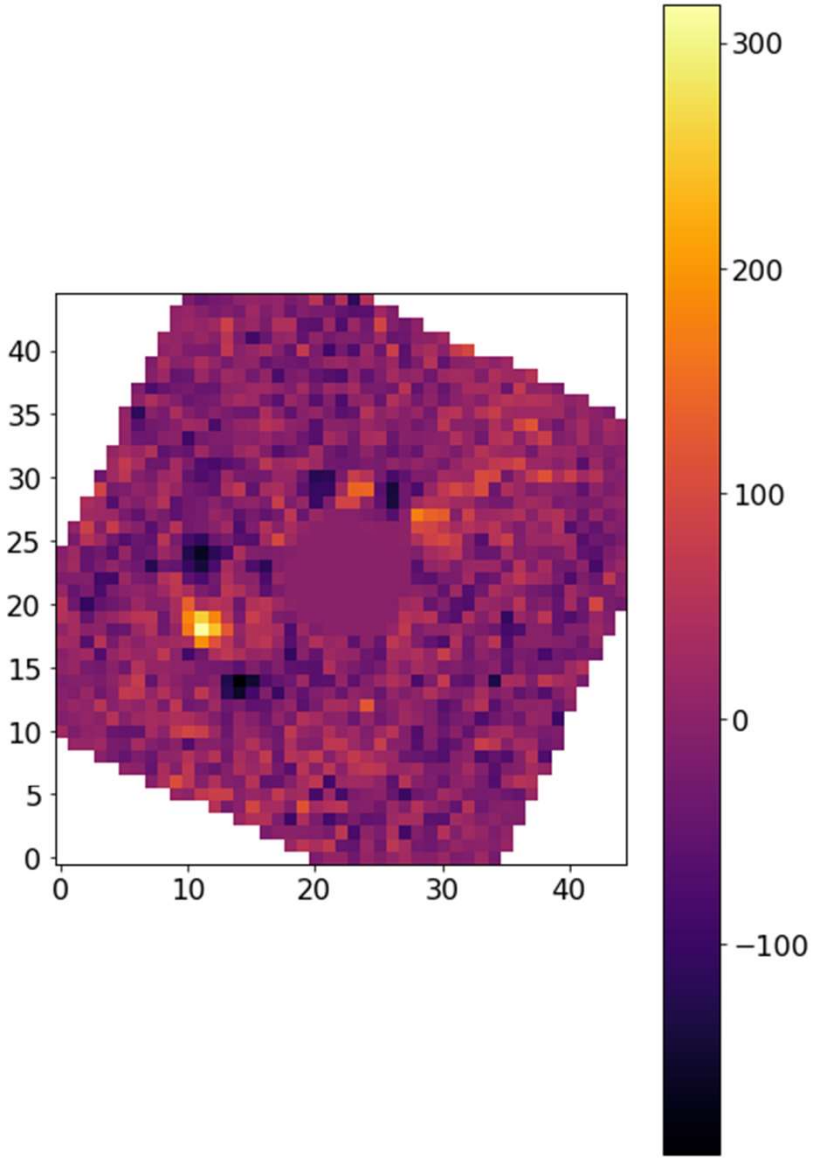
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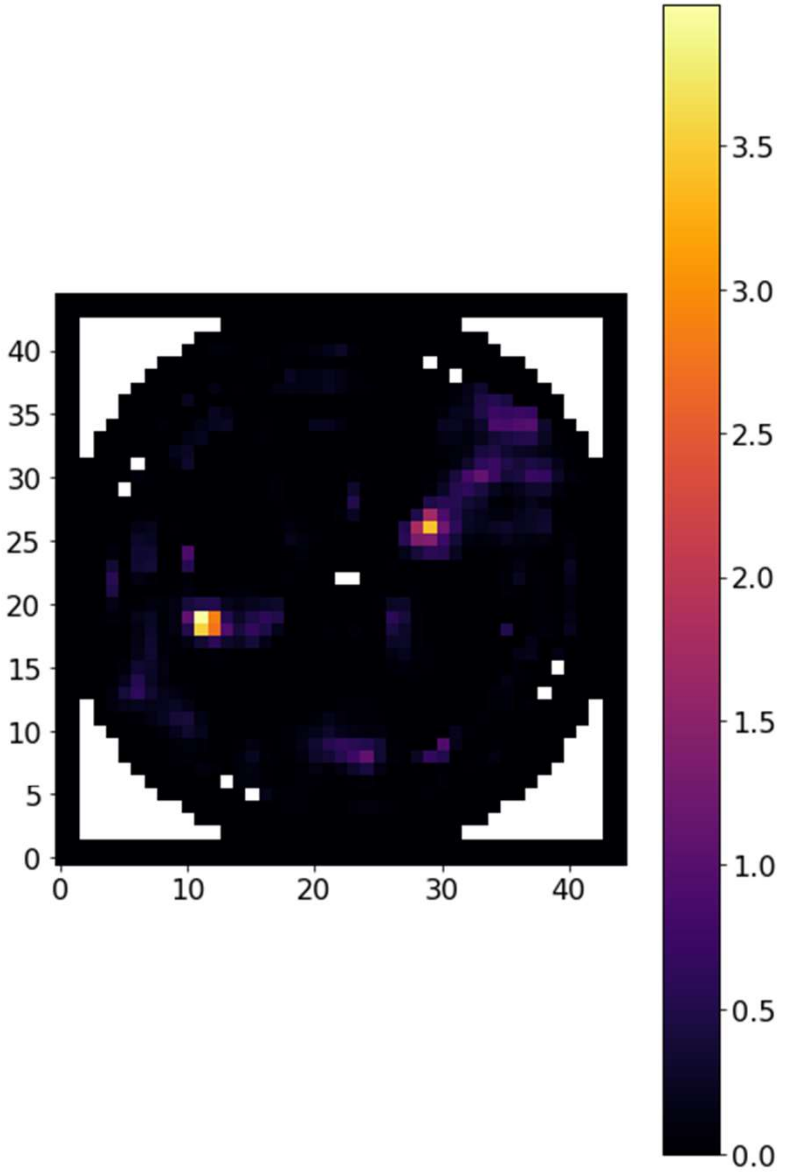
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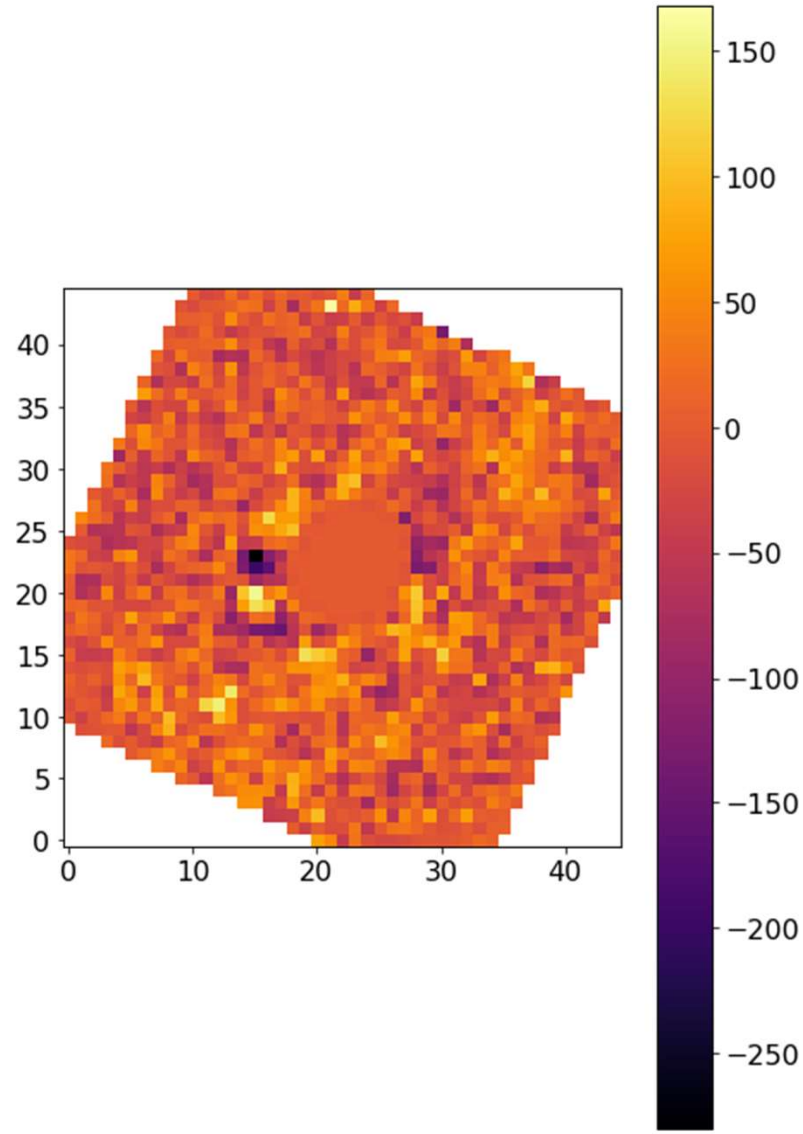
2.0
1.5
1.0
0.5
0.0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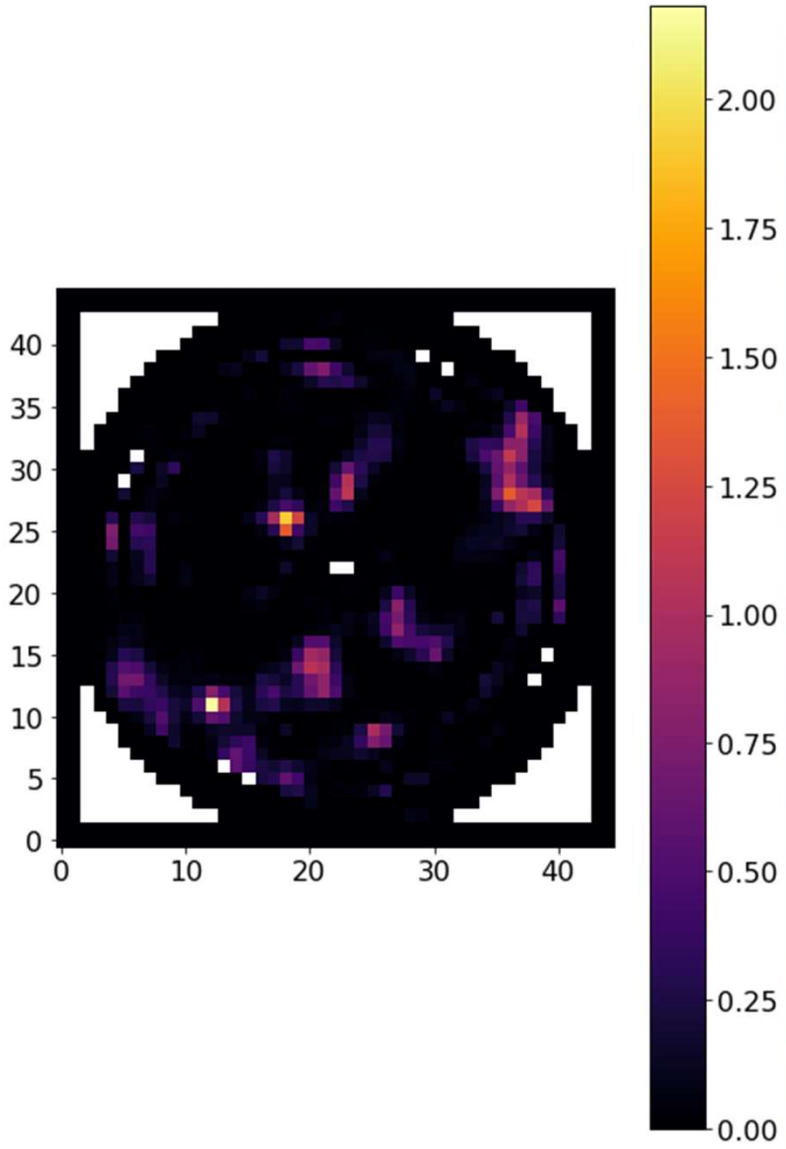
10
15
20
25
30
35
40
0
10
20
30
40
150
100
50
0
−50
−100
−150
−200
−250


0
5
10
15
20
25
30
35
40
0
10
20
30
40
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00

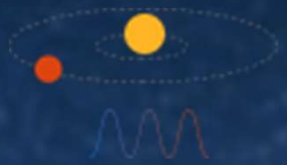
Back up slides

Ways to Find an Exoplanet

Searching for Shadows

Transit

77.7% (3117 planets)



Watching for Wobble

Radial Velocity

19.0% (764 planets)

Light in a Gravity Lens

Gravitational Microlensing

2.1% (83 planets **)**

Taking Pictures

Direct Imaging

1.2%(47 planets **)**

Ref. https://exoplanets.nasa.gov/interactable/11; https://exoplanets.nasa.gov/discoveries-overlay/

Advantages of Direct Imaging

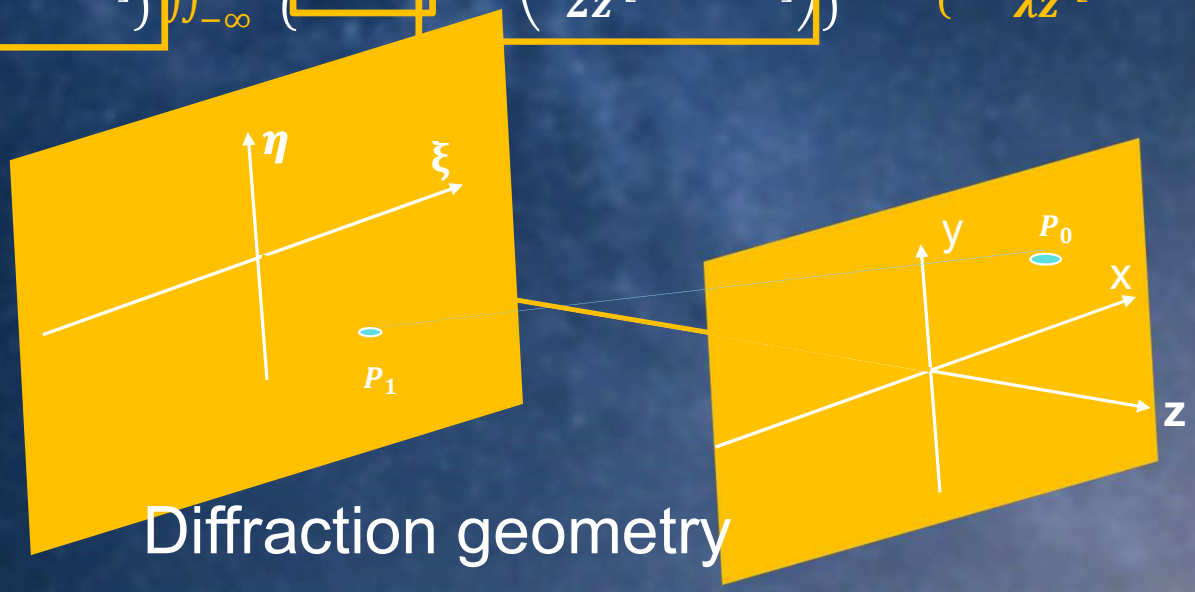
- More planets' properties revealed--Exoplanet spectra
- Biomarker--Life?
- Visual proof--"Seeing is believing"

http://www.markelowitz.com/Hyperspectral.html http://www.esa.int/Our_Activities/Space_Science/Exploring_space/Vital_signs_of_life_on_distant_worlds/(print) https://trackthestars.com/travel-telescope/

Fresnel propagation

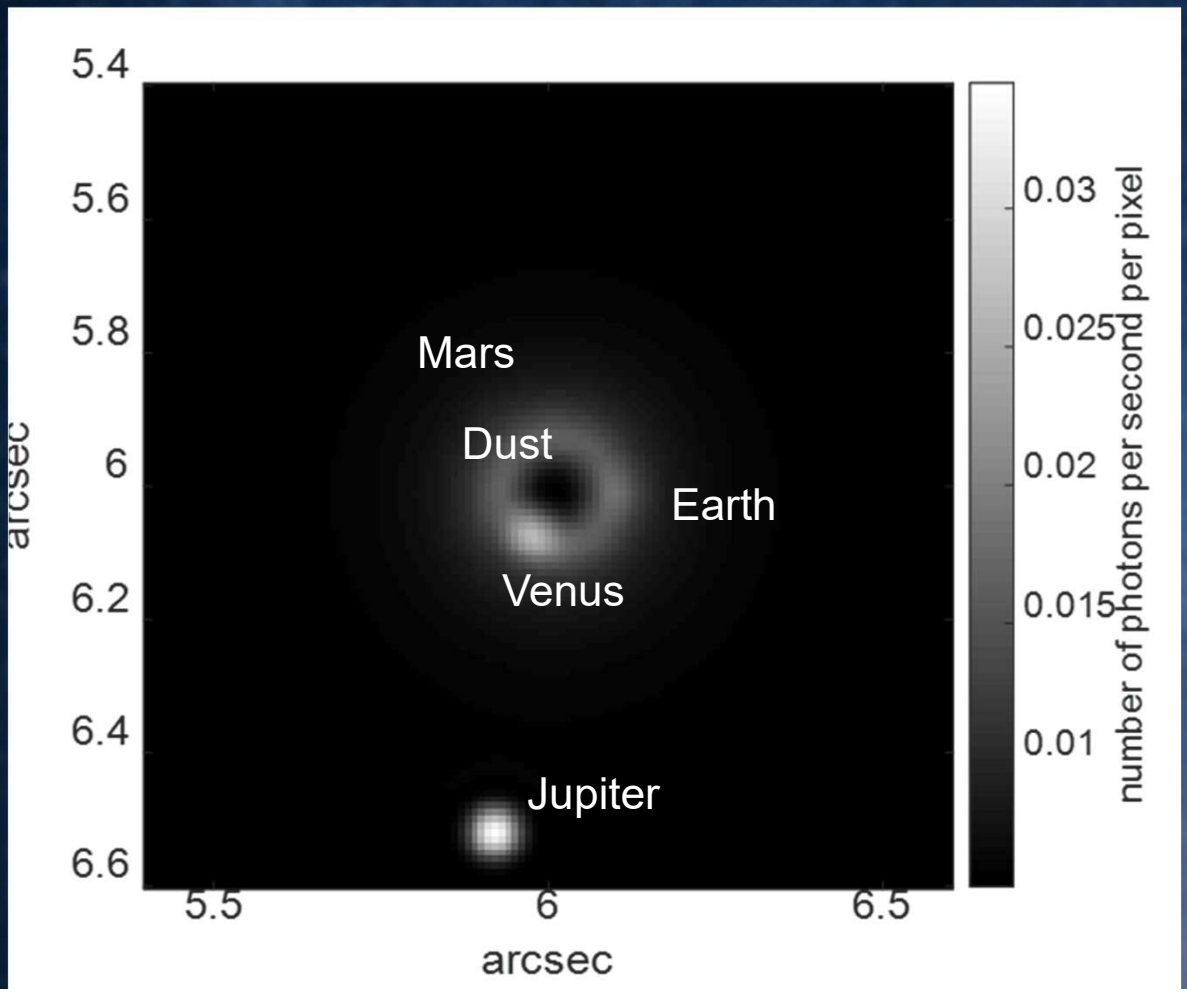
Approximation of **Huygens-Fresnel Principle**

$$U(x,y) = \frac{e^{ikz}}{i\lambda z} exp\left\{ i\frac{k}{2z}\left[x^2 + y^2\right]\right\} \iint_{-\infty}^{+\infty} \left\{ U(\xi,\eta) exp\left(i\frac{k}{2z}\left[\xi^2 + \eta^2\right]\right)\right\} exp\left\{ -i\frac{2\pi}{\lambda z}\left[x\xi \quad + y\eta \quad \right]\right\} d\xi d\eta$$



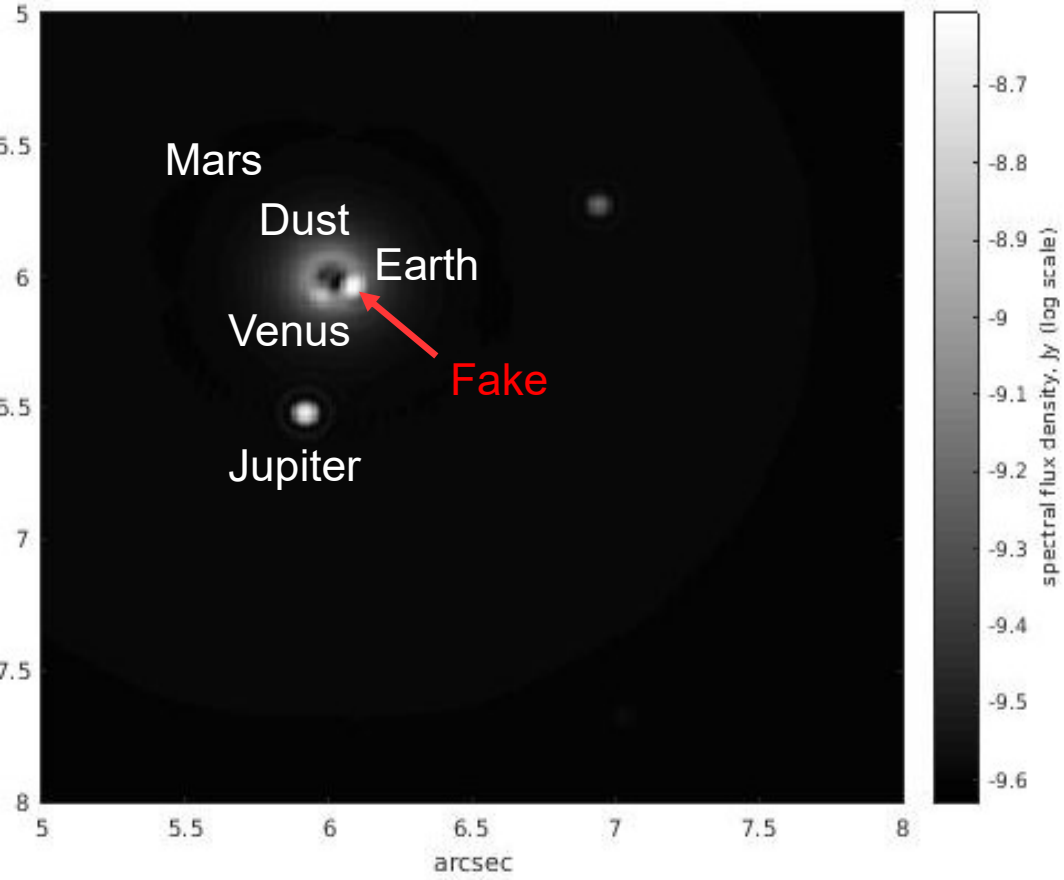
Diffraction geometry

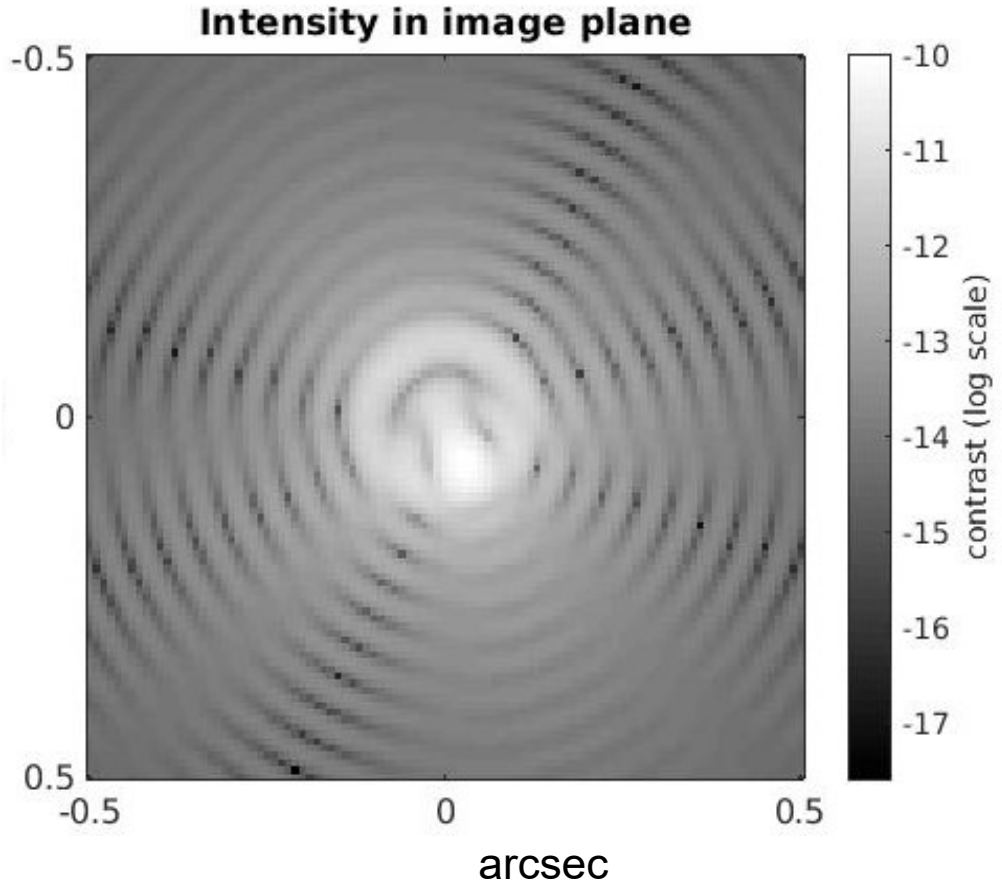
Simulation Simplification



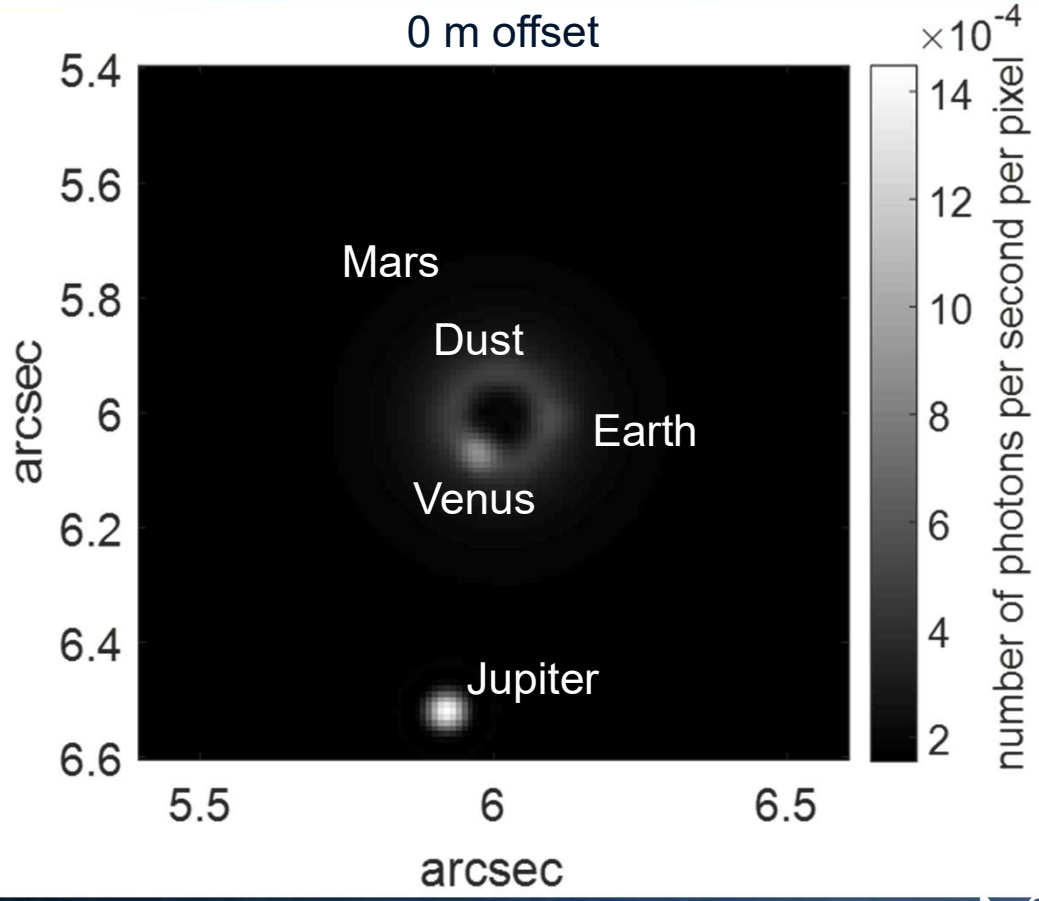
Simulated image for the perfect case *with all wavelengths (600nm~1100nm)*

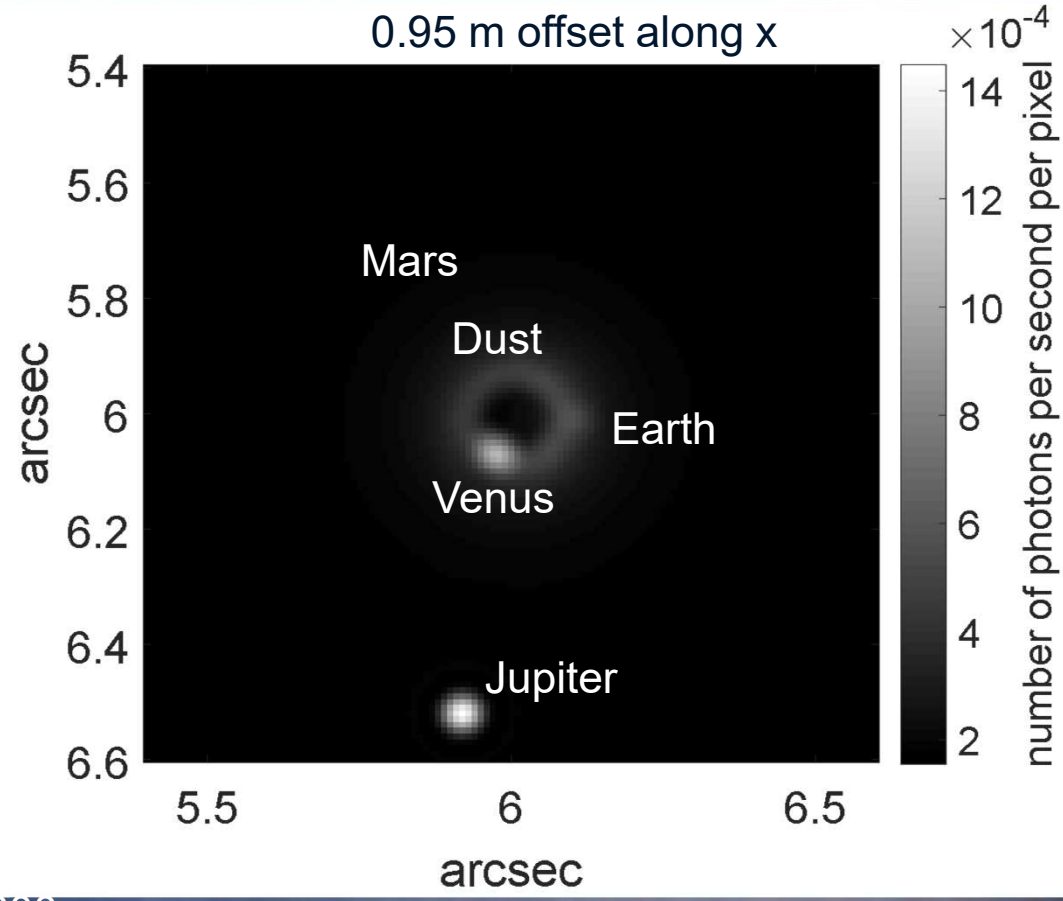
Errors On Starshade: Truncated Tip





Starshade Dynamics (Misalignment)

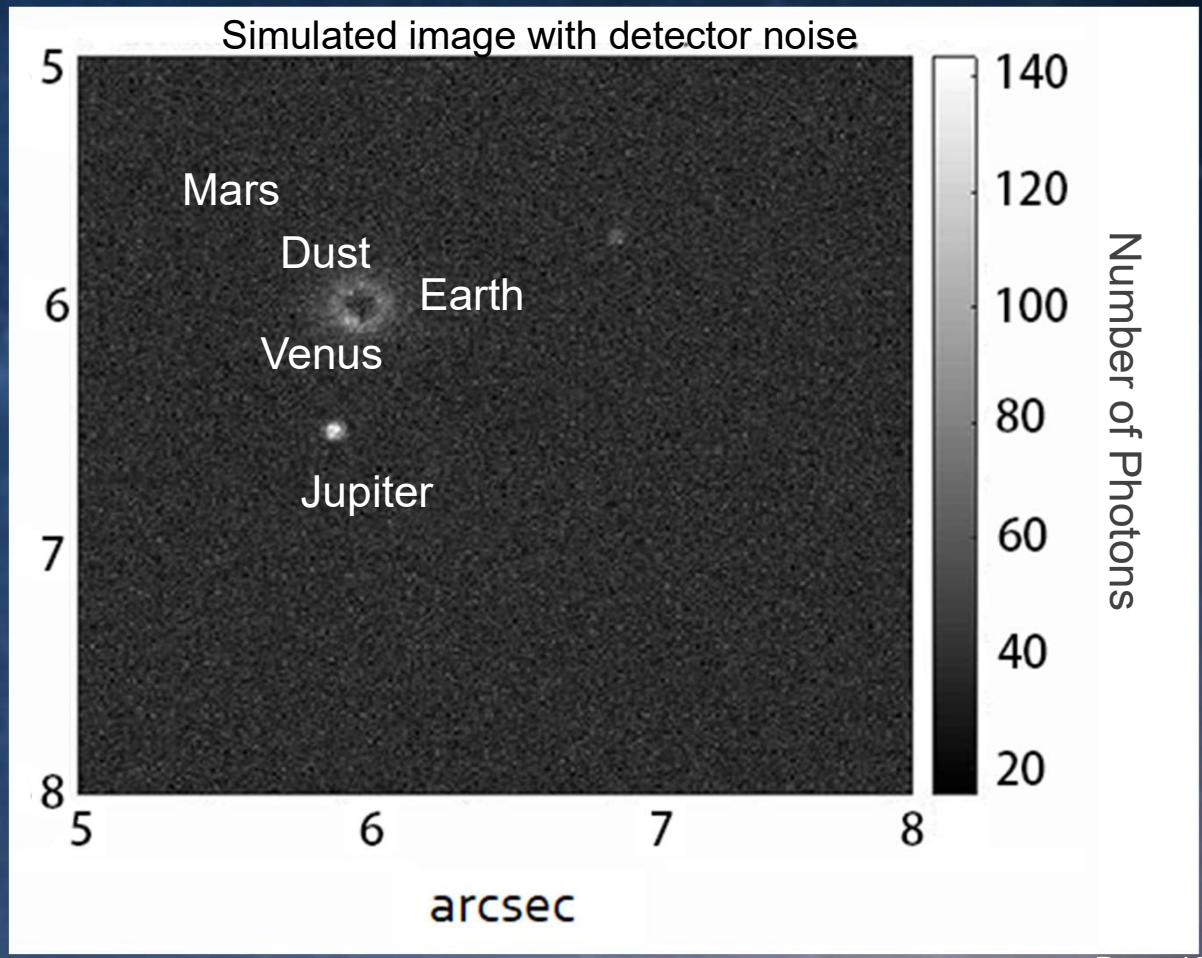




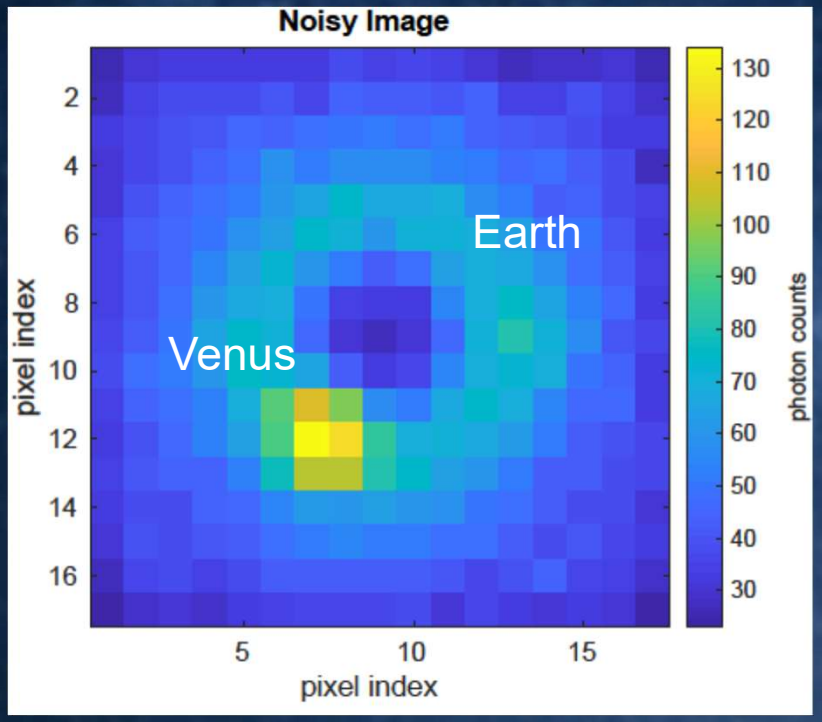
λ=600nm

Detector Model

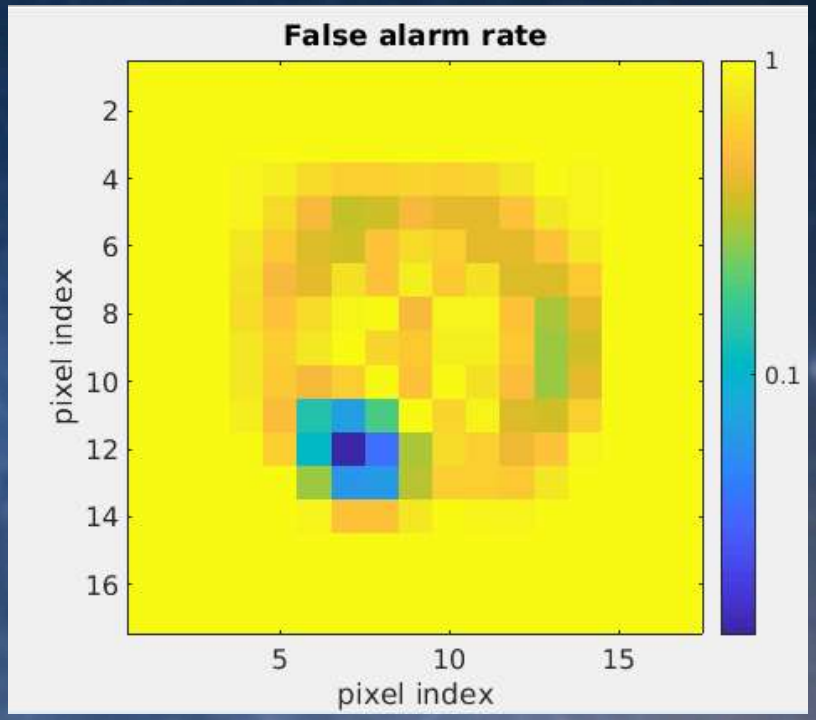
- Quantum Efficiency
- Photon counting mode
- Dark current
- Clock-induced charge
- Read noise
- Photon counting bias



System with Dust



Noisy solar system image (0.021arcsec/pixel) with perfect SS. Earth is hardly seen.



The detection result with GLRT. False alarm rate for Venus 1.6e-1; Earth 5.7e-1, smallest other 6.3e-1. We can't distinguish earth from the dust. 26% Intensity estimation error for Venus

GLRT with Expectation–maximization (EM) Algorithm

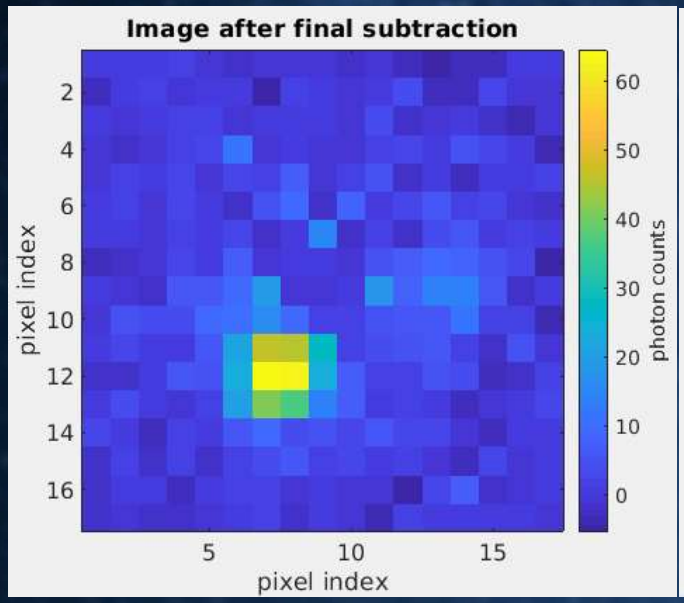
Isub_backg (image after subtracting estimated background) = *Iorig* (the original image) - *Imedian (*the median value of pixels with the exactly same distance of *Iorig)*

Detect and estimate the value of planets' signals *Ip* in *Isub_backg*

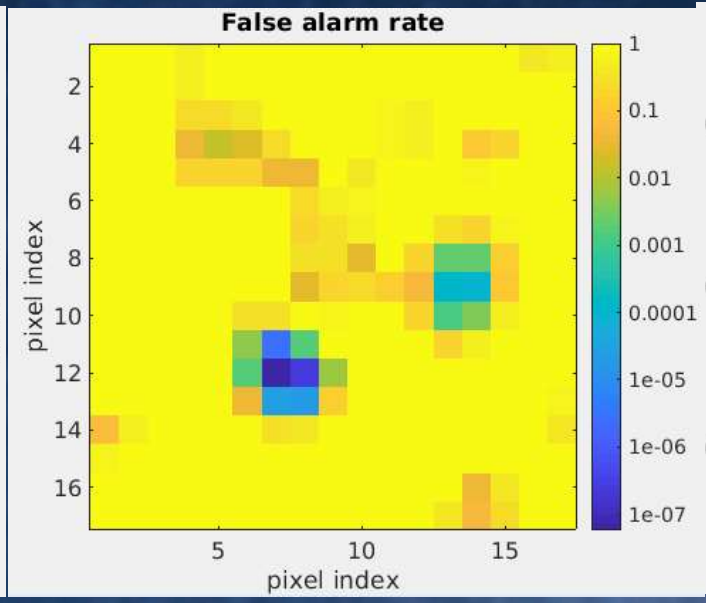
Ibackg (image after subtracting estimated planets' signal) = *Iorig - Ip*

Isub_backg = *Iorig* – *Imedian_backg* (the median value of pixels with the exactly same distance of *Ibackg*)

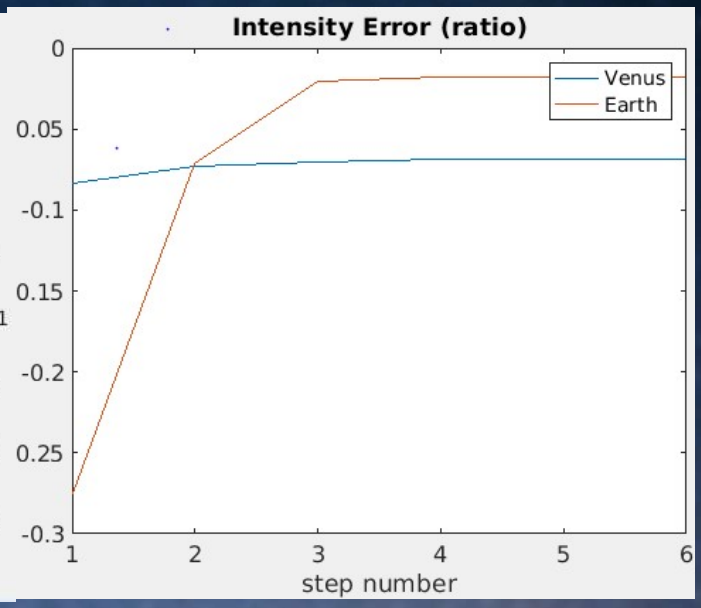
GLRT with Expectation–maximization (EM) Algorithm



Final image after iteratively subtracting the estimation of the background.



The detection result with GLRT. We can detect both Venus with FAR of 7.2e-4 and earth with FAR of 1.9e-2. The subtraction caused FAR as small as 2.2e-1 in other area.

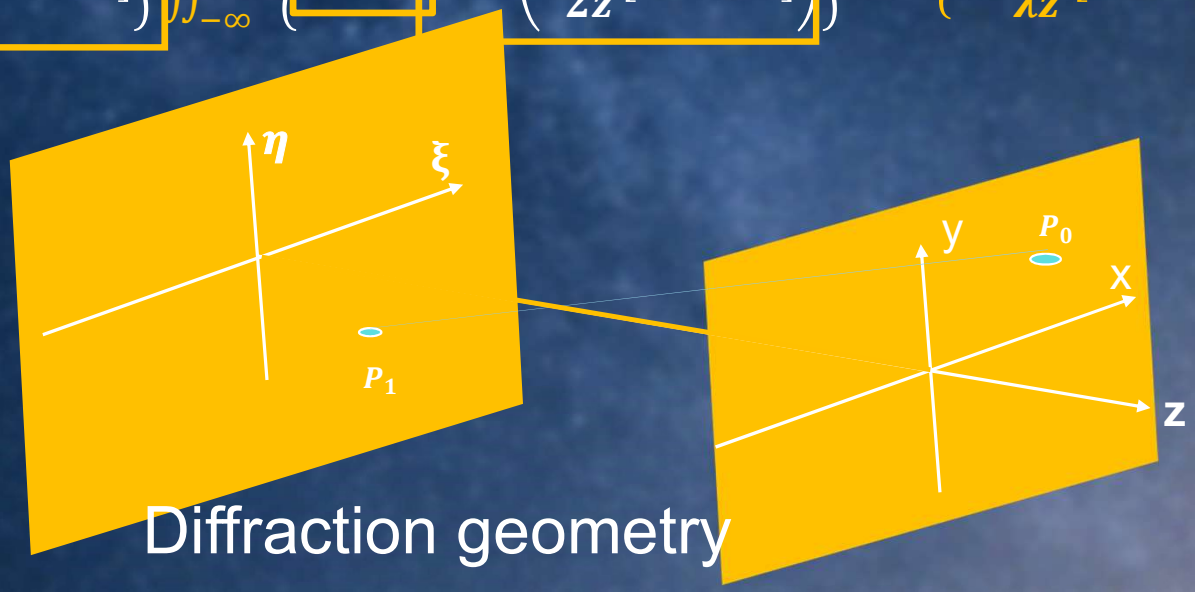


IEE in each iteration. The final IEE for Venus is -6.9% and Earth -1.8%.

Fresnel propagation

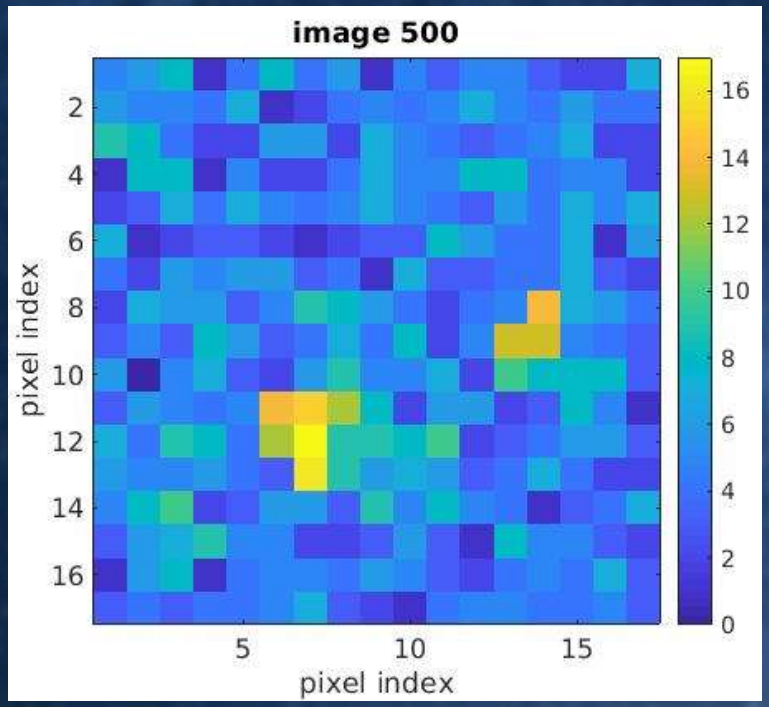
Approximation of **Huygens-Fresnel Principle**

$$U(x,y) = \frac{e^{ikz}}{i\lambda z} exp\left\{ i\frac{k}{2z}\left[x^2+y^2\right]\right\} \iint_{-\infty}^{+\infty} \left\{ U(\xi,\eta) exp\left(i\frac{k}{2z}\left[\xi^2+\eta^2\right]\right)\right\} exp\left\{ -i\frac{2\pi}{\lambda z}\left[x\xi + y\eta\right]\right\} d\xi d\eta$$

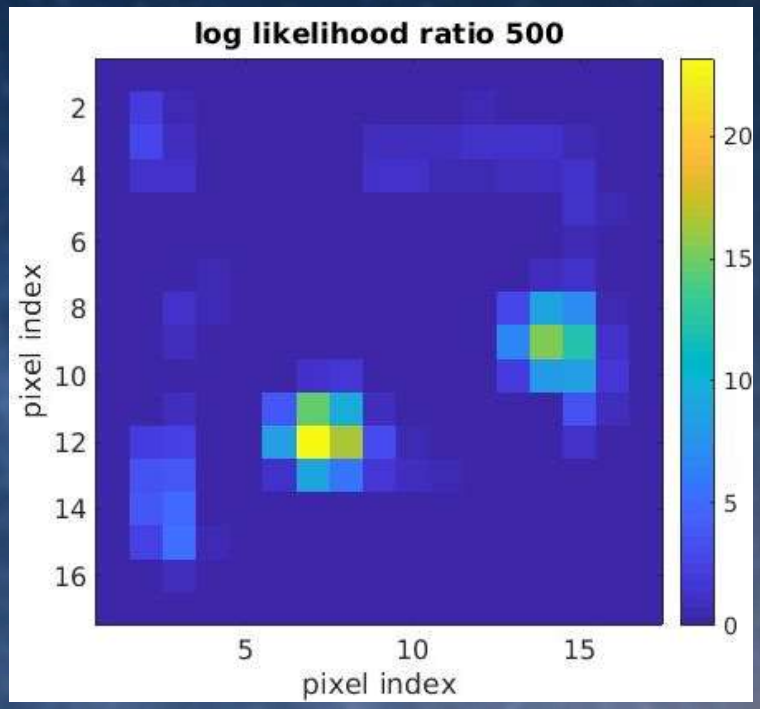


Diffraction geometry

Sequential Generalized Likelihood Ratio Test

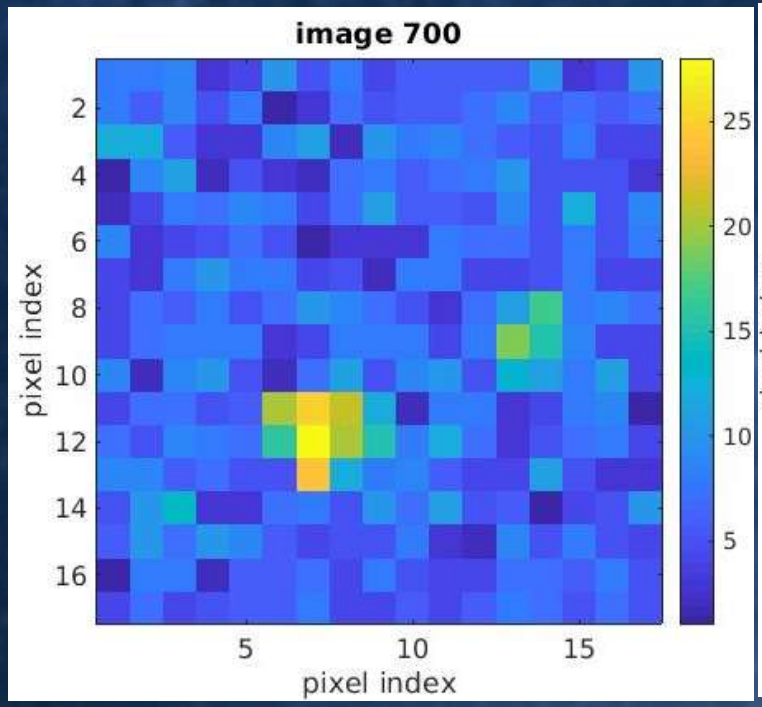


coadded image with 500 sequential PC images

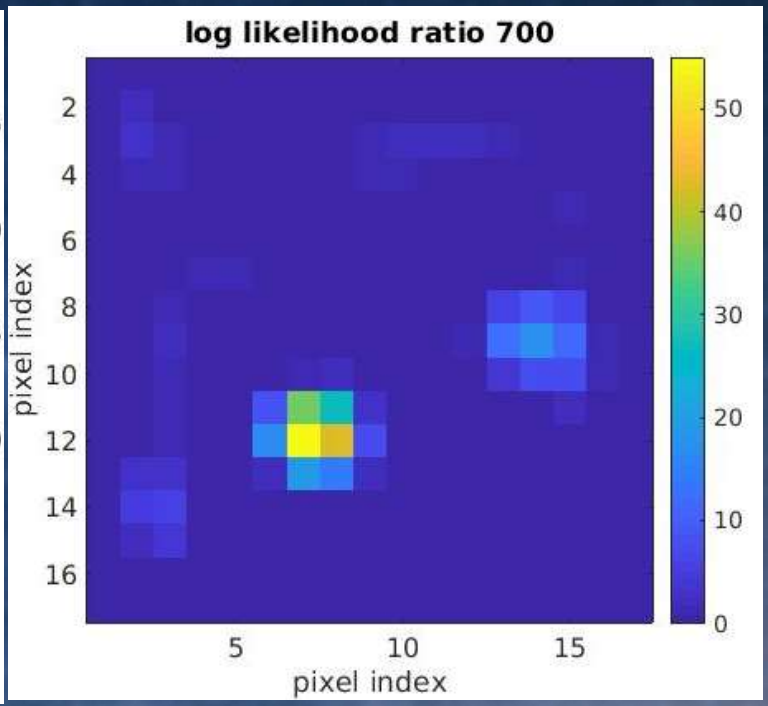


False alarm rate of each pixel

Sequential Generalized Likelihood Ratio Test



coadded image with 700 sequential PC images



False alarm rate of each pixel