

# From Wobbles to Worlds: Exploring the Orbital Landscape of Exoplanet TTVs

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1: Columbia University, 2: University of Washington, 3: SwRI Boulder

ExoExplorers Science Series  
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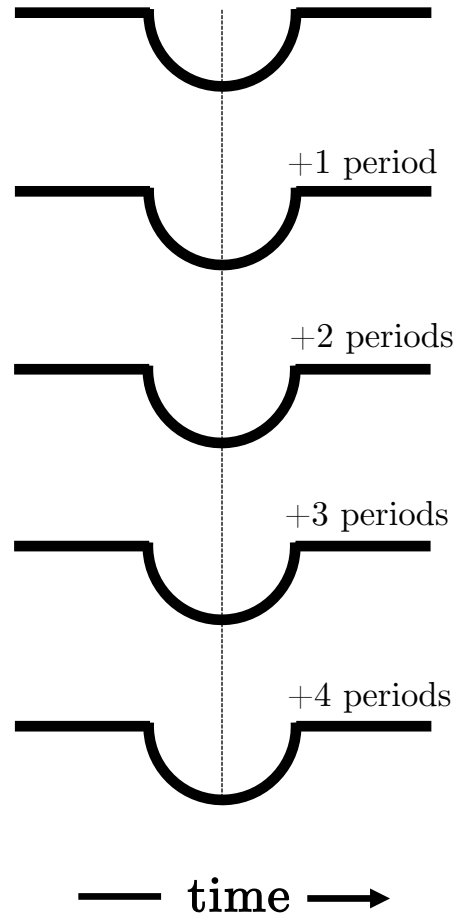
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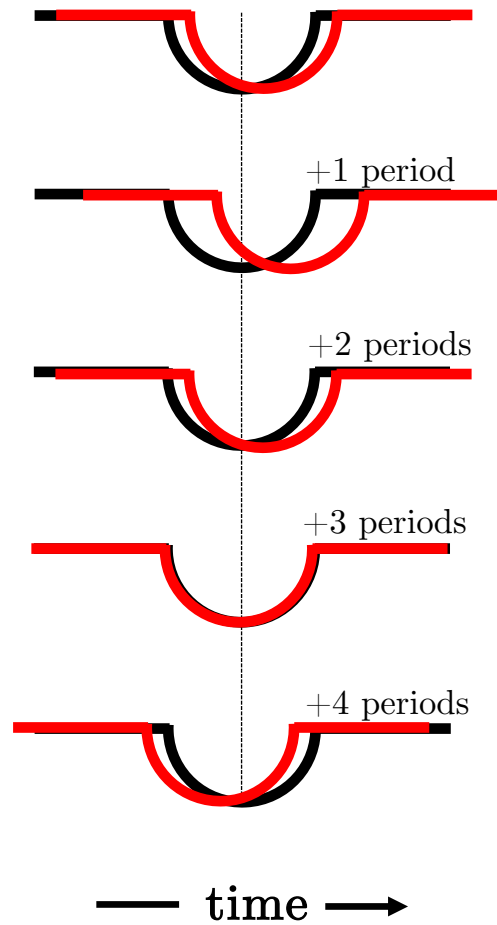
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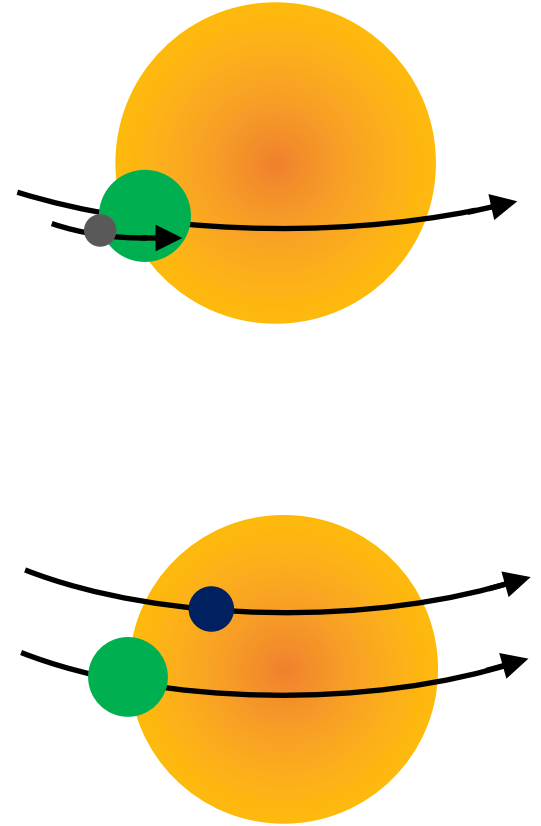
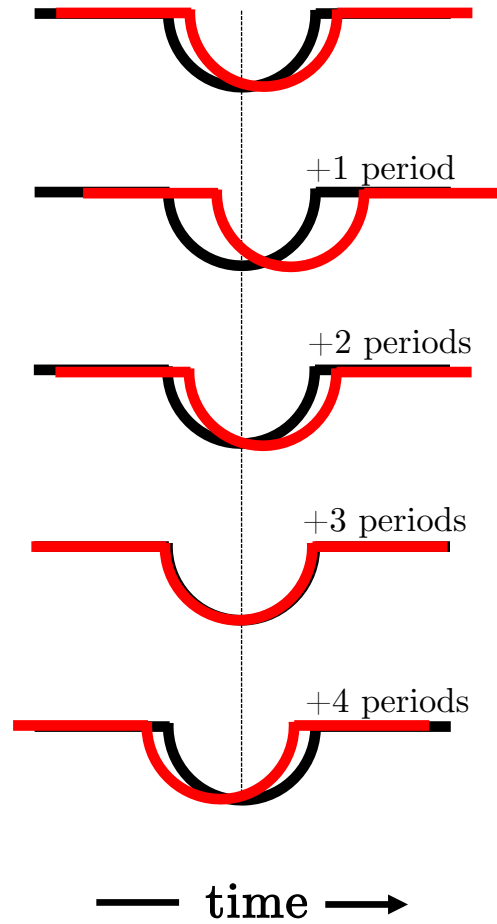
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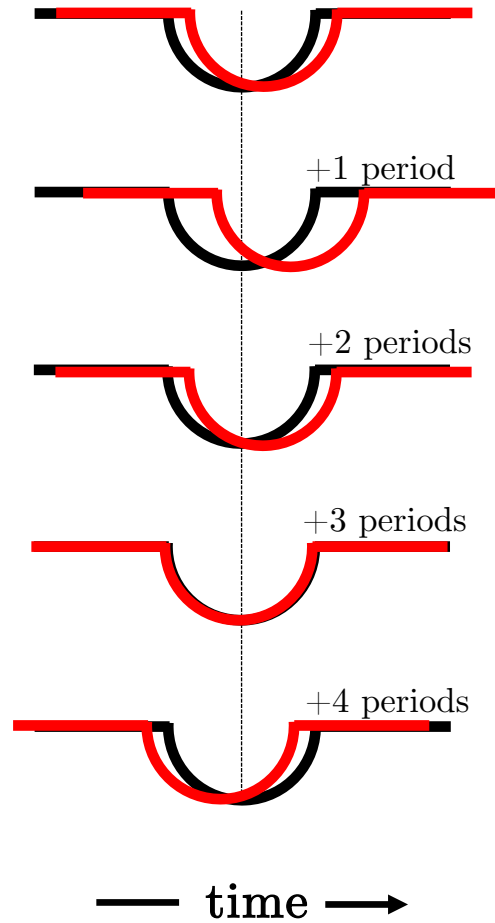
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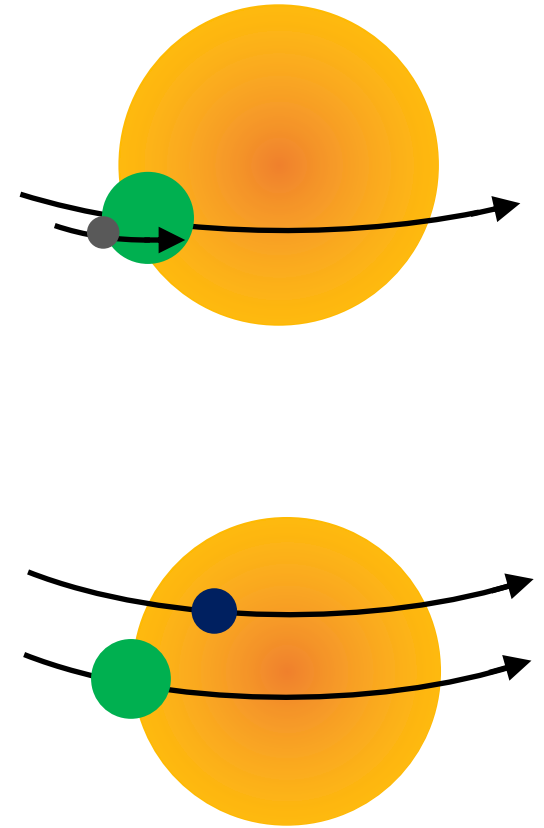
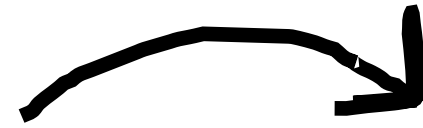
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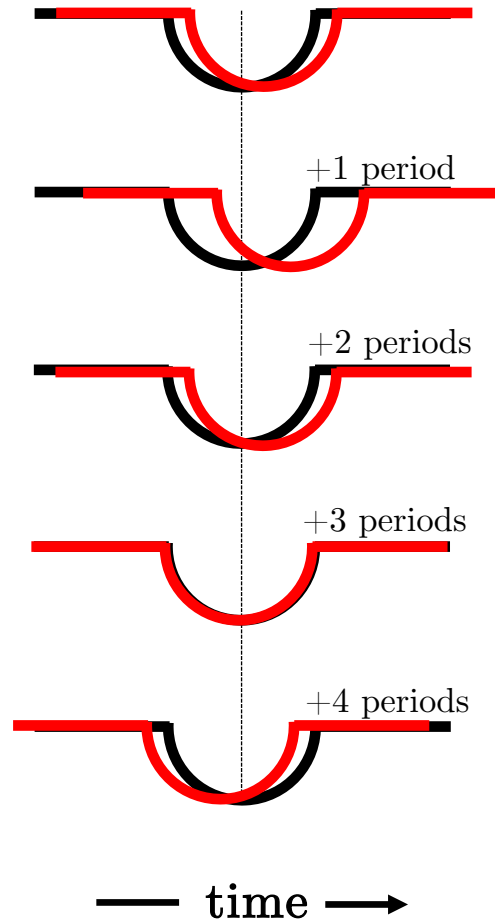
exomoon  
corridor



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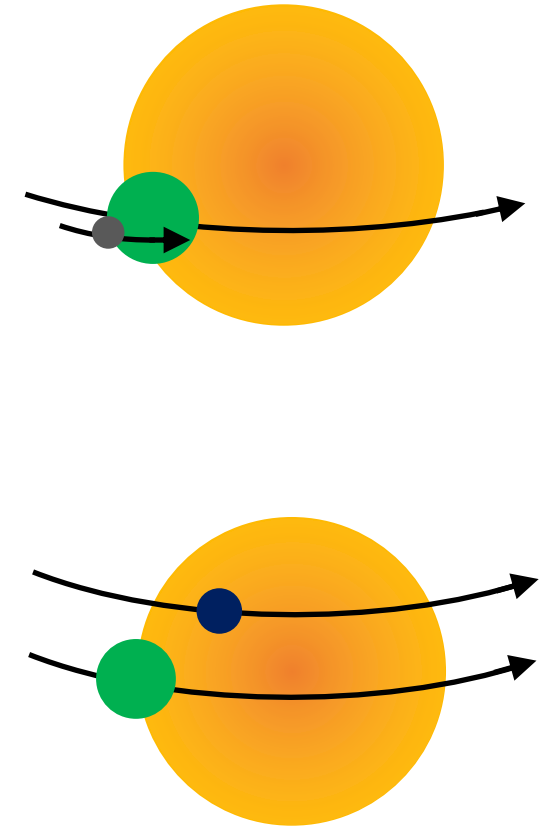
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exomoon  
corridor

exoplanet  
edge

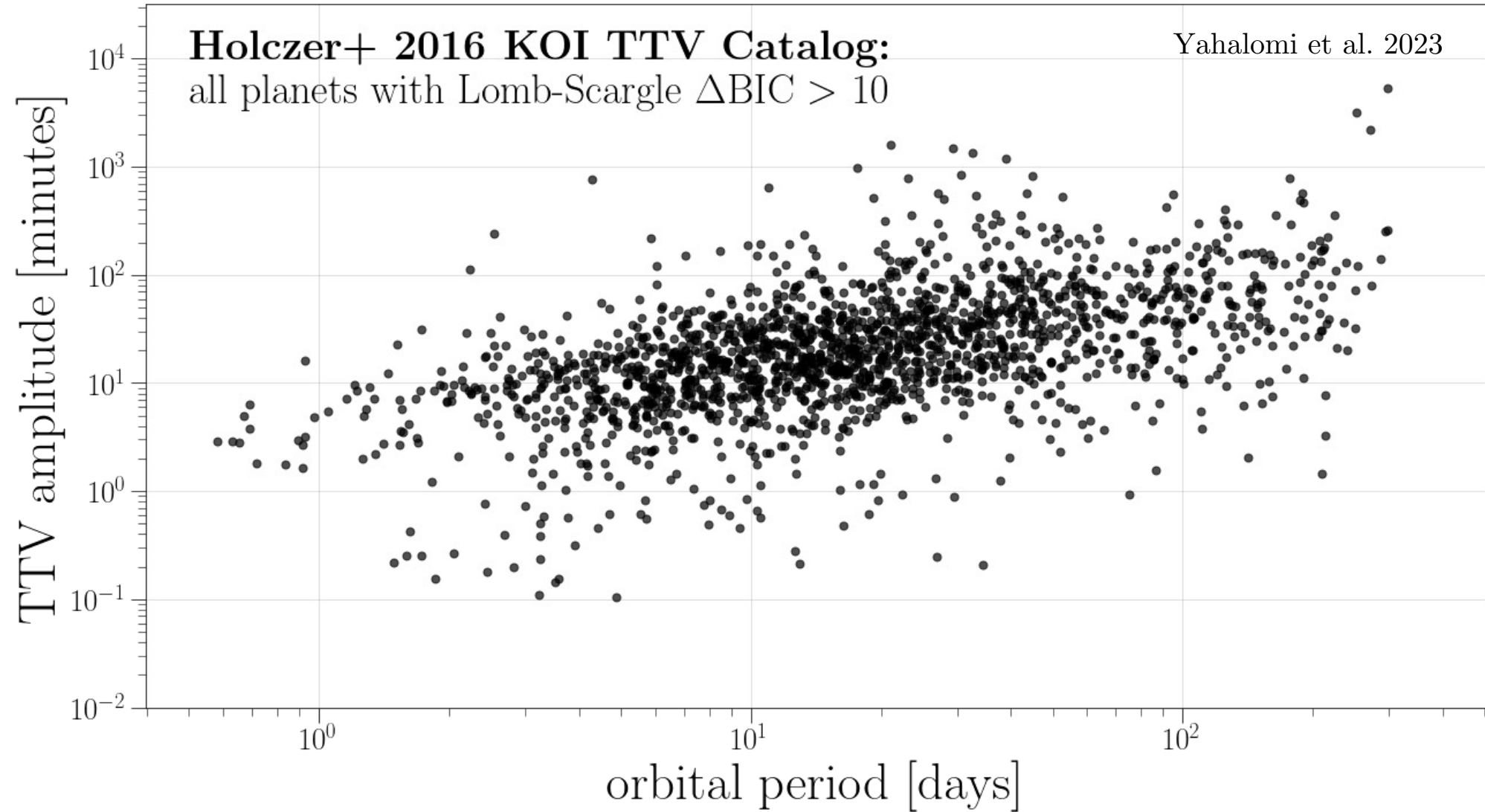




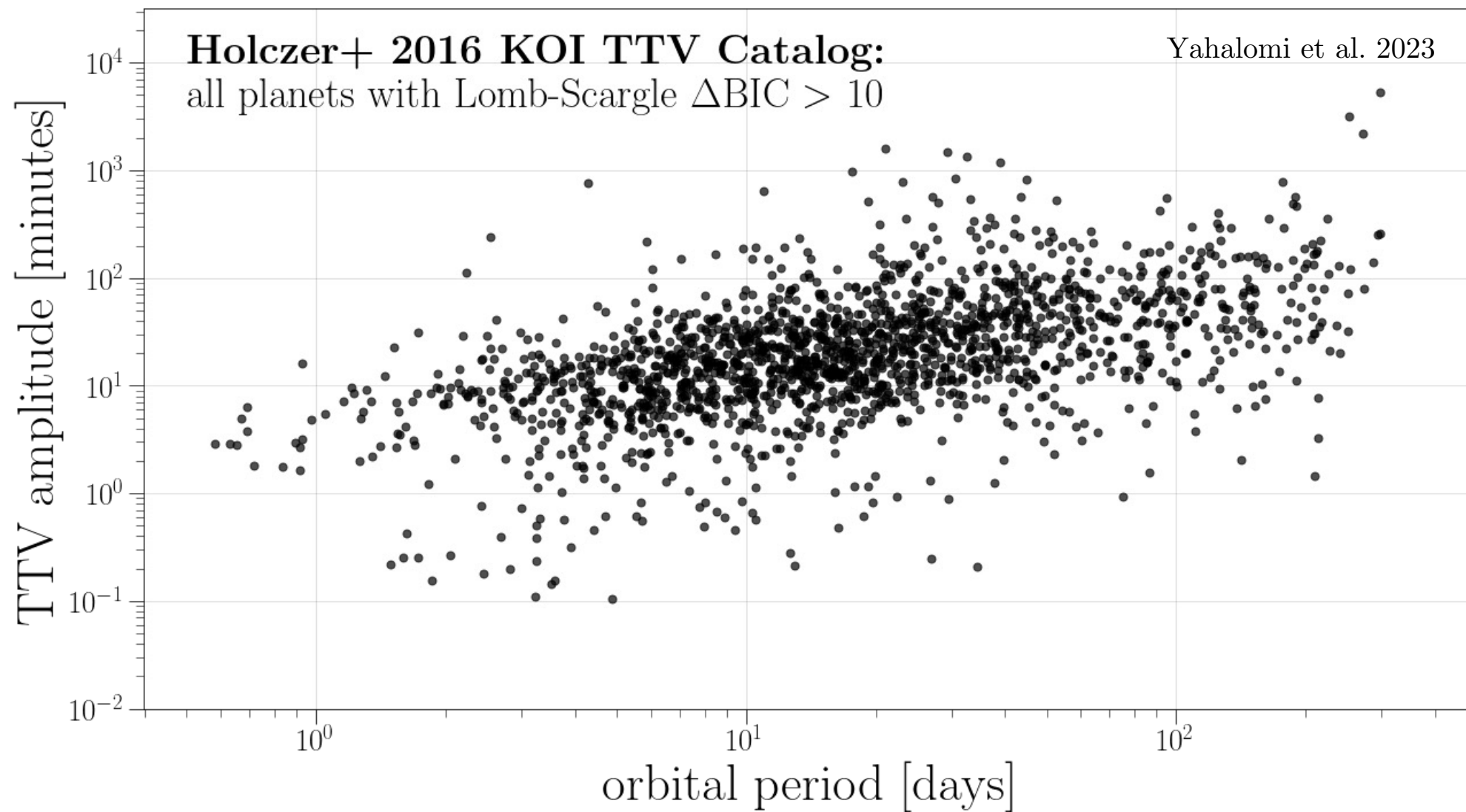


**TTVs are ubiquitous!**

# TTVs are ubiquitous!

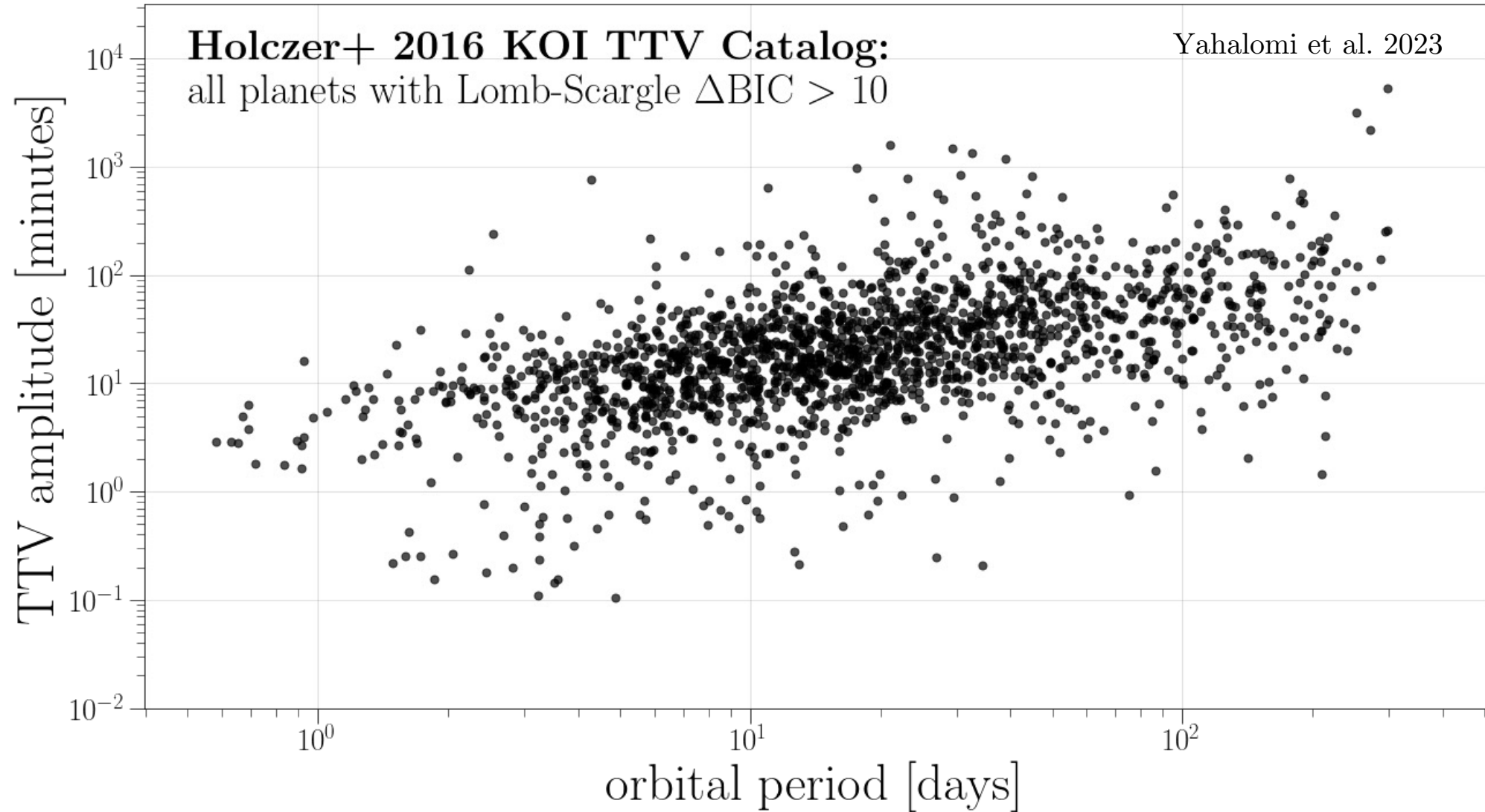


1889 KOIs “strongly favor” ( $\Delta\text{BIC} > 10$ ) sinusoidal TTV model over linear ephemeris



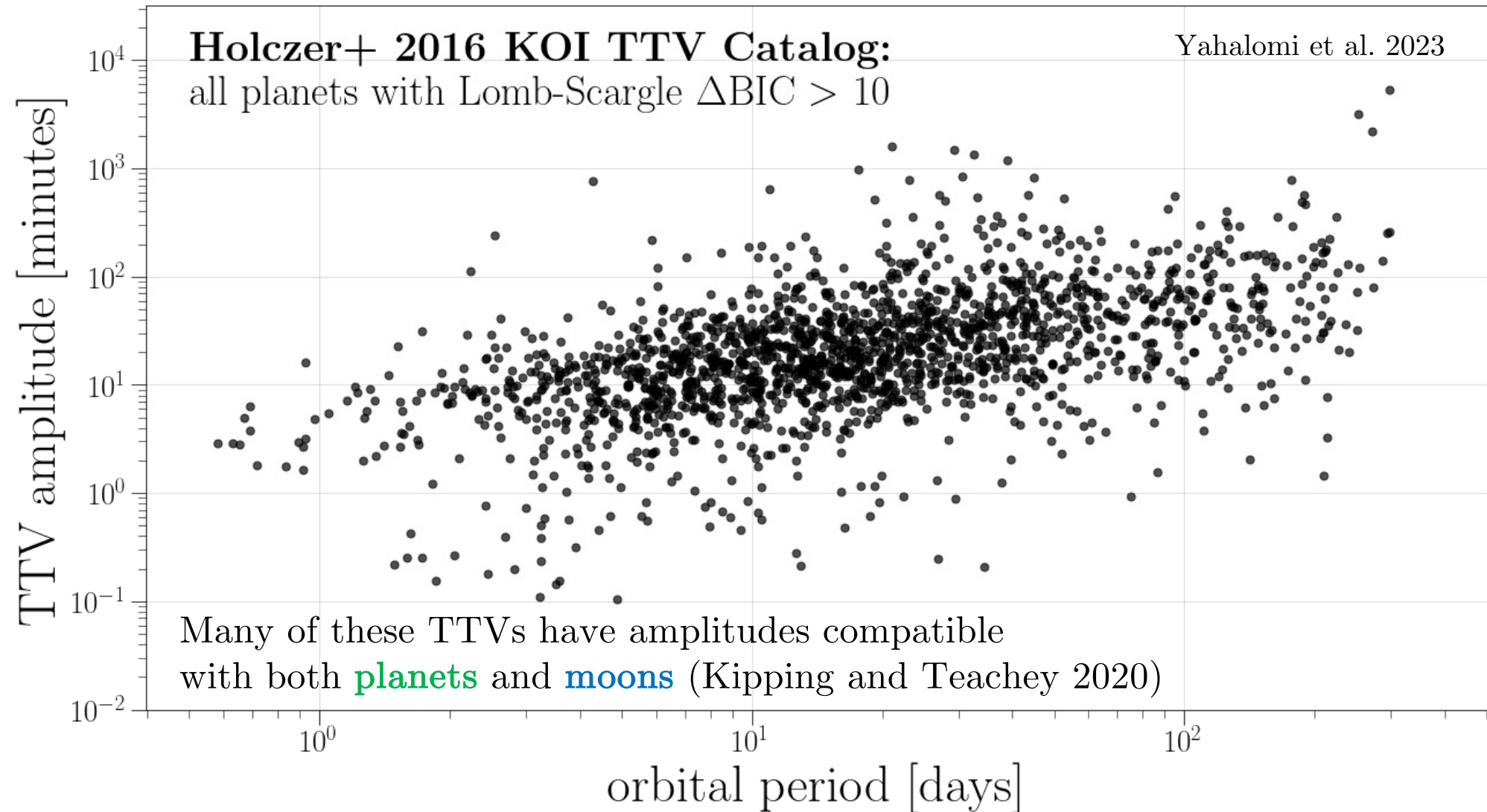
(nearly 75% of the systems in Holczer+ 2016 catalog)

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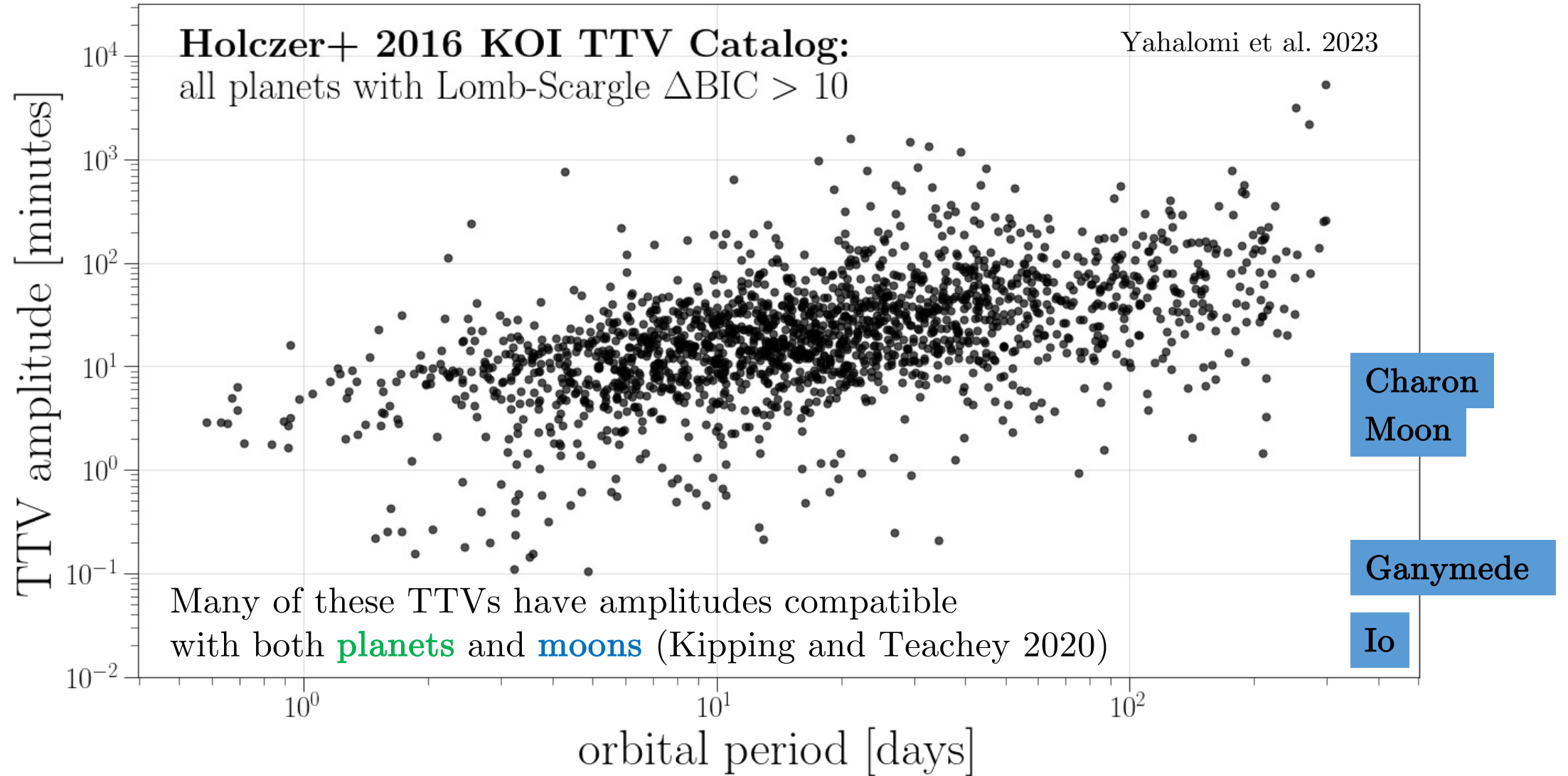
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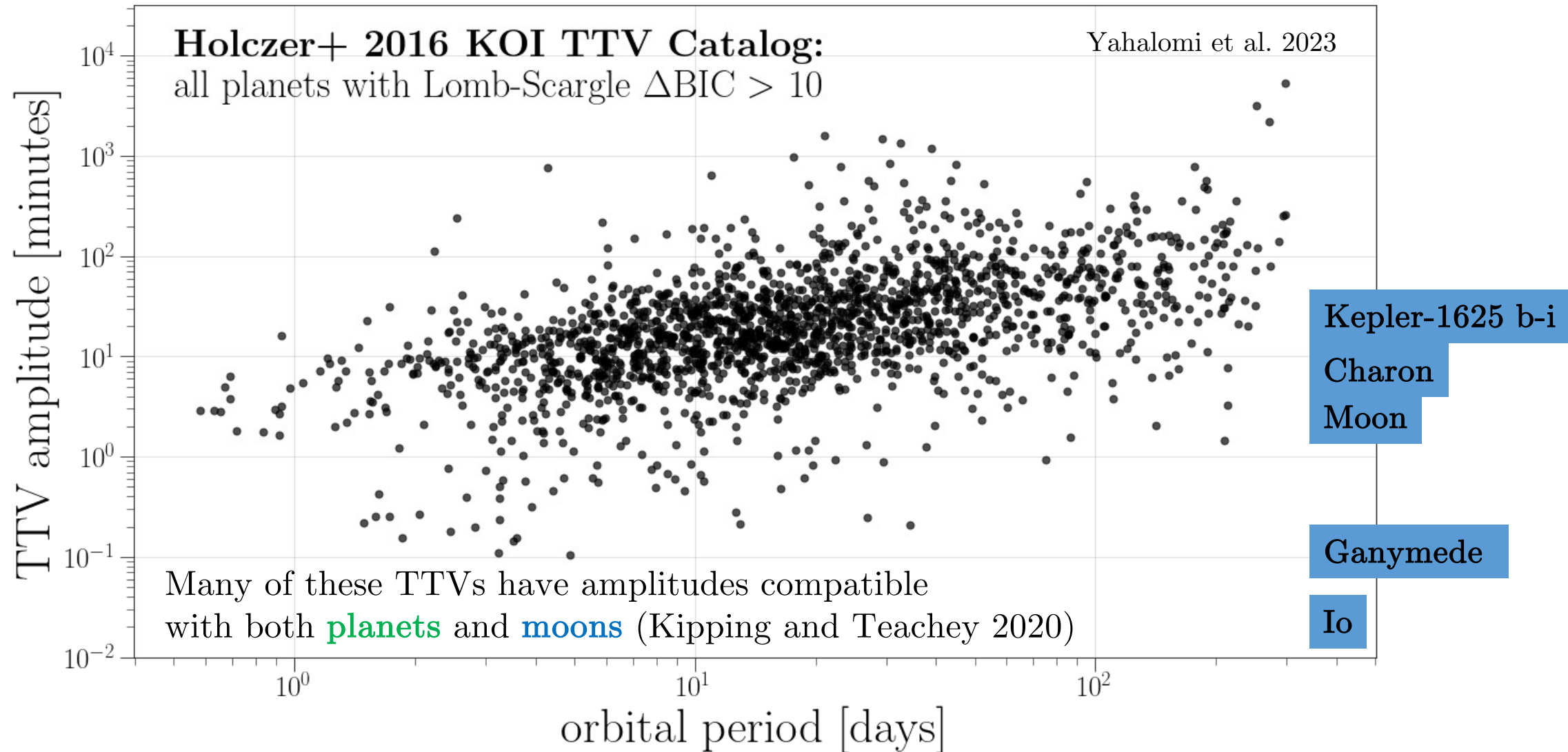
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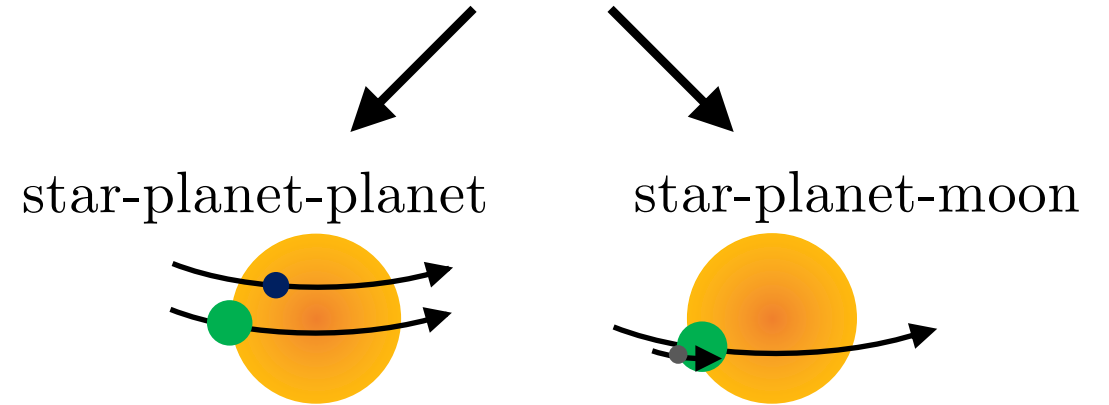
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TTVs alone are **ambiguous**

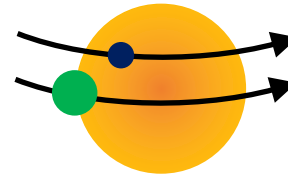
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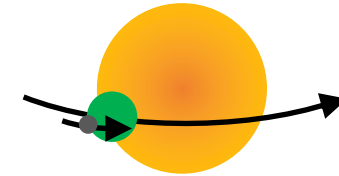
Early days of planet hunting, a transit alone was **ambiguous**

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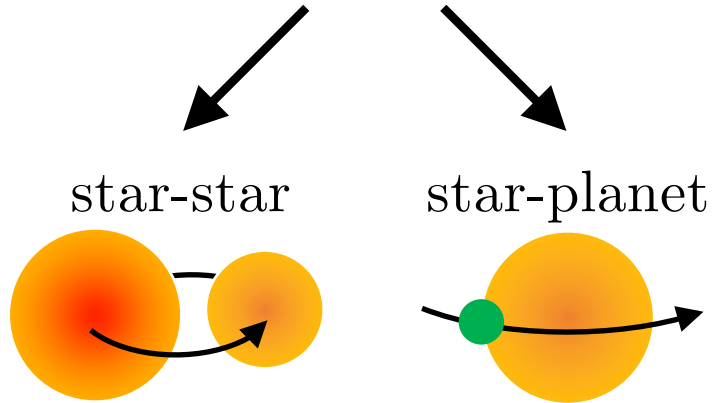
star-planet-planet



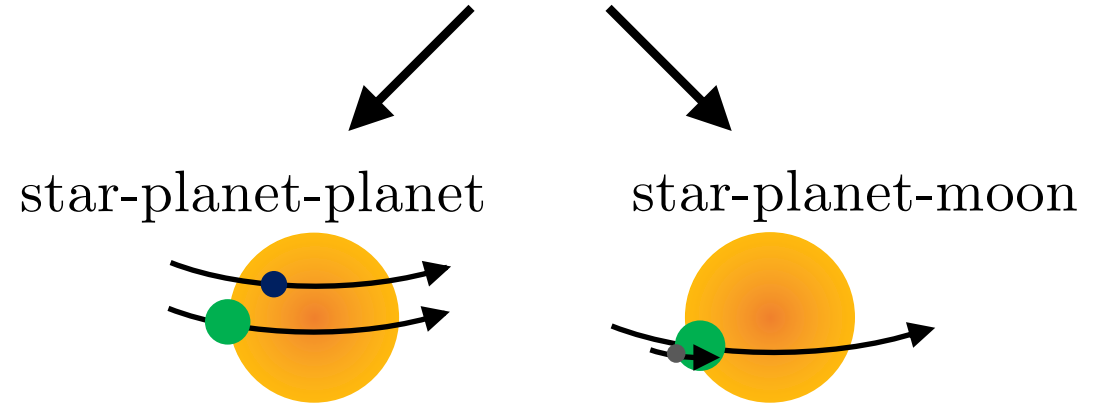
star-planet-moon



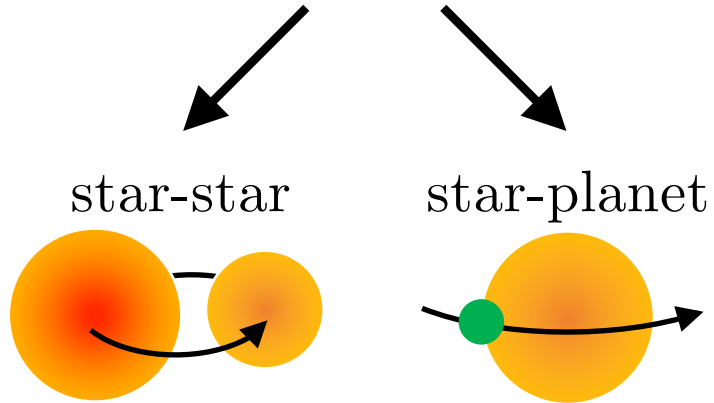
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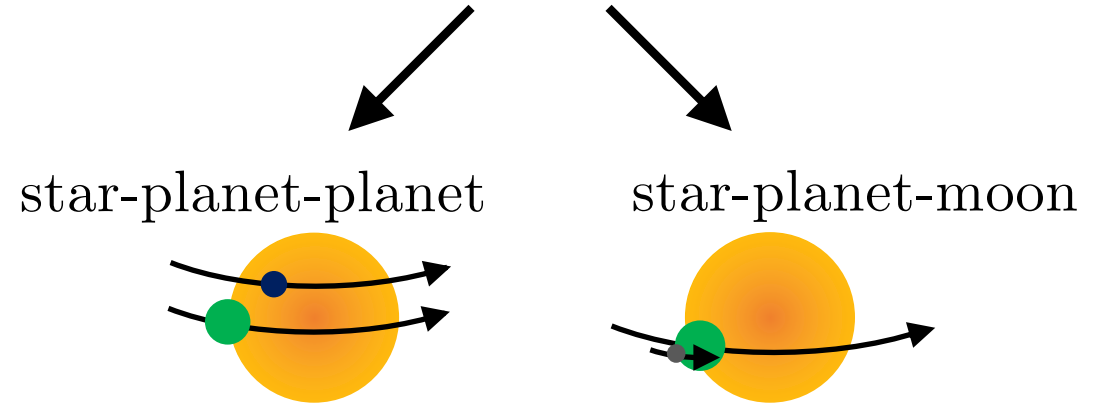


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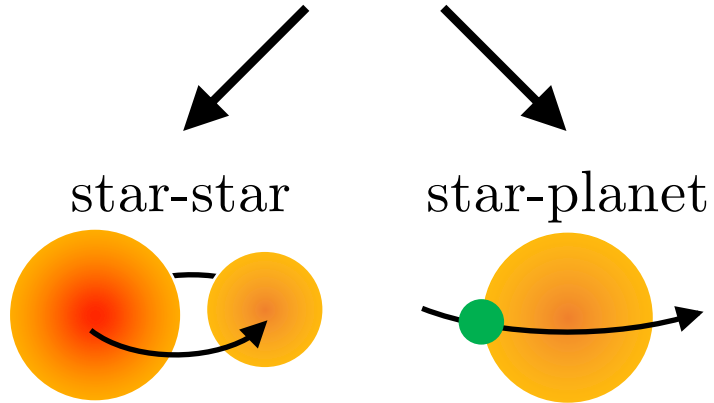


Statistical validation techniques that analyze properties of the transit shape now allow for **transit model selection**:  
(BLENDER: Torres et al. 2004, 2011)  
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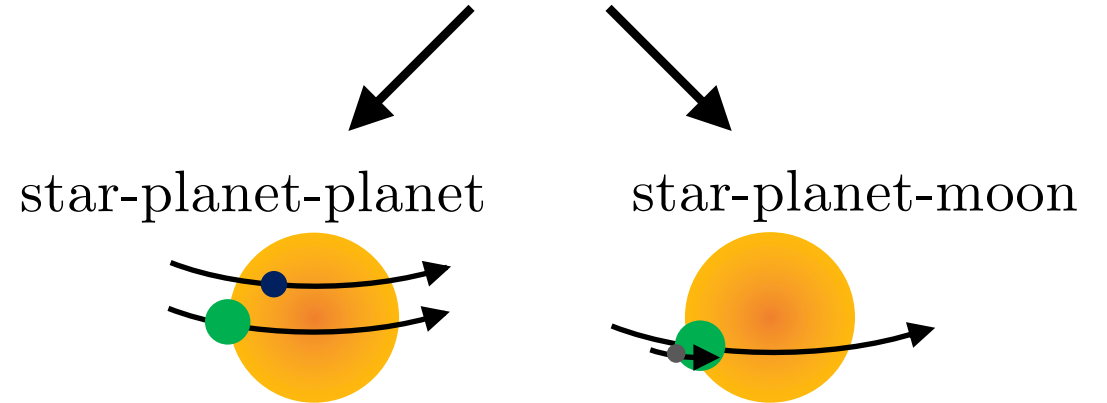


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We are developing similar tools for **TTV model selection** by analyzing the orbital landscape of exoplanet TTVs

## a (brief) history of planet-moon TTVs

- Sartoretti and Schneider (1999) were the first to suggest looking for satellites via transit timing variations.

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- Moon-planet TTV amplitude is proportional to satellite-to-planet mass ratio multiplied by the moon's semi-major axis
  - TTV amplitude can be anywhere from less than a second to up to an hour (Kipping & Teachey 2020).



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- Therefore, we expect the TTV period = aliasing of moon period

# a (brief) history of planet-moon TTVs

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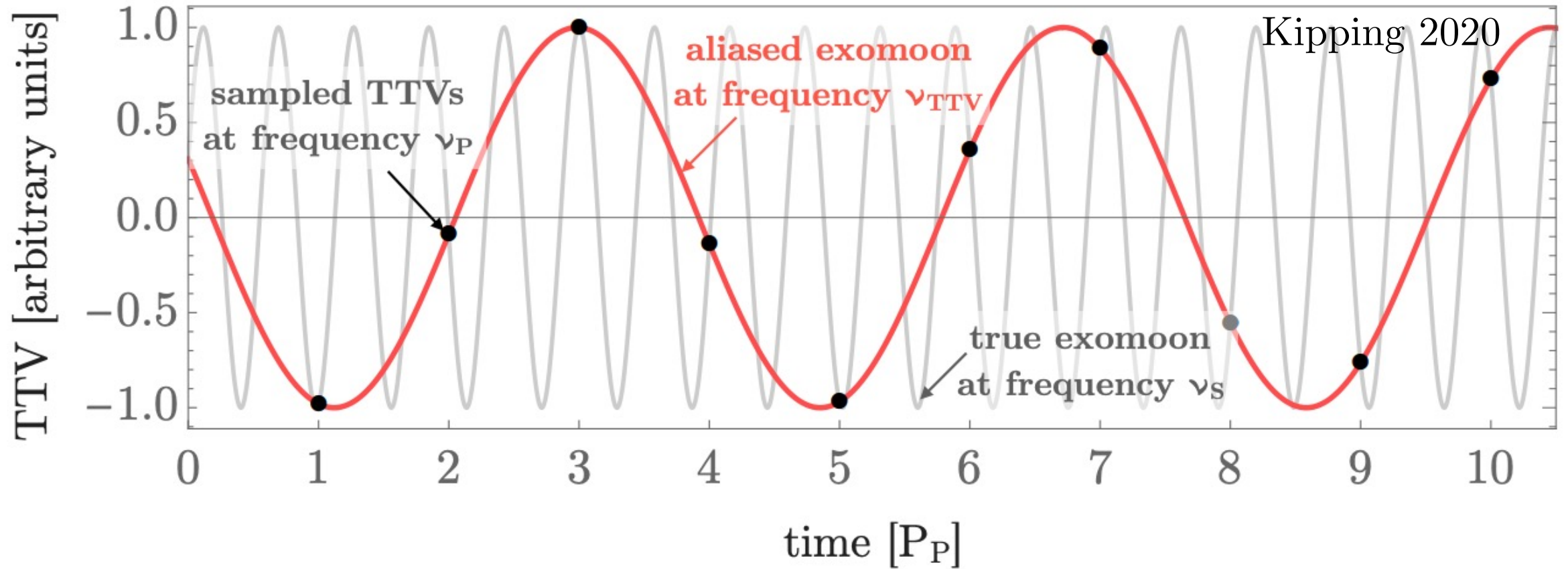
1. It is

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aliased TTV period of the Earth-Moon system

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$$P_{\text{TTV}} = \frac{1}{\frac{1}{P_S} - \text{round}\left[\frac{P_P}{P_S}\right] \frac{1}{P_P}} \quad \text{Kipping 2020}$$



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- Earth period = 365.25 days

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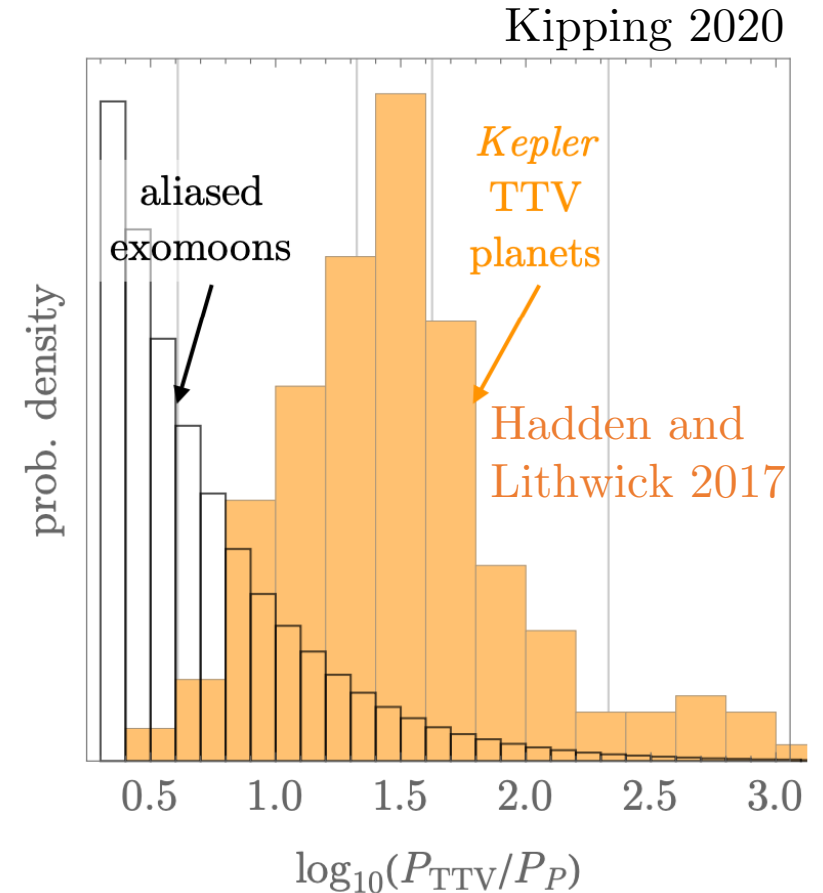
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- Moon period = 27.3 days
- Earth period = 365.25 days
  
- True Earth-Moon induced TTV period on = 27.3 days
- Observable (aliased) Earth-Moon induced TTV period = 963.41 days
- Observable (aliased) Earth-Moon induced TTV period =  $2.64 P_{\text{Earth}}$

- These aliases are more likely to manifest near the Nyquist rate
  - $f_{\text{Nyquist}} = 1/2 f_{\text{sampling}} = 1/2 f_{\text{transit}}$
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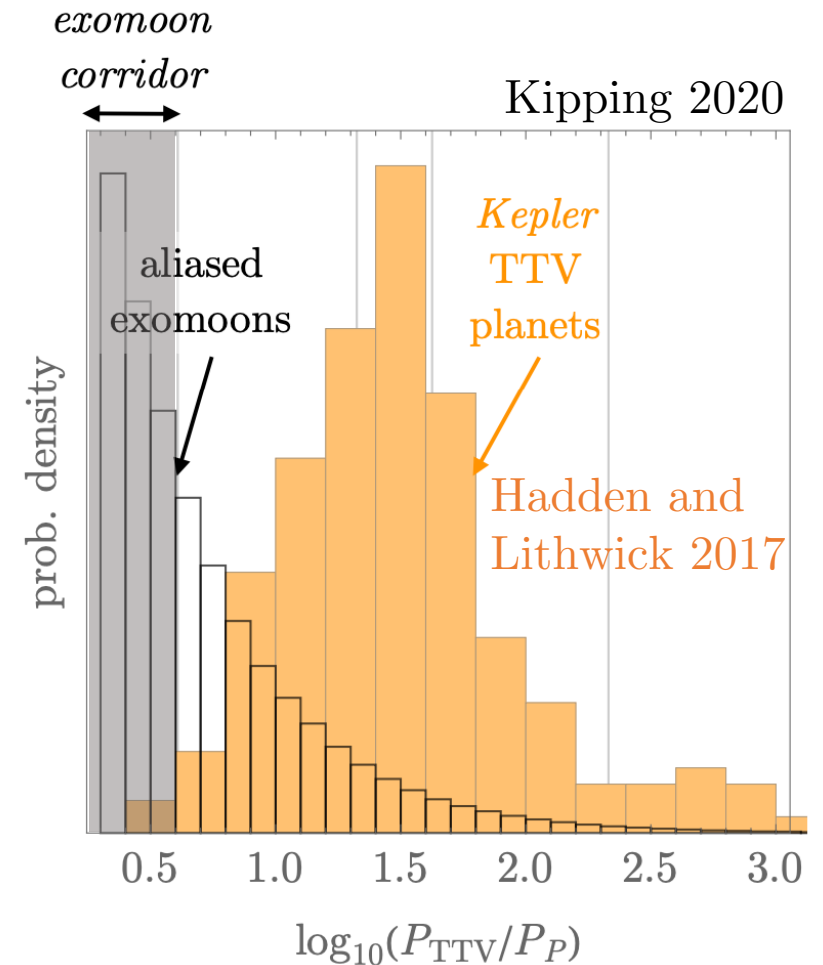
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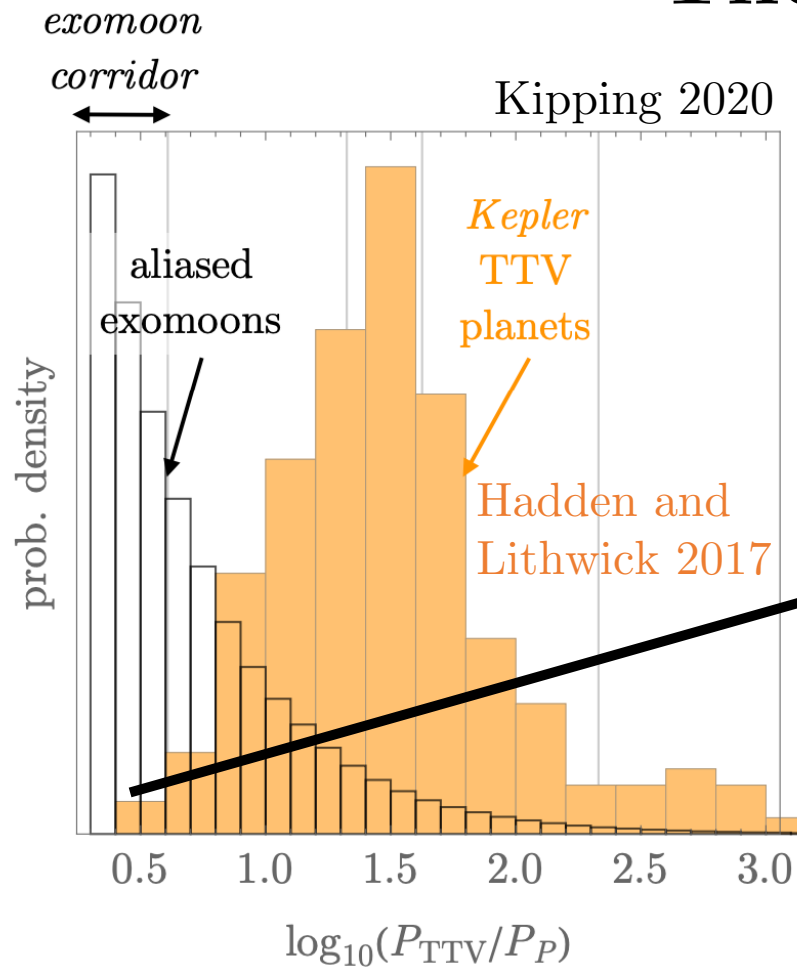


# The Exomoon Corridor

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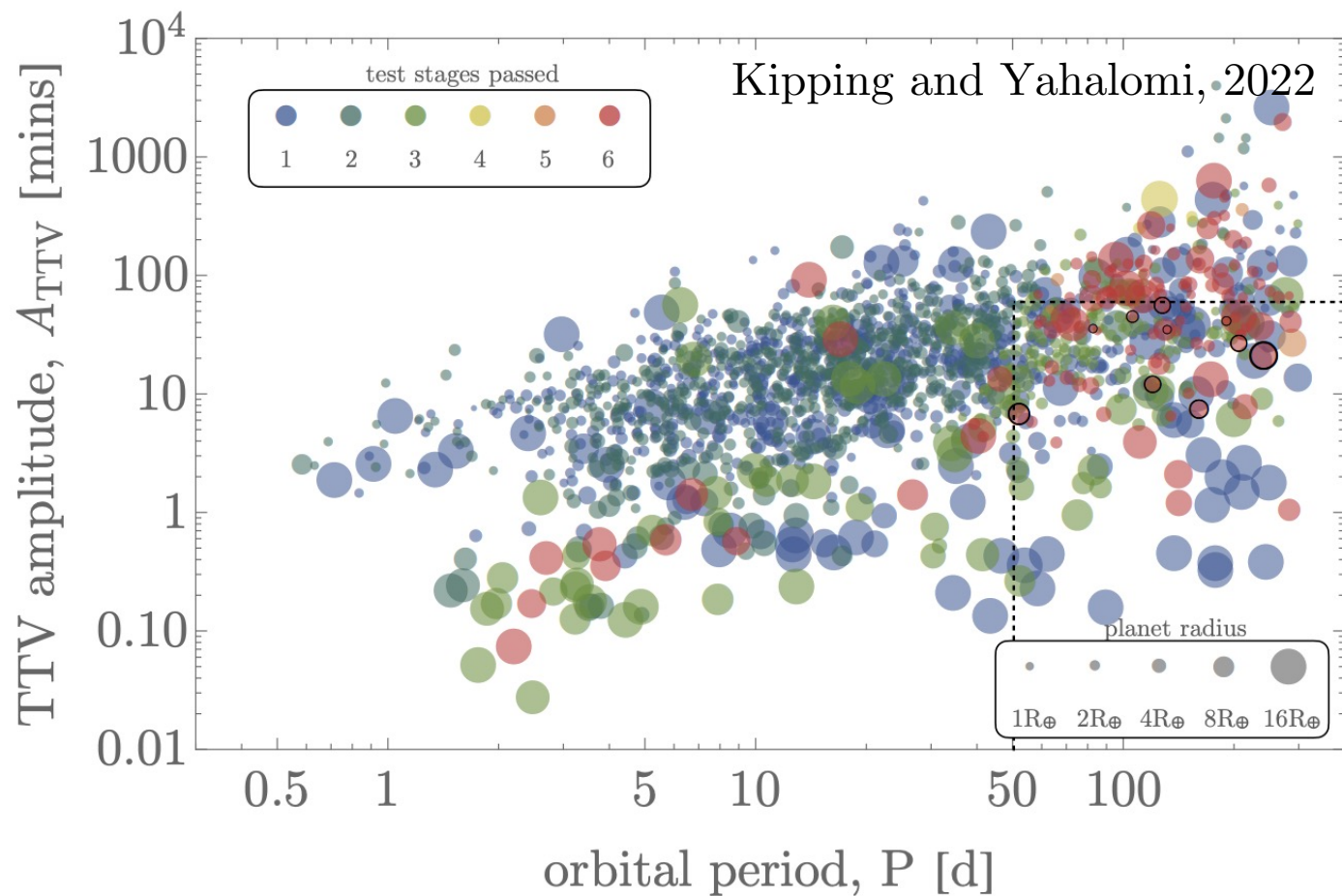
let's search the exomoon corridor of Kepler data for possible new moons!

(Kipping and Yahalomi, 2022)

# A search for transit timing variations within the exomoon corridor using *Kepler* data

David Kipping<sup>1\*</sup> and Daniel A. Yahalomi<sup>1</sup>

<sup>1</sup>Dept. of Astronomy, Columbia University, 550 W 120th Street, New York NY 10027

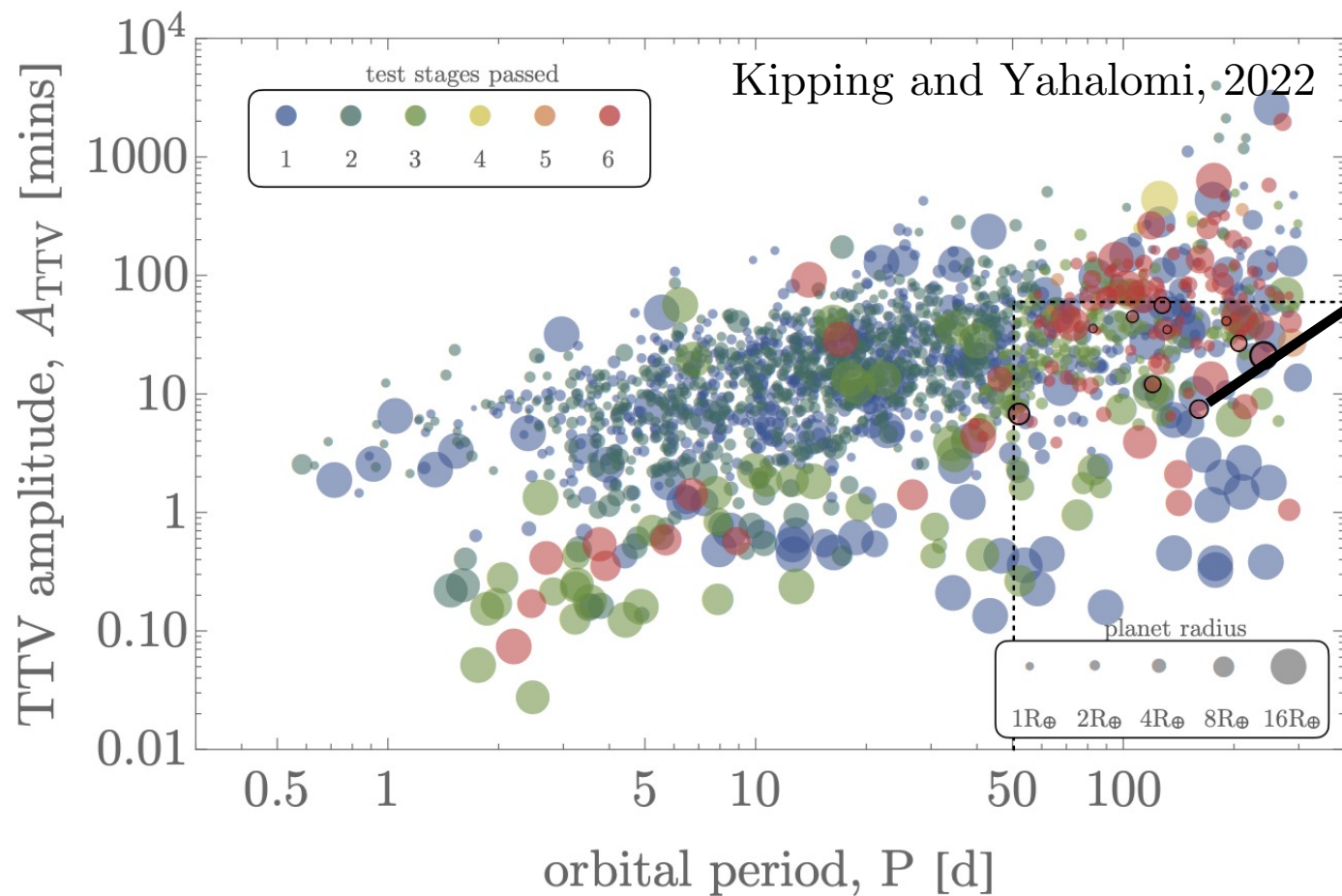




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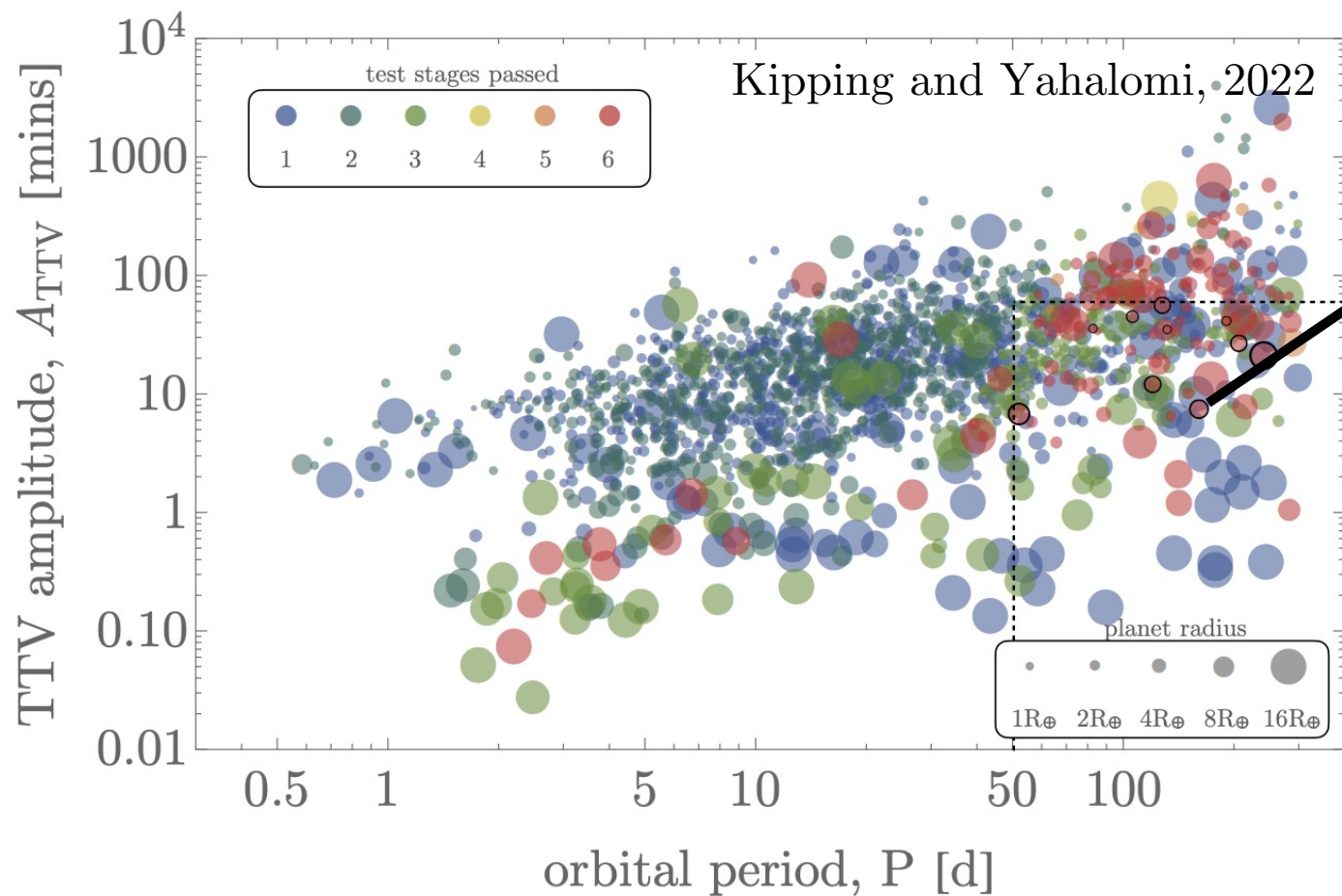
**Kepler-1513b**

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## Kepler-1513b

(a gas giant on  $\sim 160$ -day orbit)

is the unseen companion to  
Kepler-1513b  
a planet or a moon?

(Yahalomi et al. 2023)

# Kepler-1513b

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- Critical step in transit analysis is removal of systematics in time-series photometry.

# Kepler-1513b

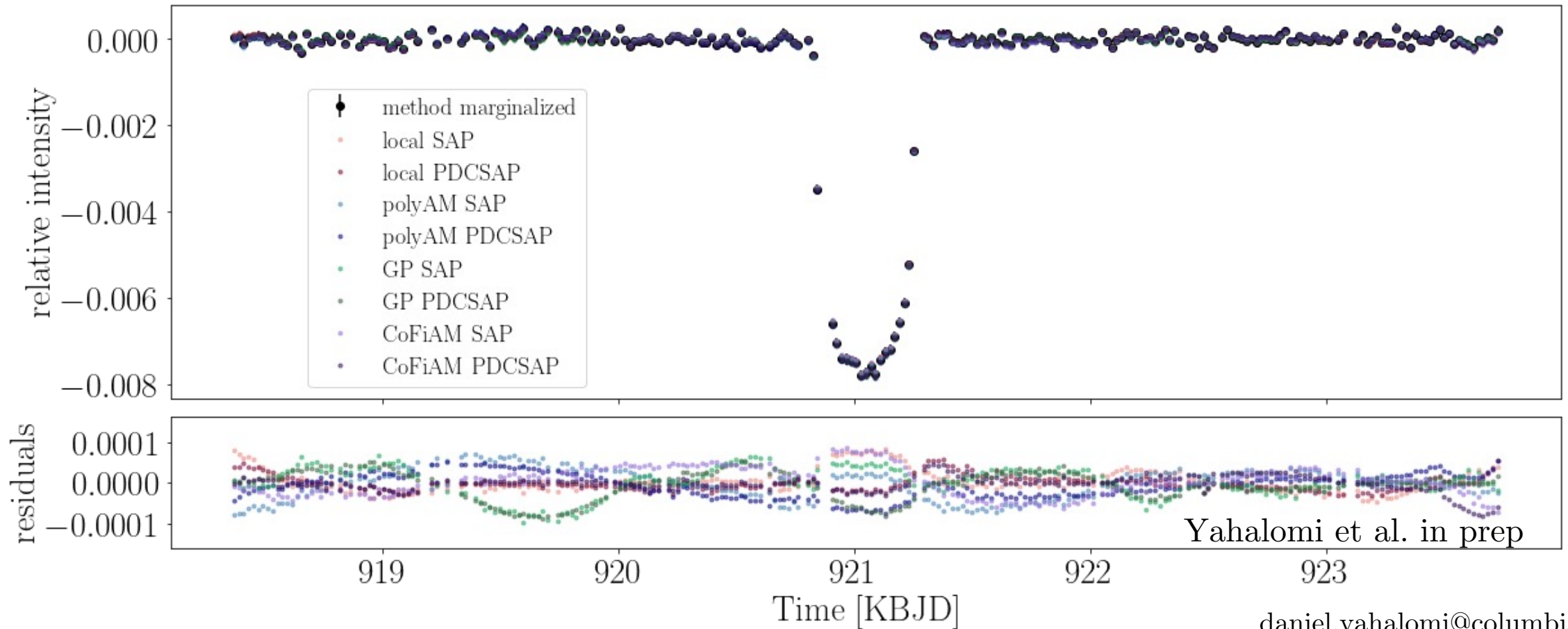
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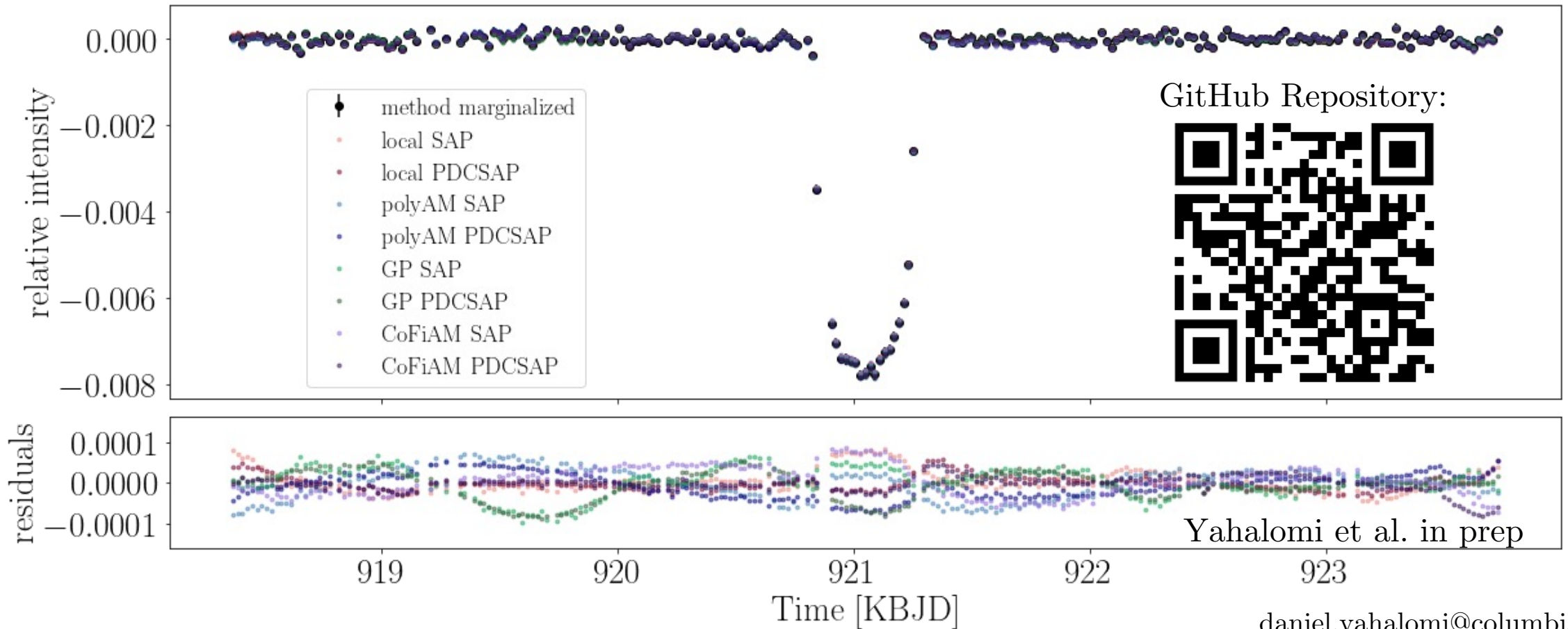
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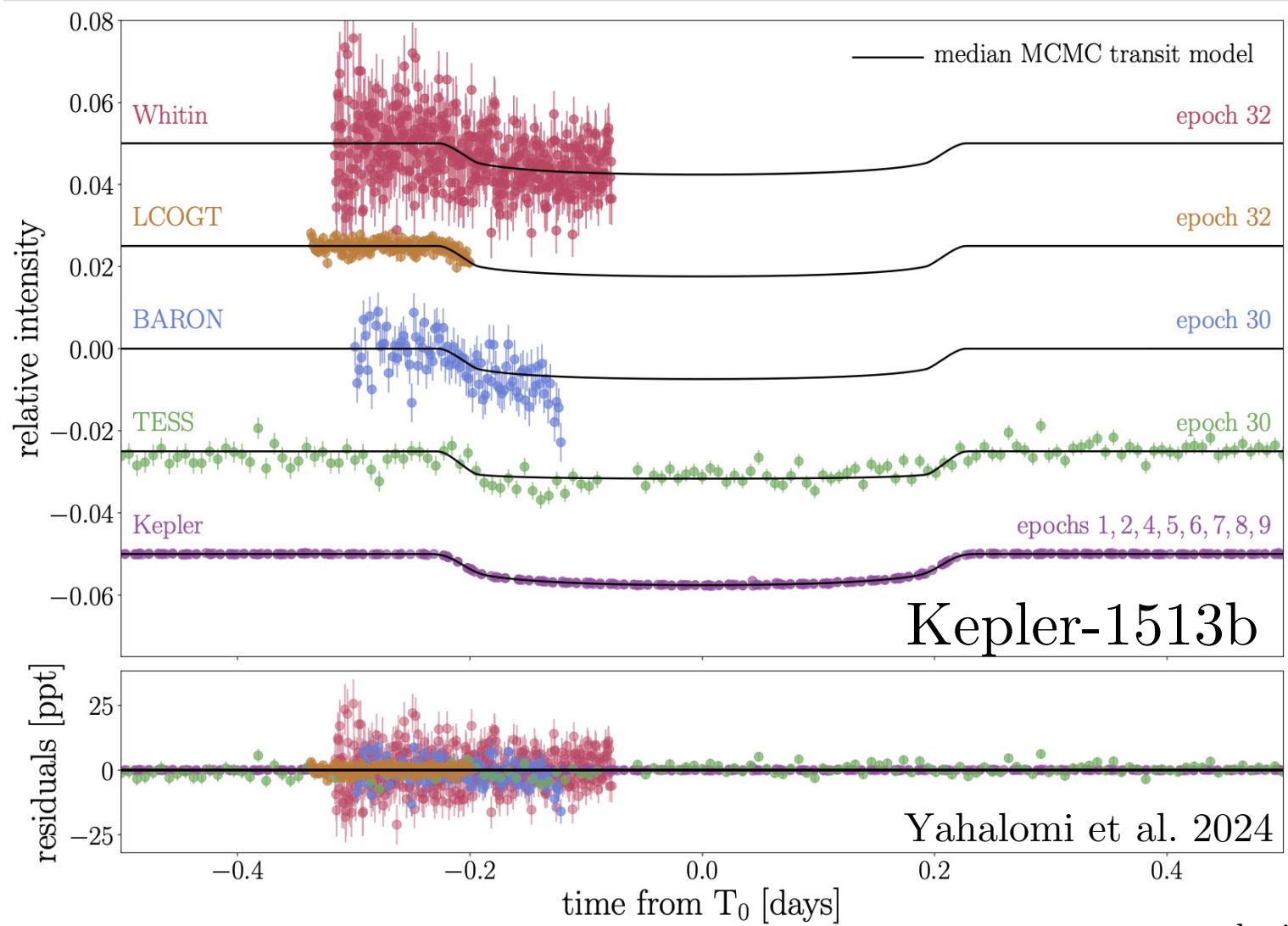
Yahalomi et al. in prep

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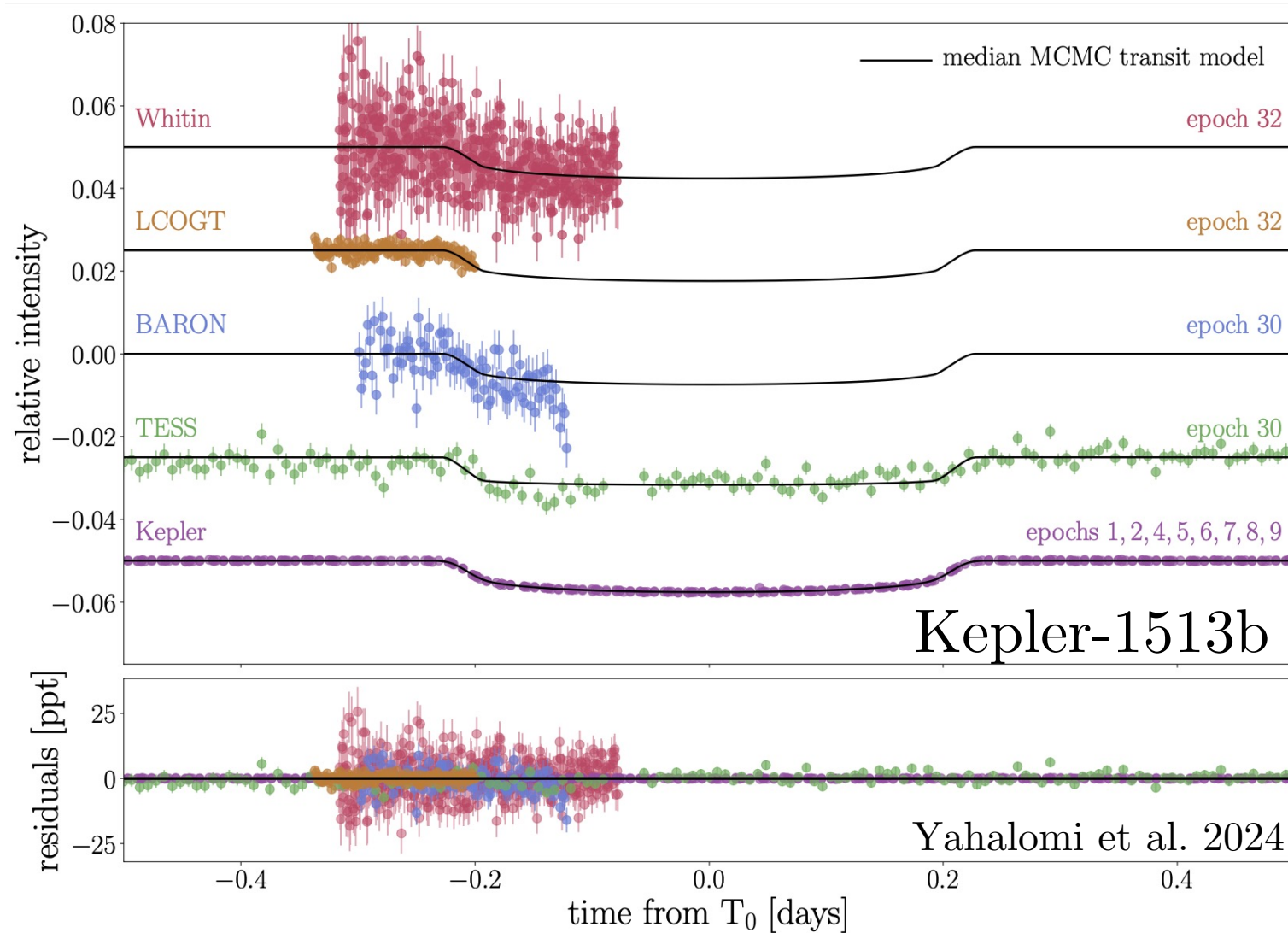
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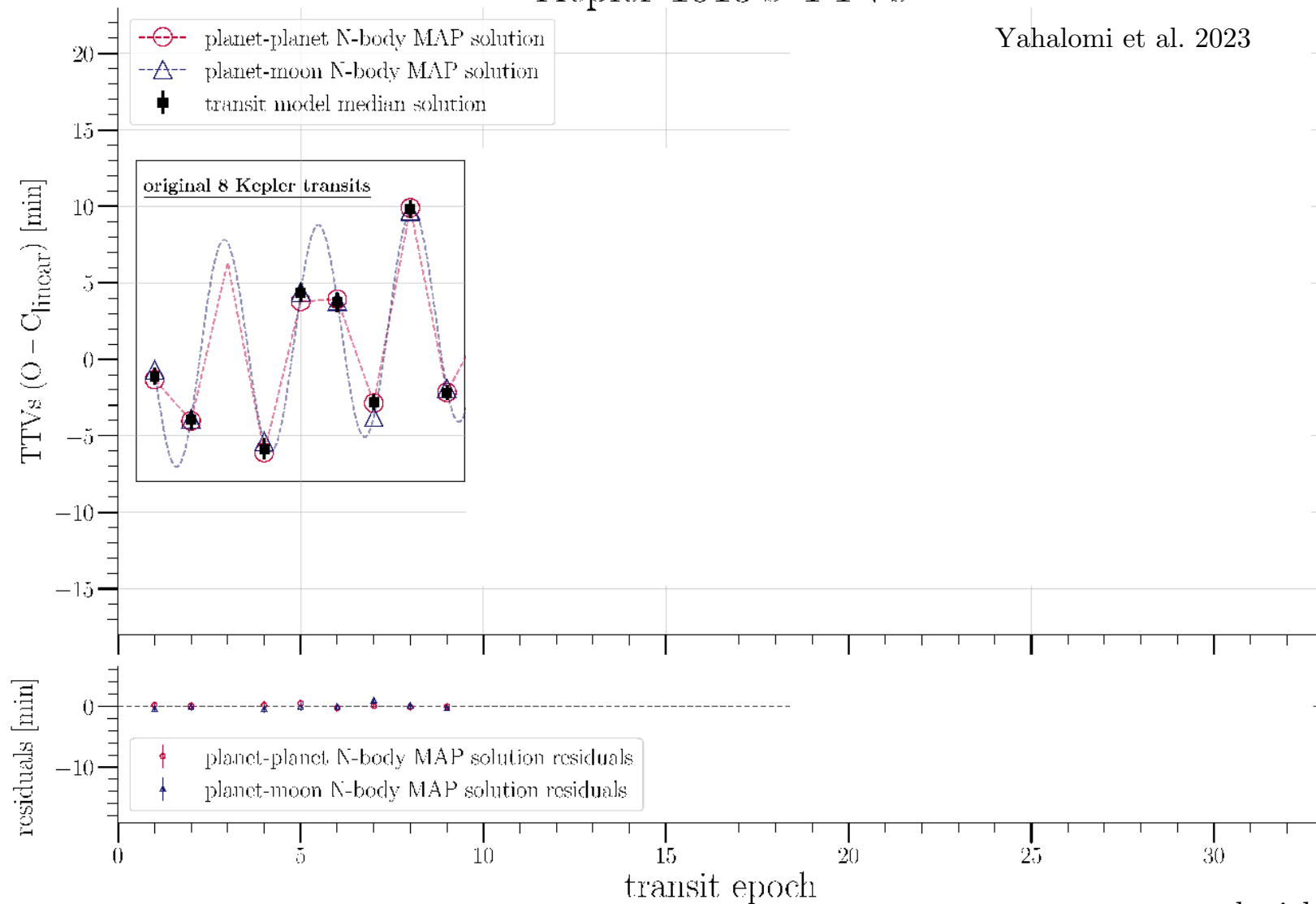


we observed 2 additional transits  $\sim 9$  years after the last Kepler transit



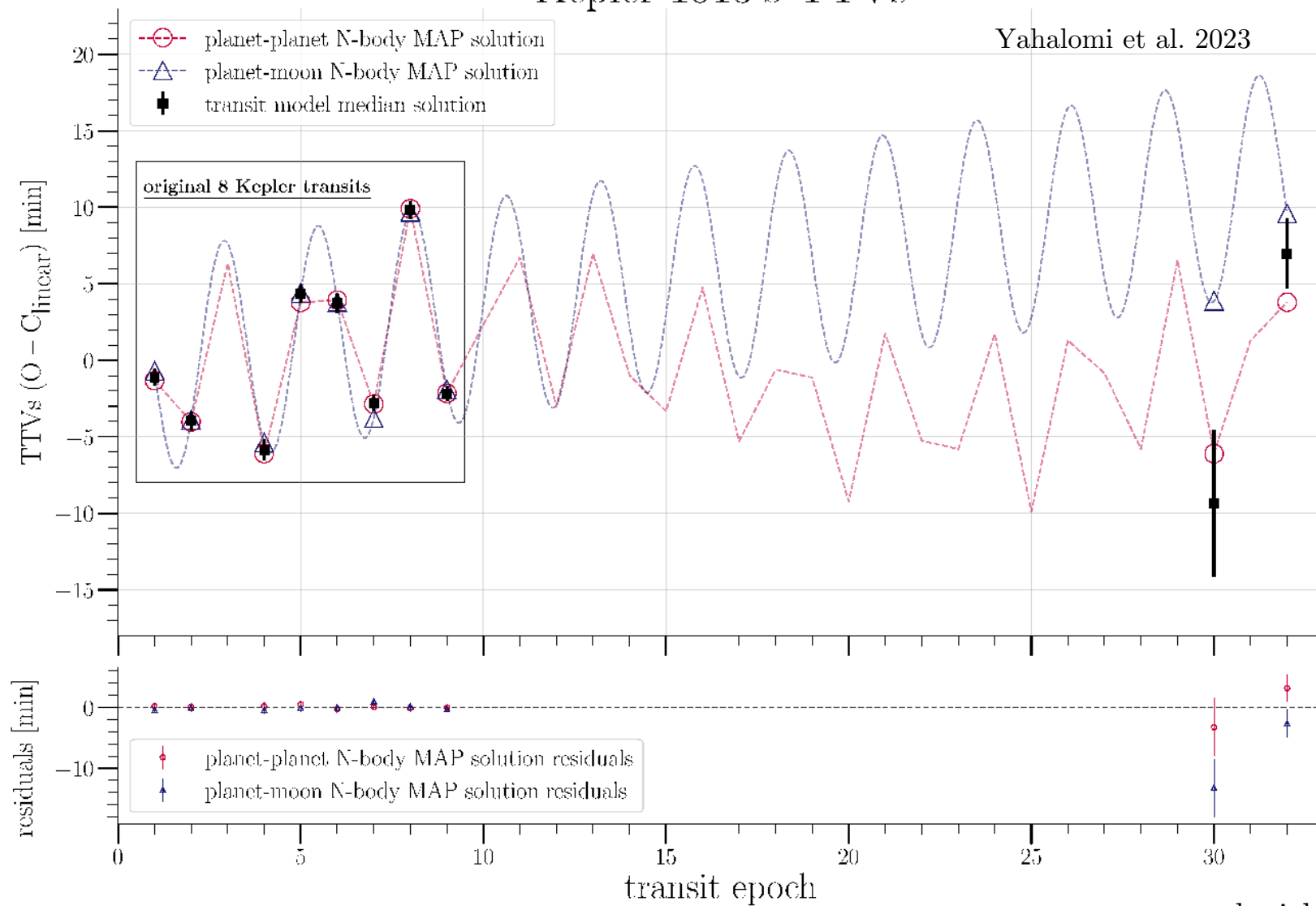
# Kepler-1513 b TTVs

Yahalomi et al. 2023



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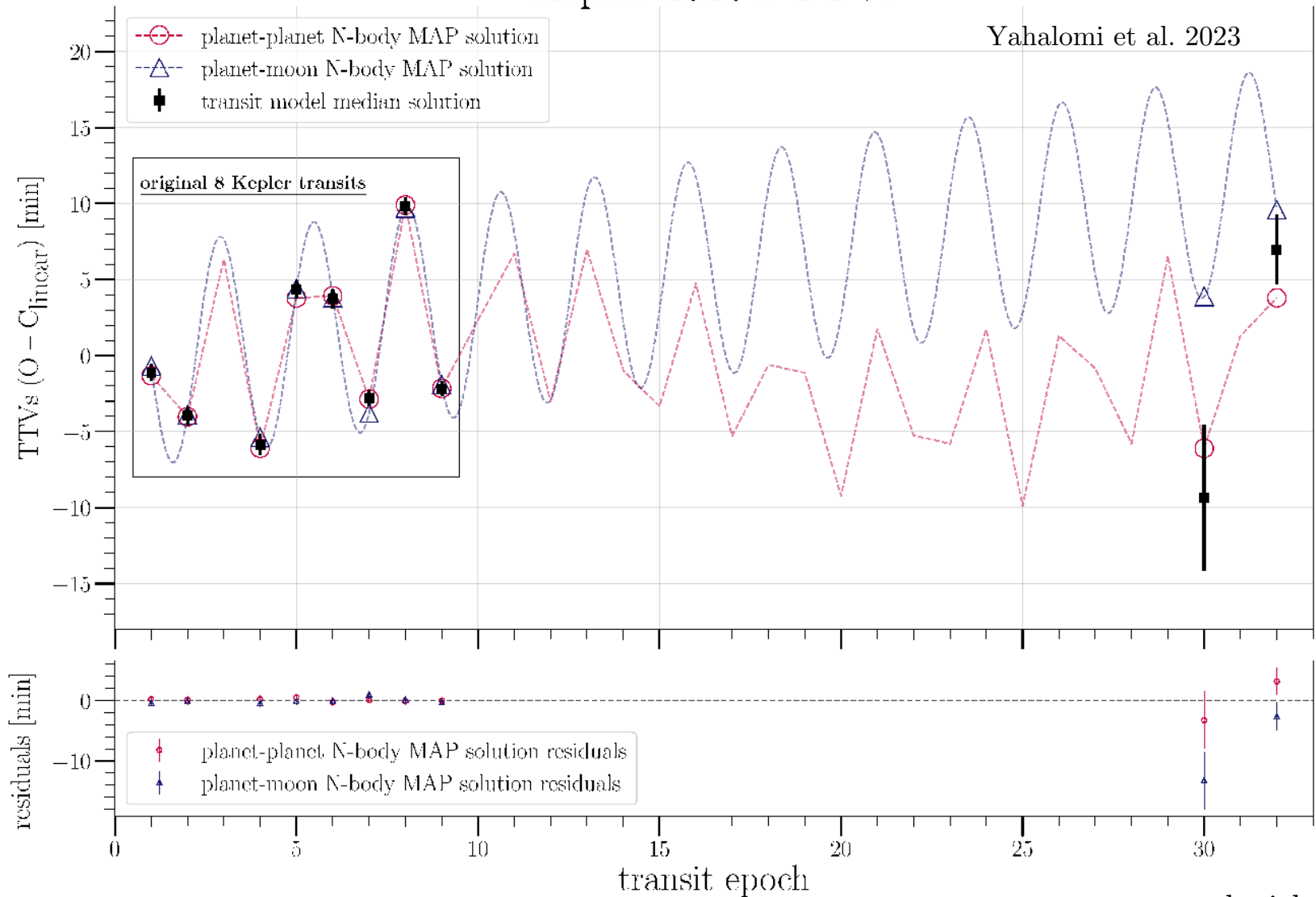
Yahalomi et al. 2023




follow-up transit observations reveal second, slow-period TTV...

### Kepler-1513 b TTVs

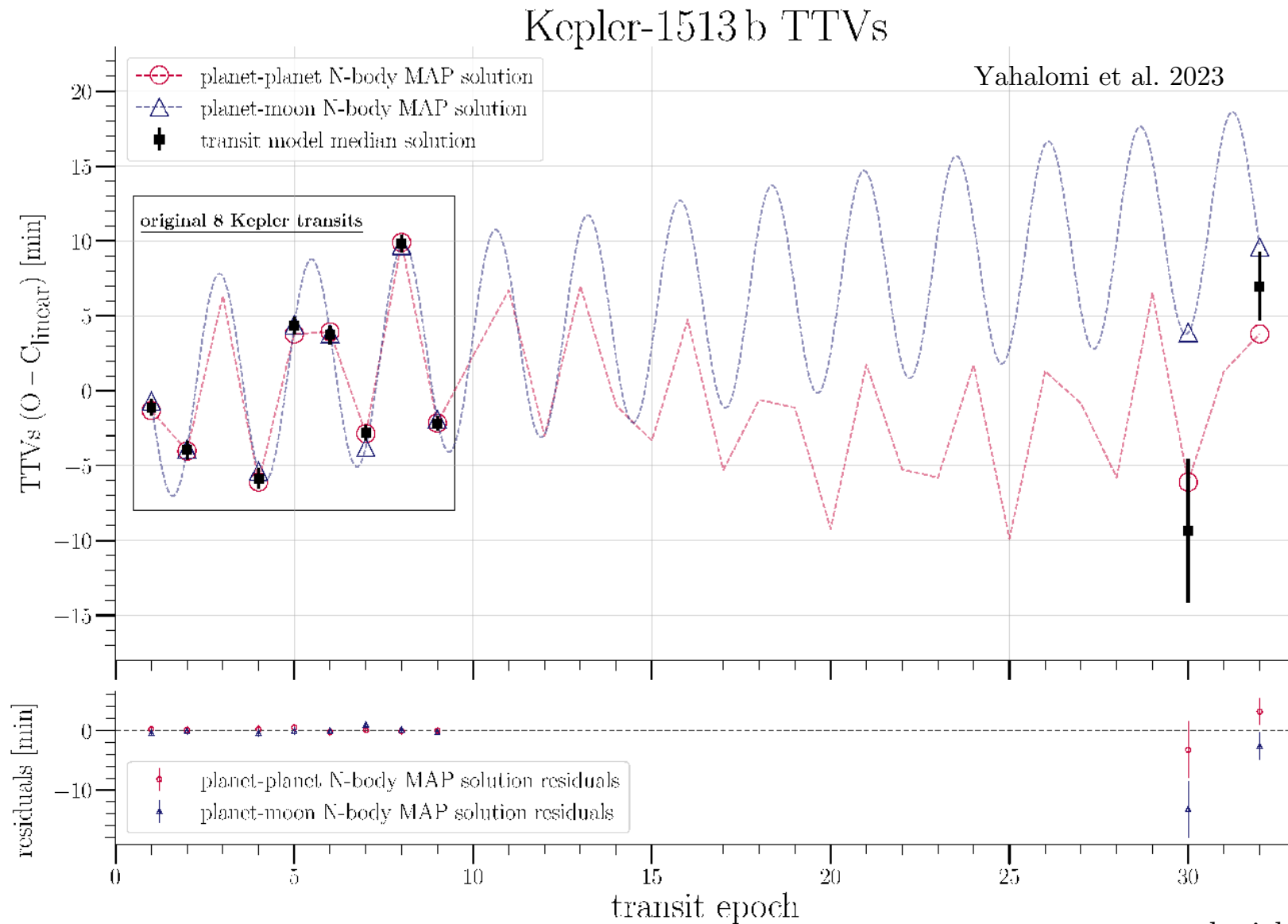
Yahalomi et al. 2023





*That's no moon.*

Kepler-1513c is most likely a Saturn that is  $\sim 5\%$  outside a 5:1 period ratio

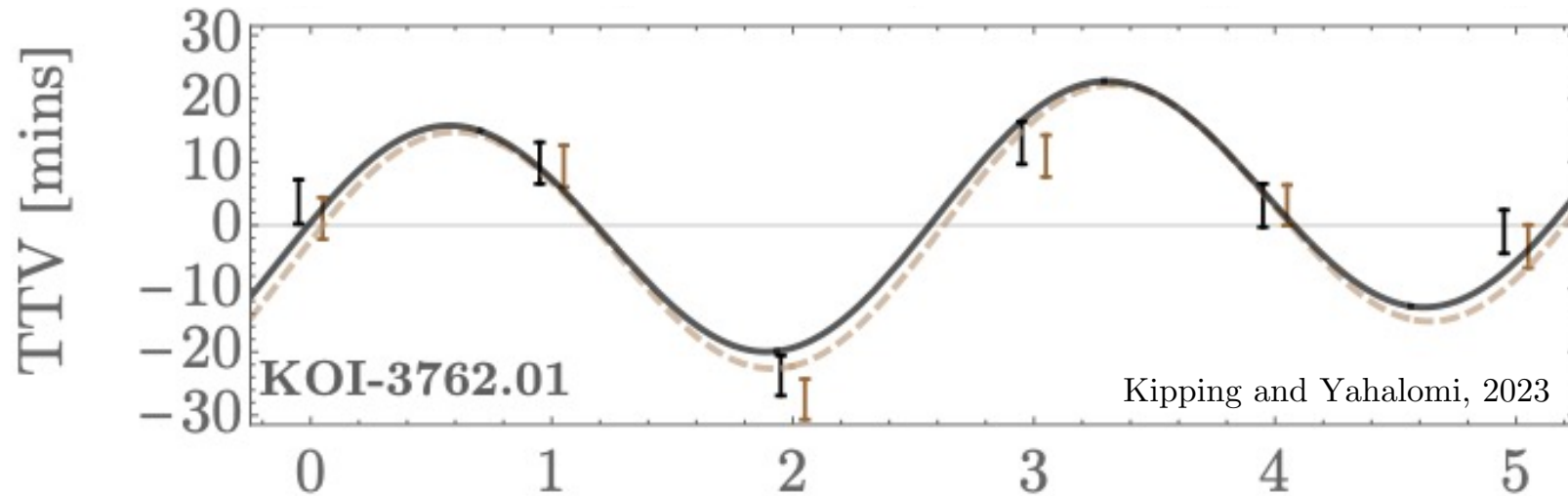


Additional targets in the exomoon corridor:



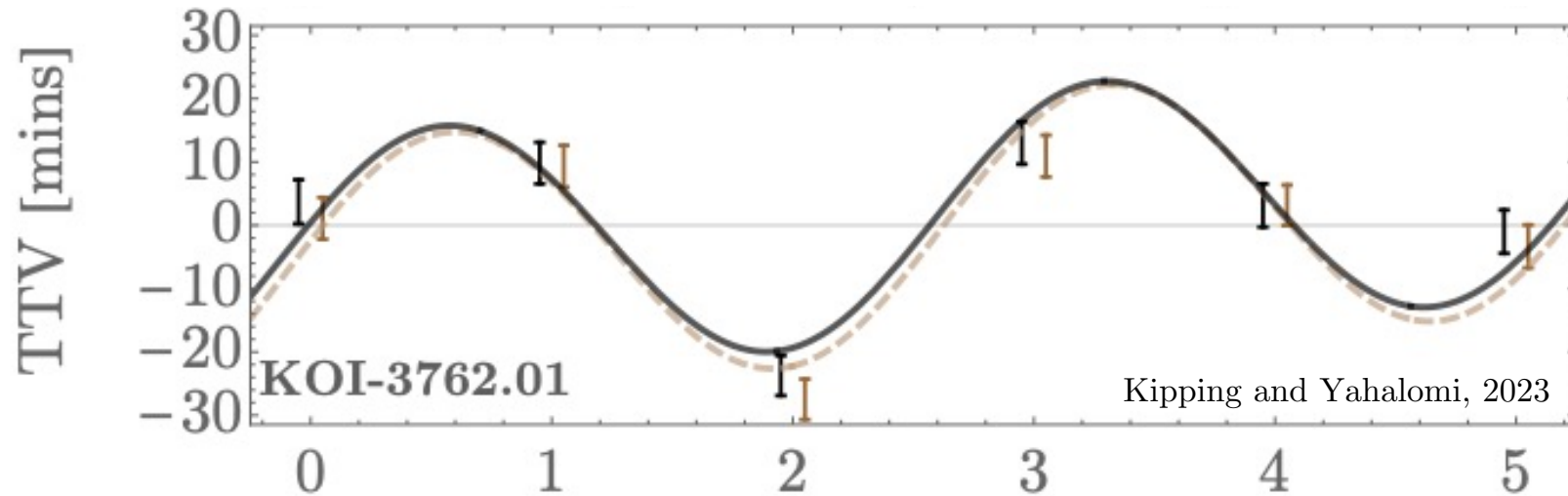
## Additional targets in the exomoon corridor:

- KOI-3762b is a  $\sim 10 R_{\text{Earth}}$  gas giant at  $\sim 241$  days.



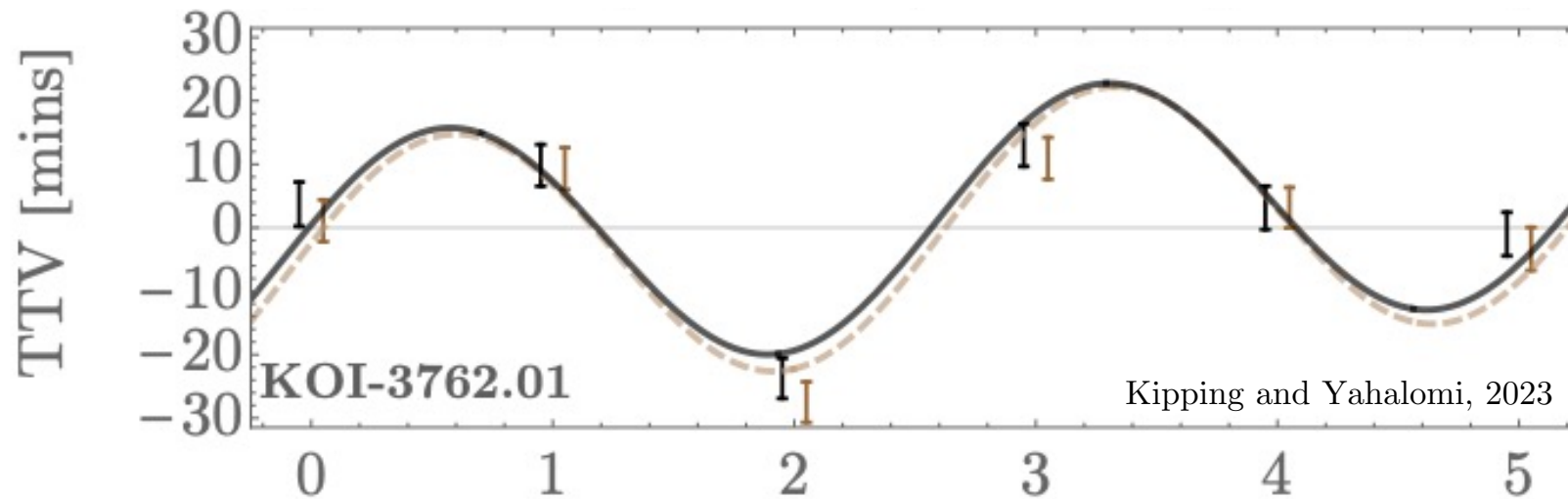
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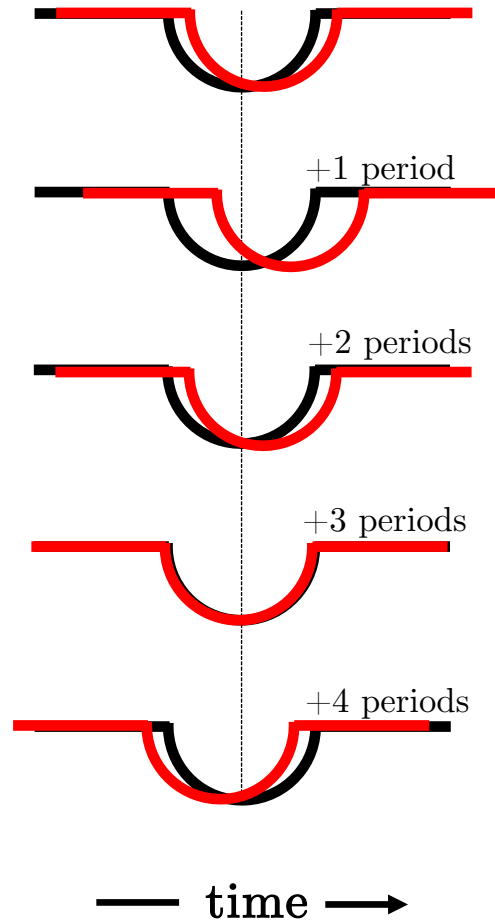
- KOI-3762b is a  $\sim 10 R_{\text{Earth}}$  gas giant at  $\sim 241$  days.
- Currently, short-period TTV signal dominated by single epoch...
- Transit opportunity in June!



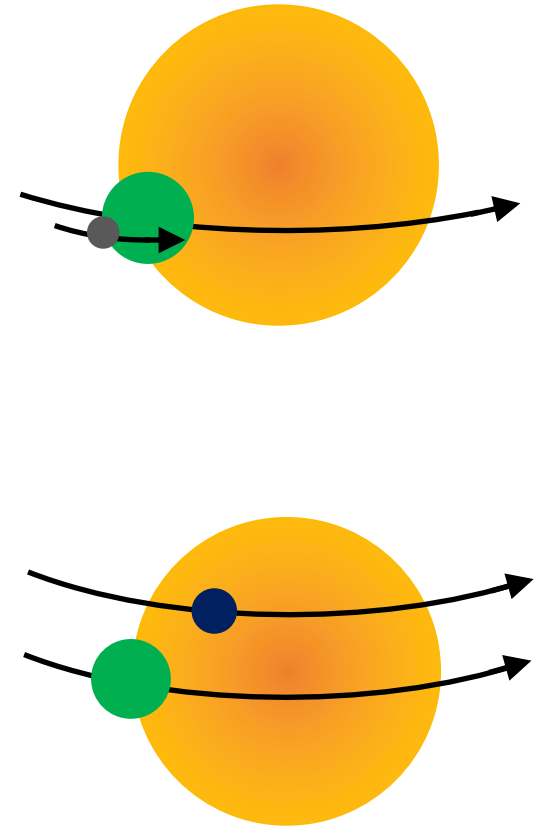
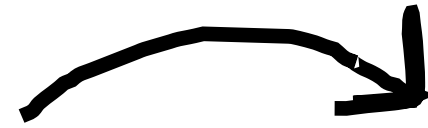
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exomoon  
corridor





# planet-planet TTVs characteristic periods

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near MMR induced TTVs

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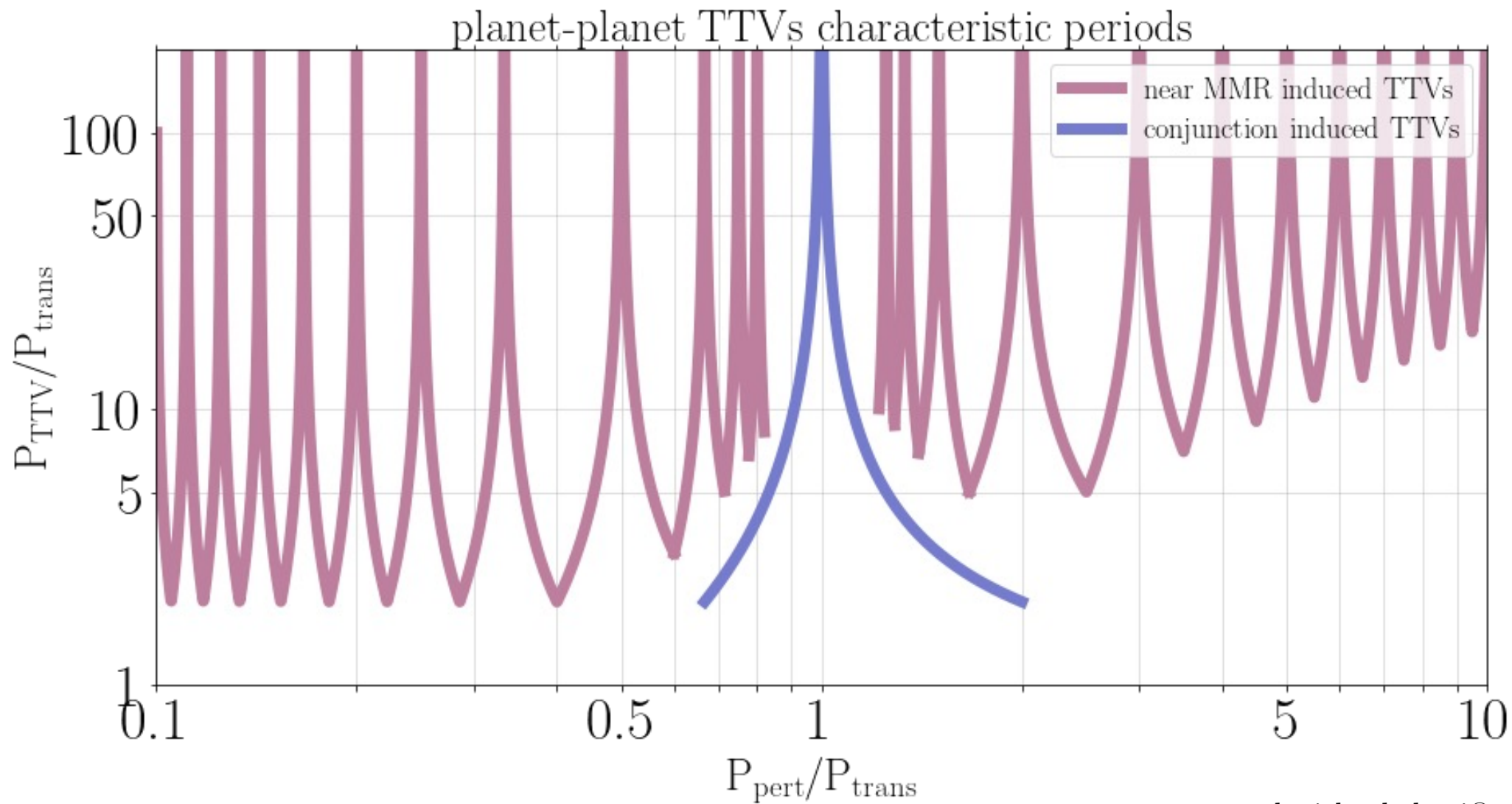
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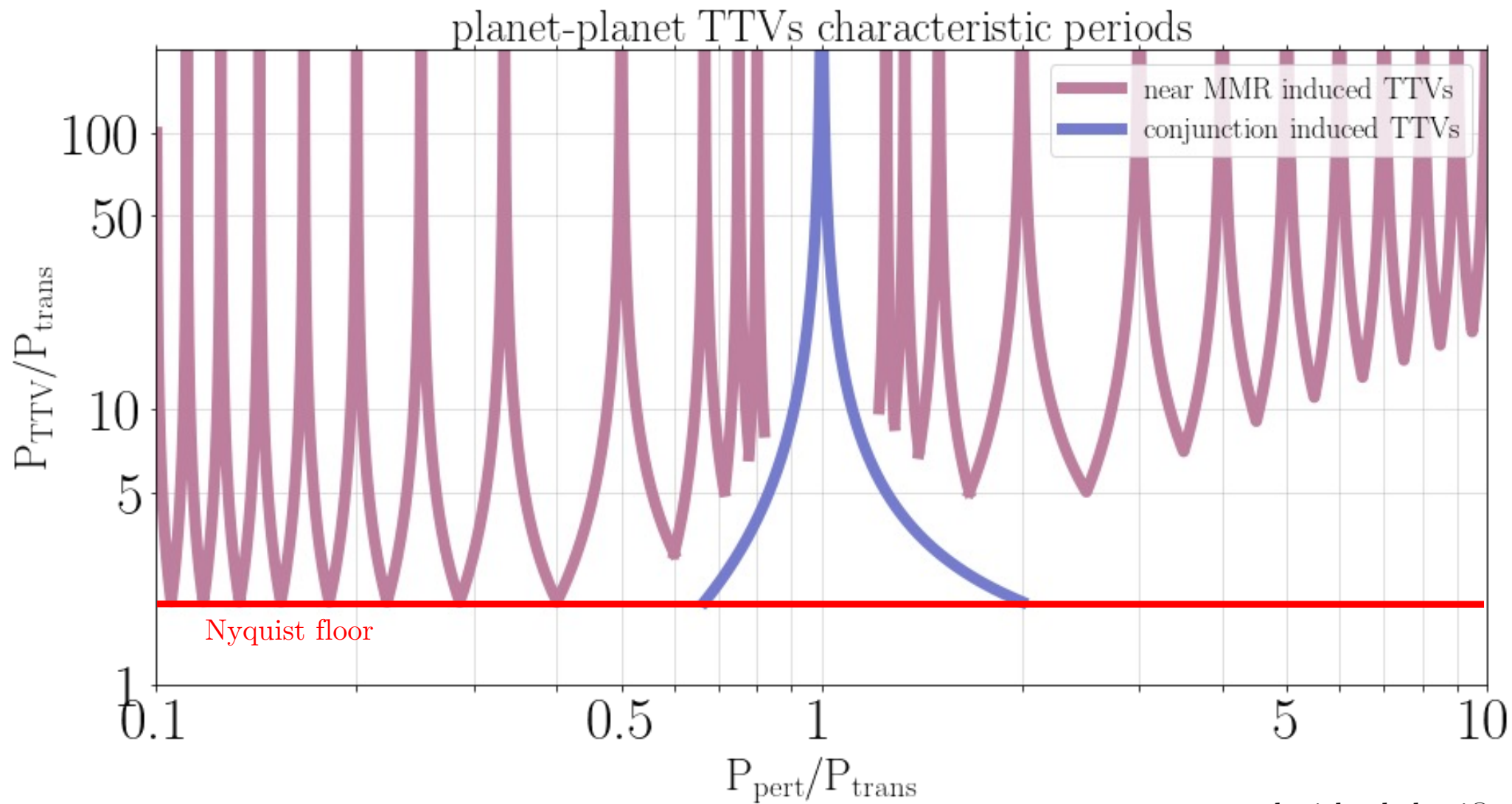
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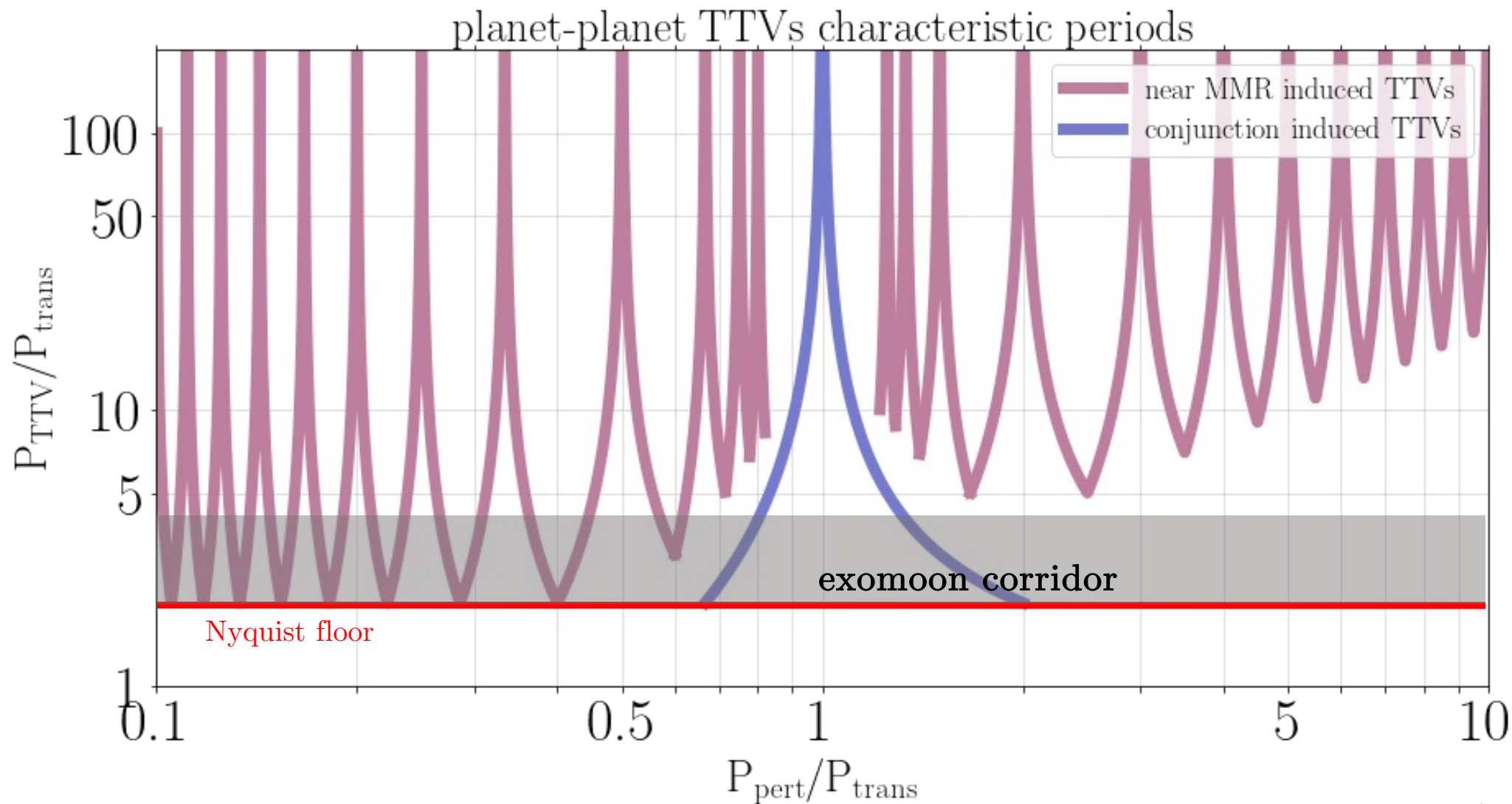
conjunction induced TTVs

$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

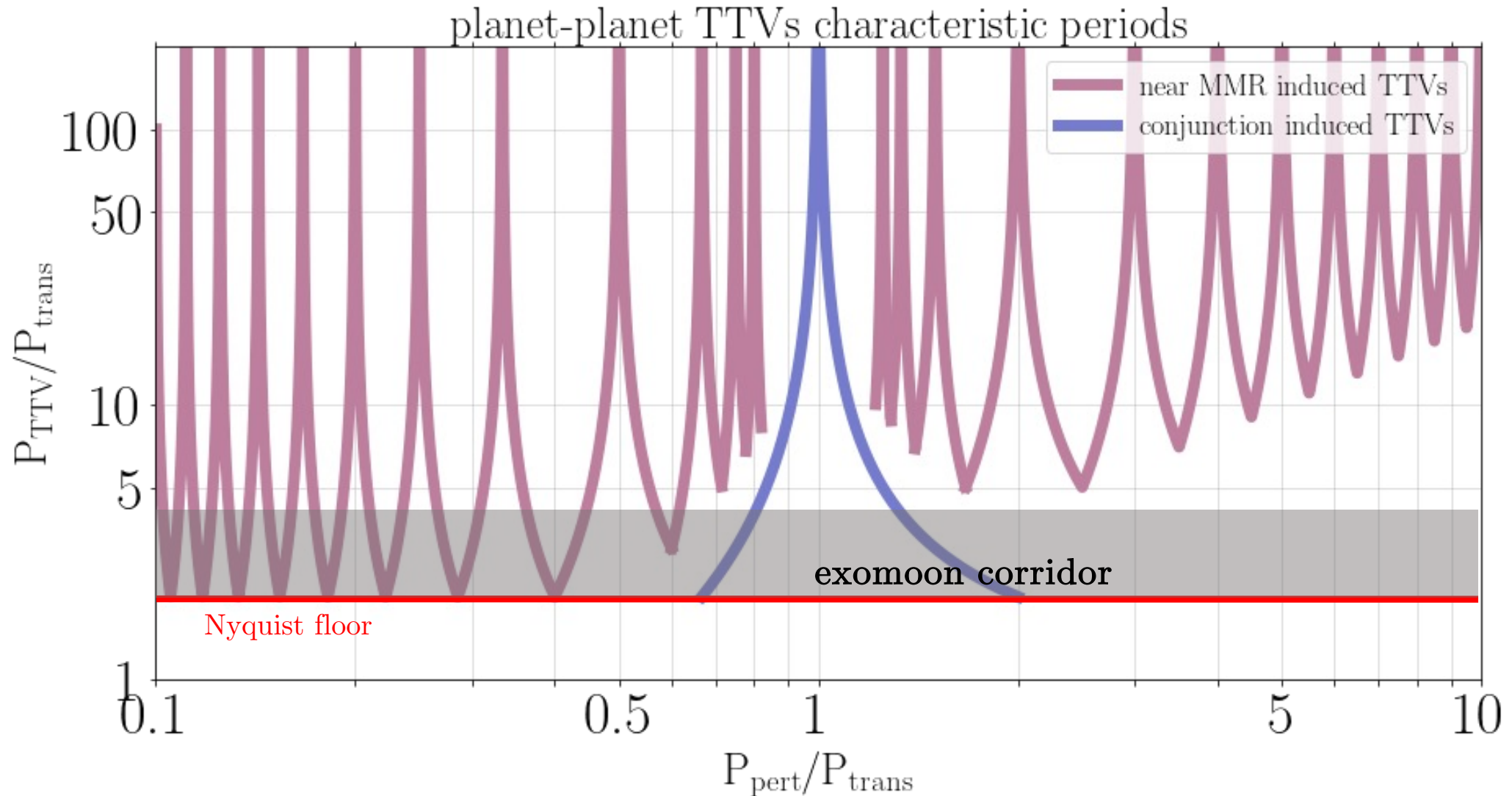




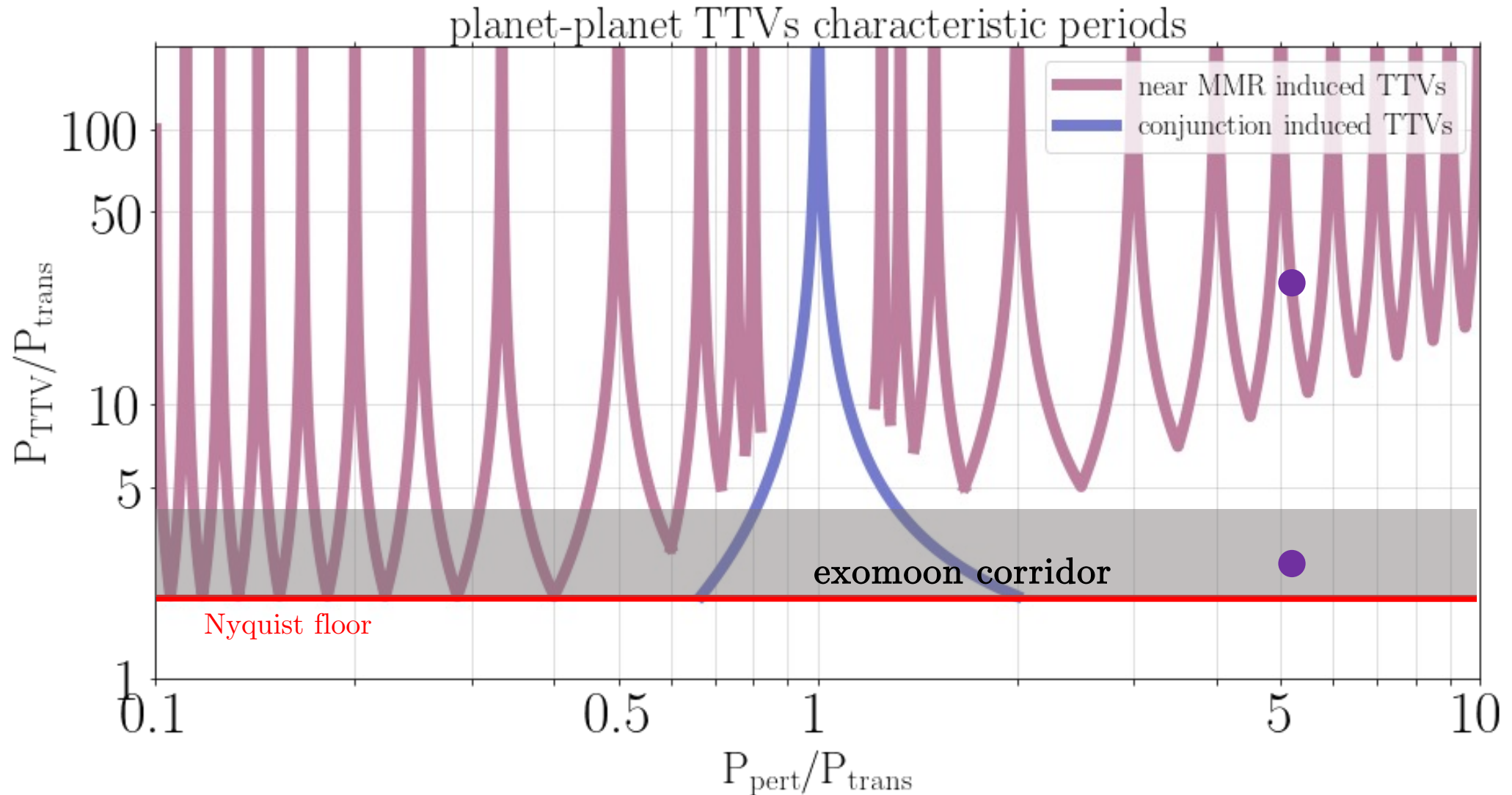




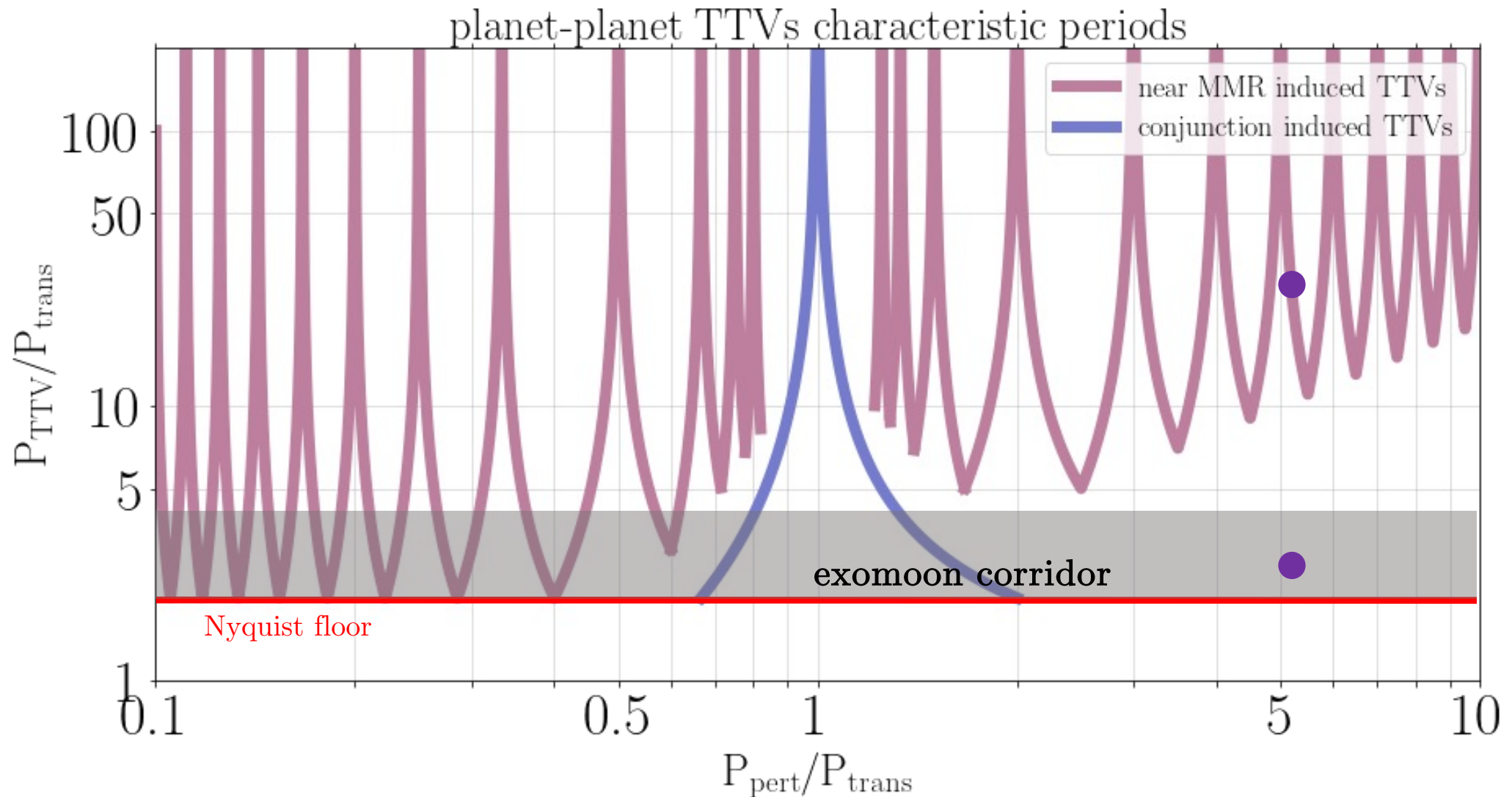
# Where do Kepler-1513 b and c live on this diagram?



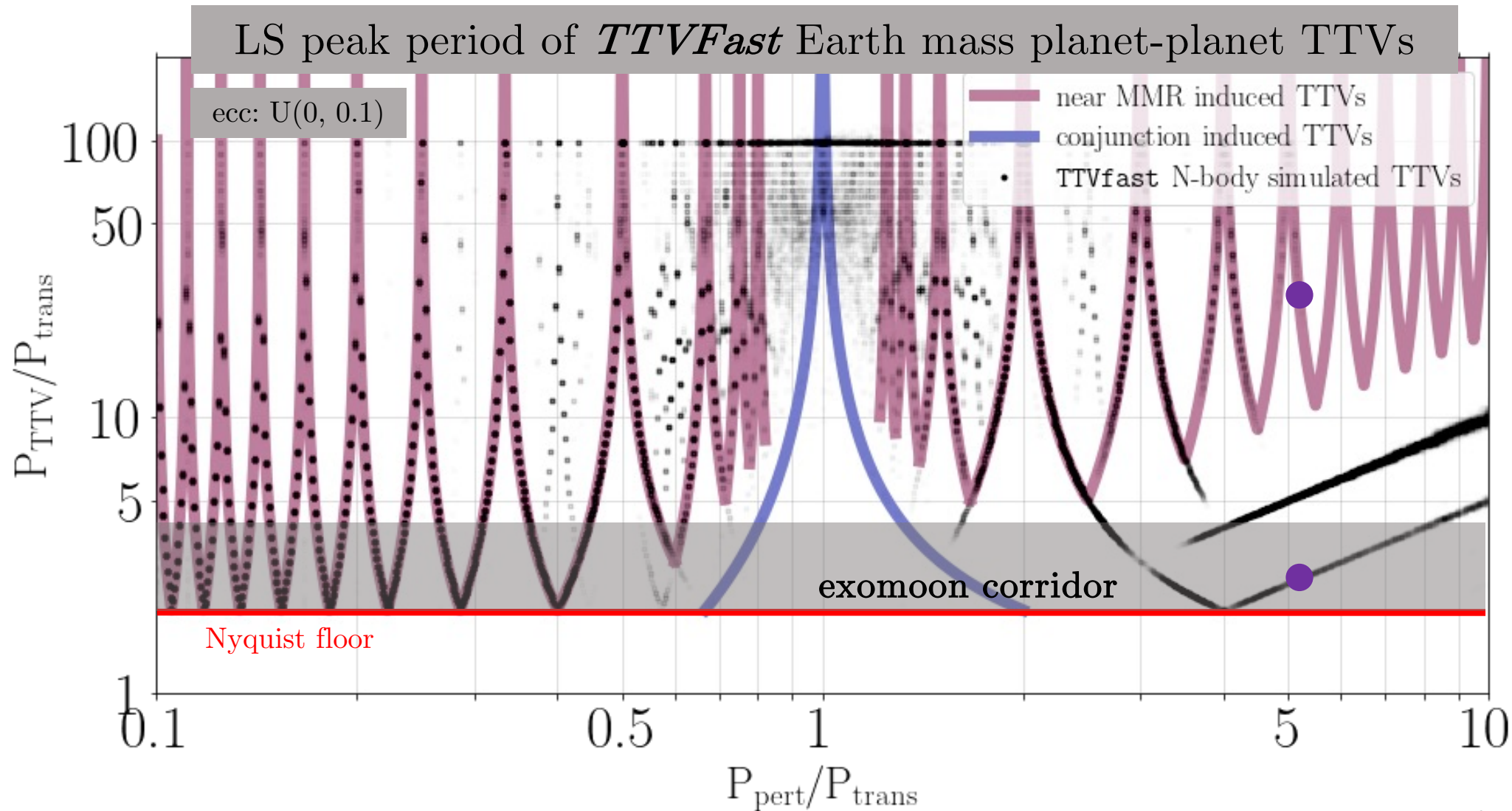
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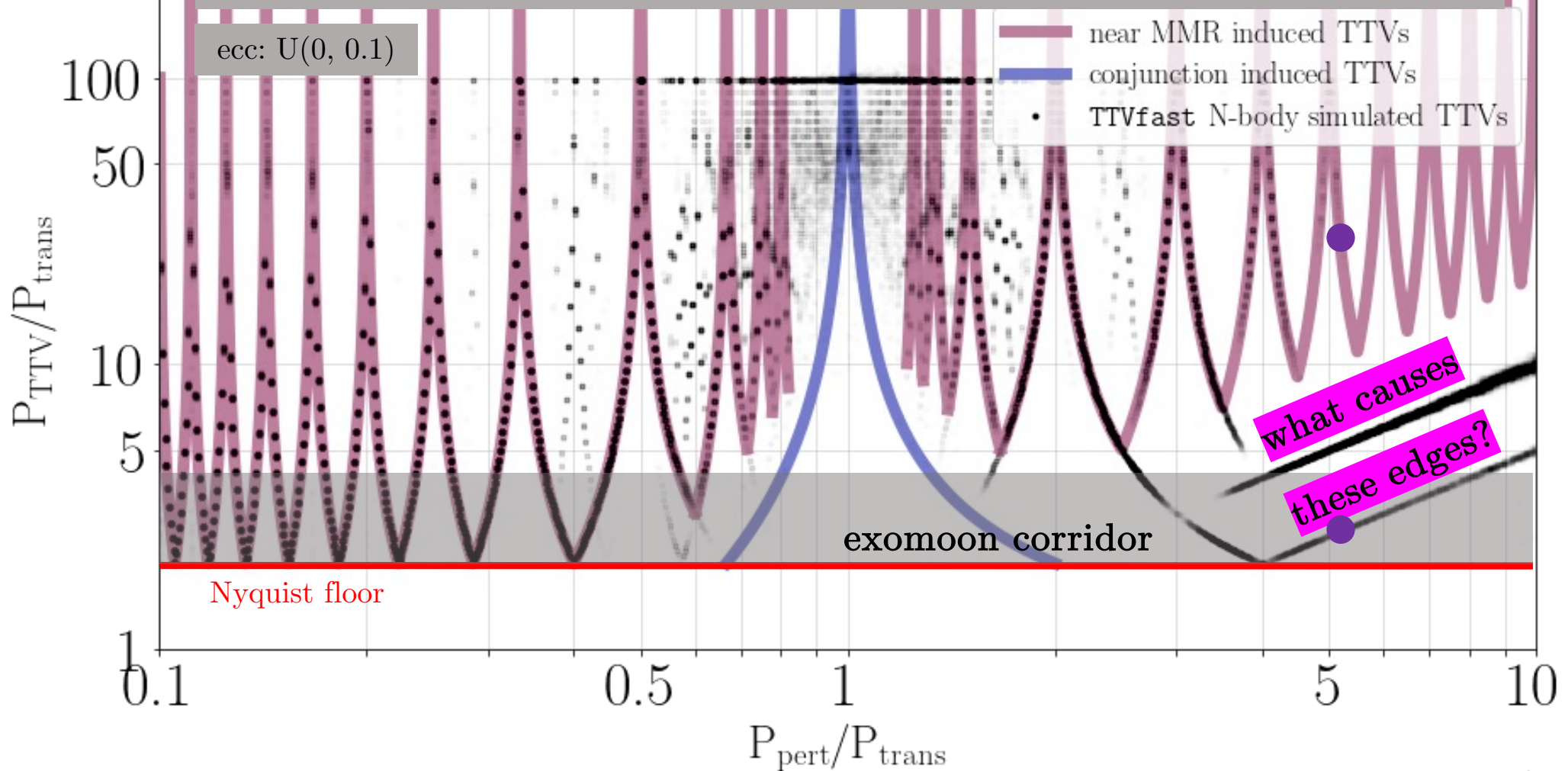
We can test these characteristic TTV periods via N-body simulations with *TTVfast* (Deck et al. 2014) and peak period determination with LS periodograms





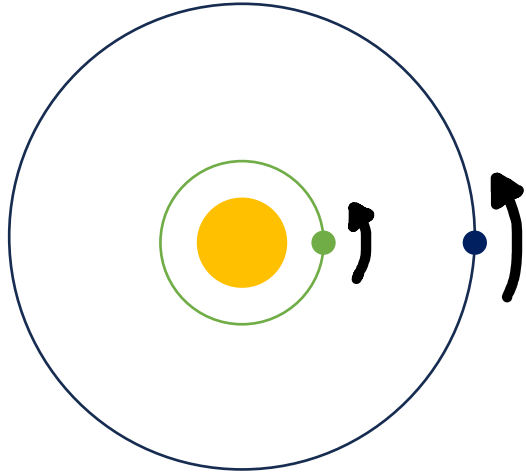


# LS peak period of *TTVfast* Earth mass planet-planet TTVs

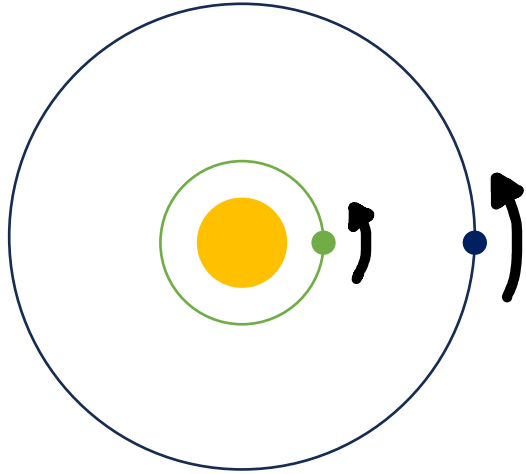




# conjunction induced TTVs

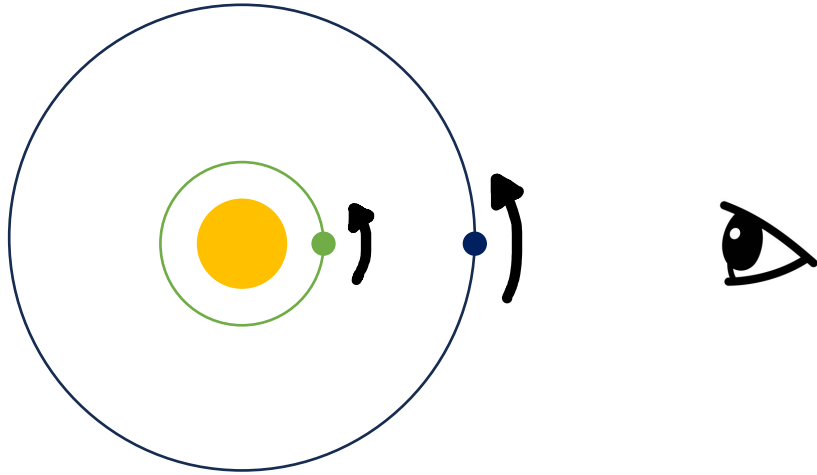


## conjunction induced TTVs



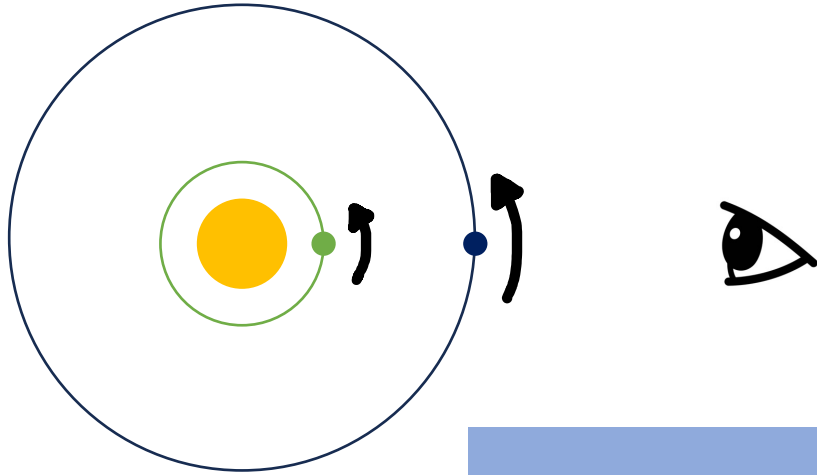
$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

## conjunction induced TTVs



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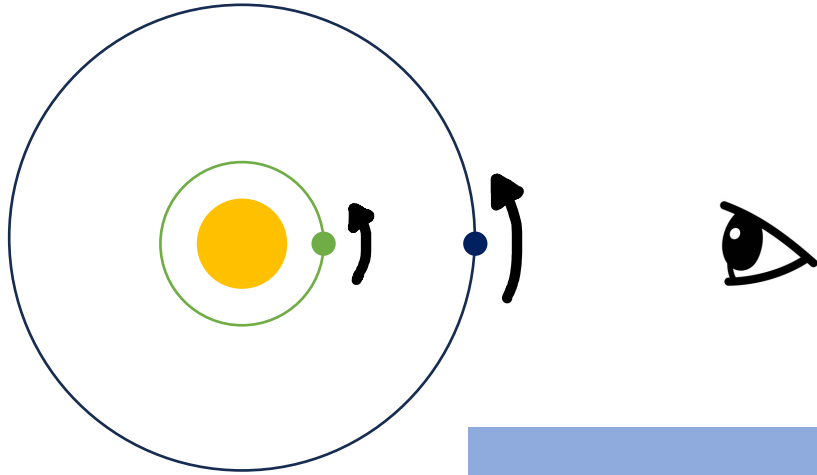
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we only observe transits

## conjunction induced TTVs

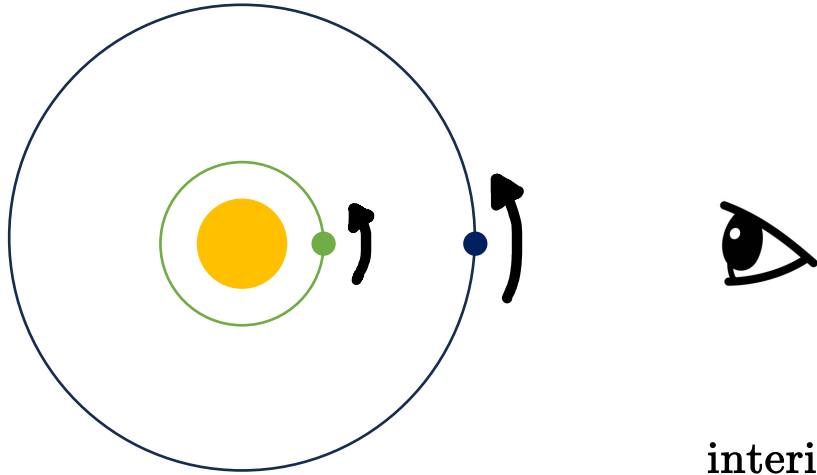


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we only observe transits

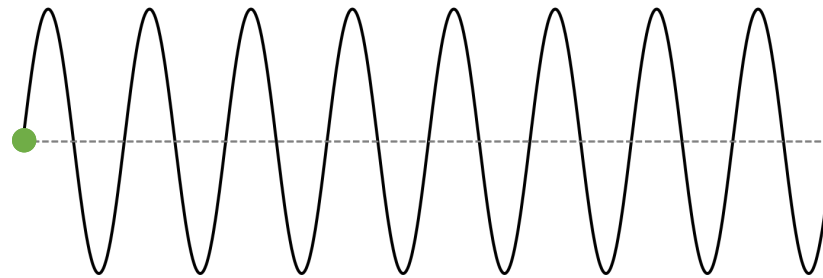
what do the TTVs  
look like during transits?

# conjunction induced TTVs



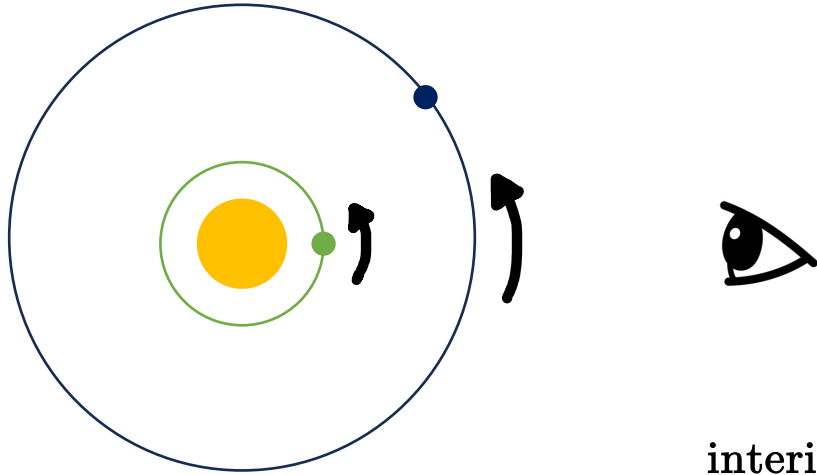
$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

interior planet TTVs



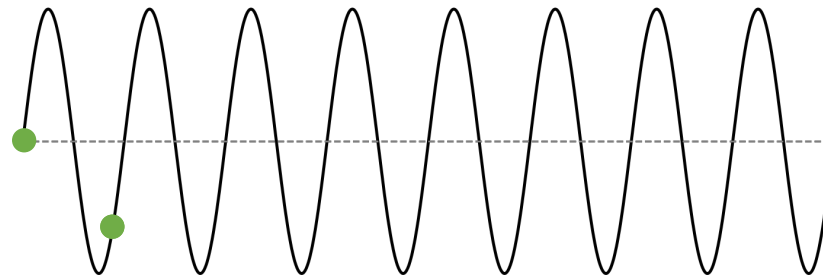
— time →

# conjunction induced TTVs



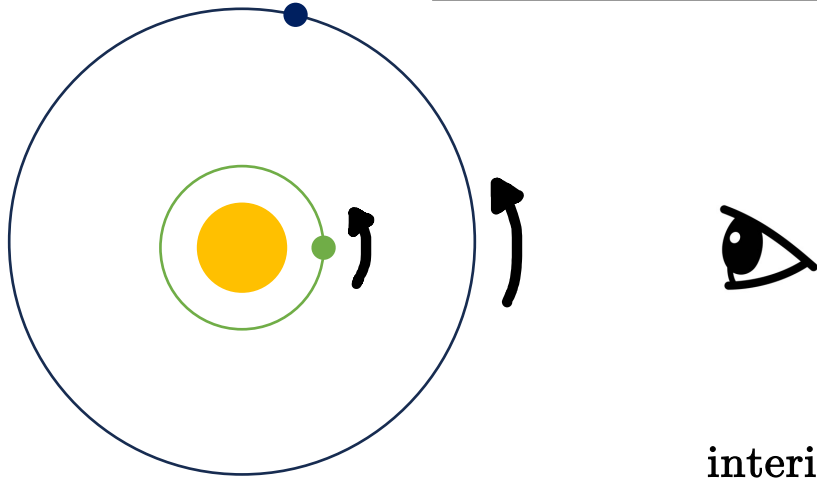
$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

interior planet TTVs



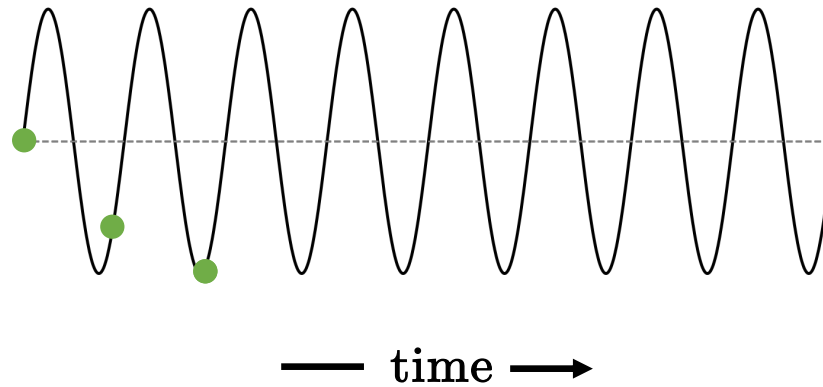
— time —>

# conjunction induced TTVs



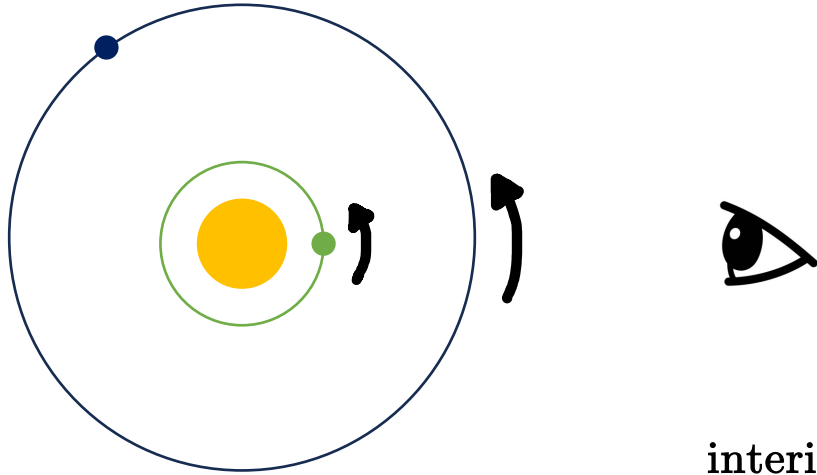
$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

interior planet TTVs



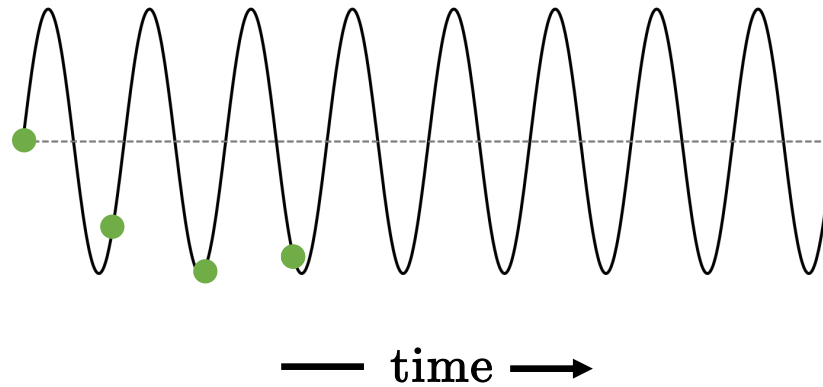


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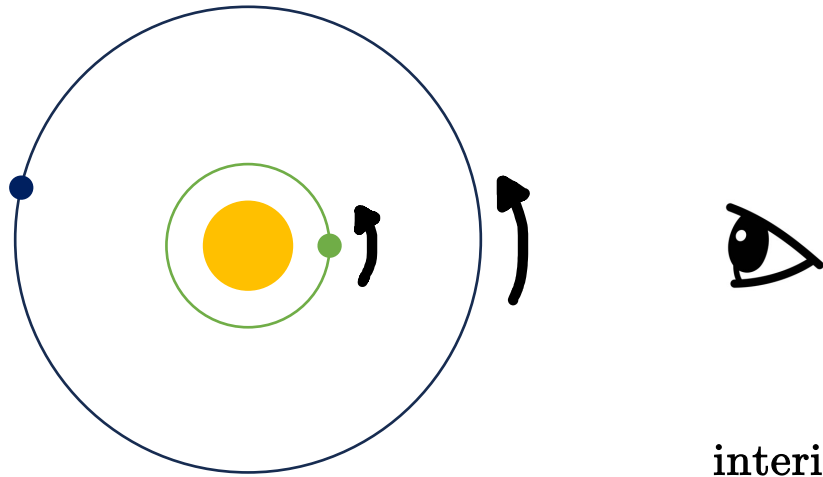


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interior planet TTVs

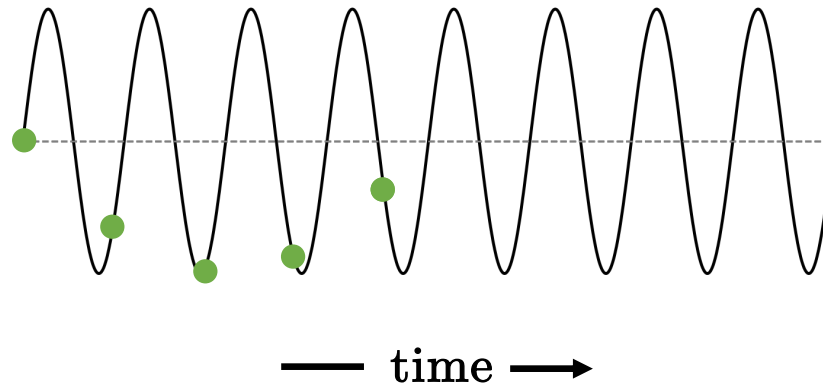


# conjunction induced TTVs

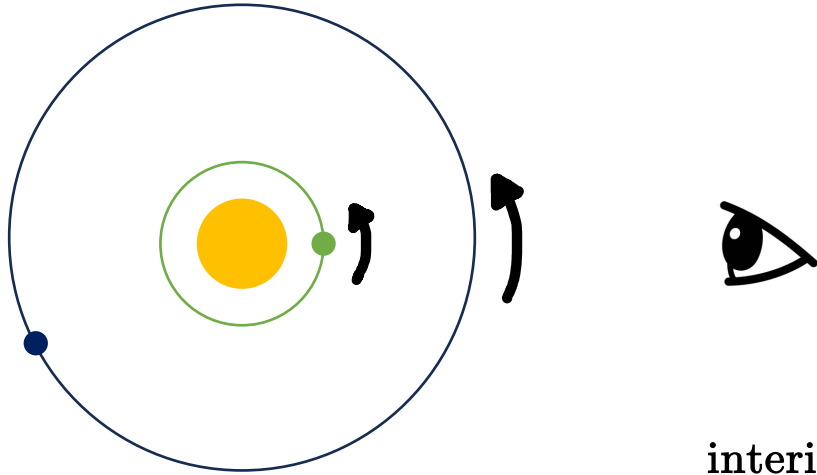


$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

interior planet TTVs

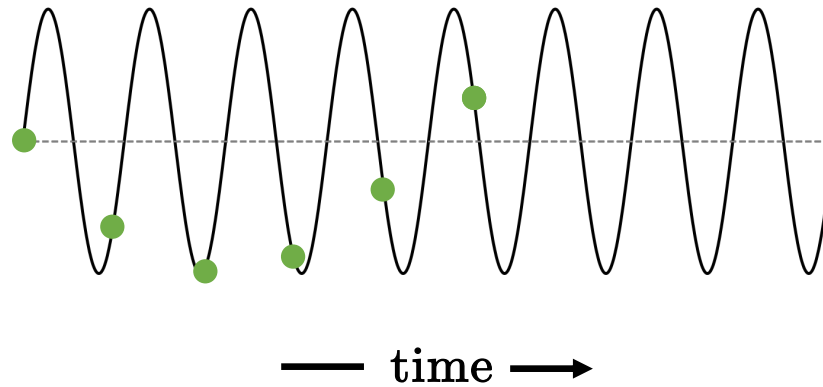


# conjunction induced TTVs

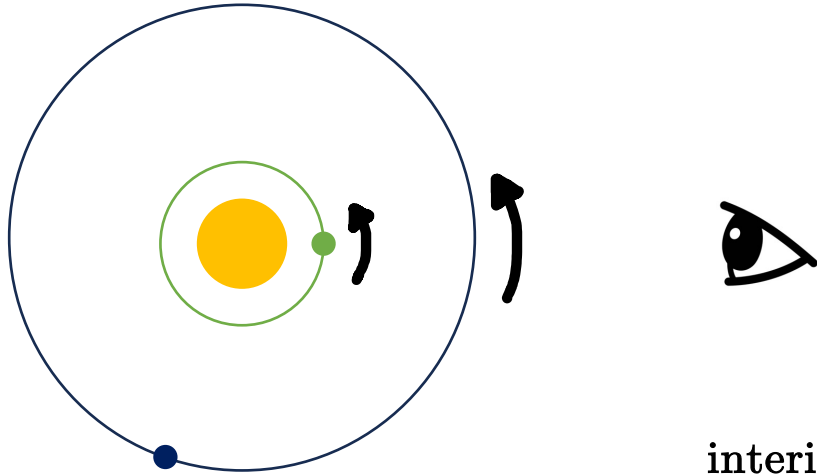


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interior planet TTVs

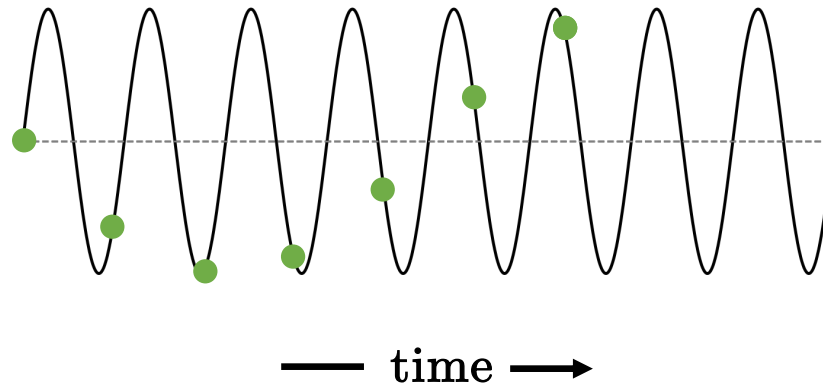


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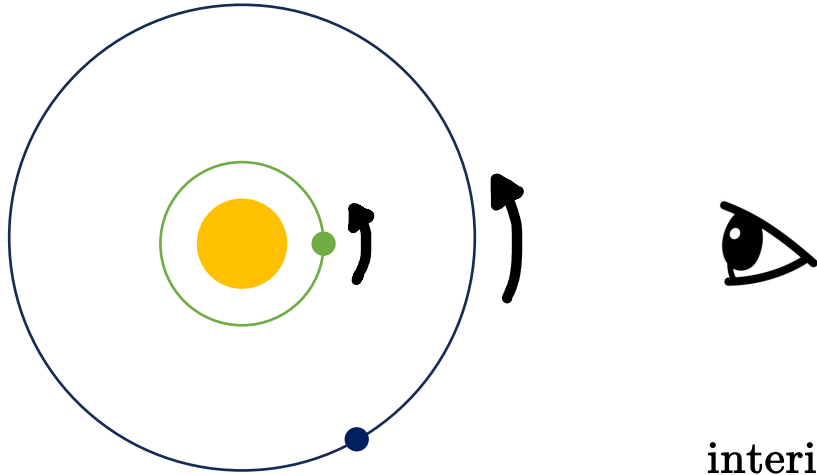


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interior planet TTVs

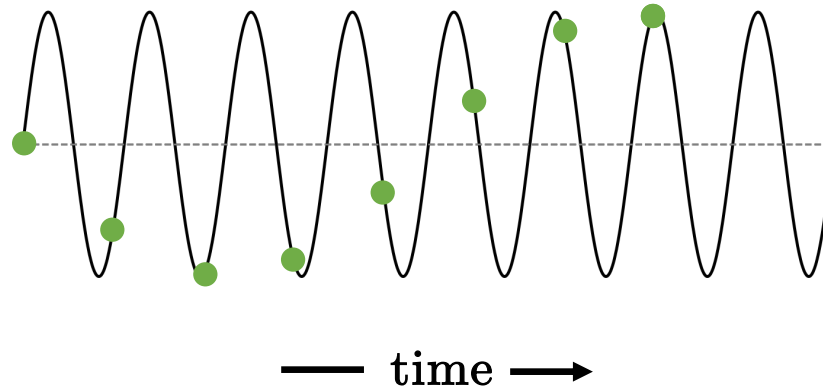


# conjunction induced TTVs

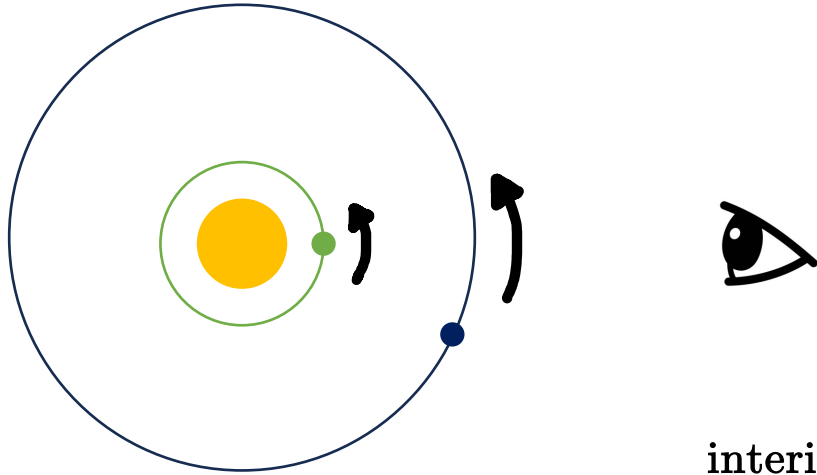


$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$

interior planet TTVs

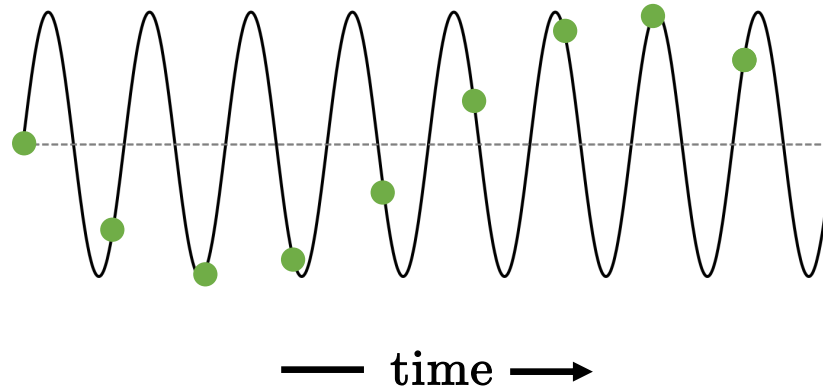


# conjunction induced TTVs



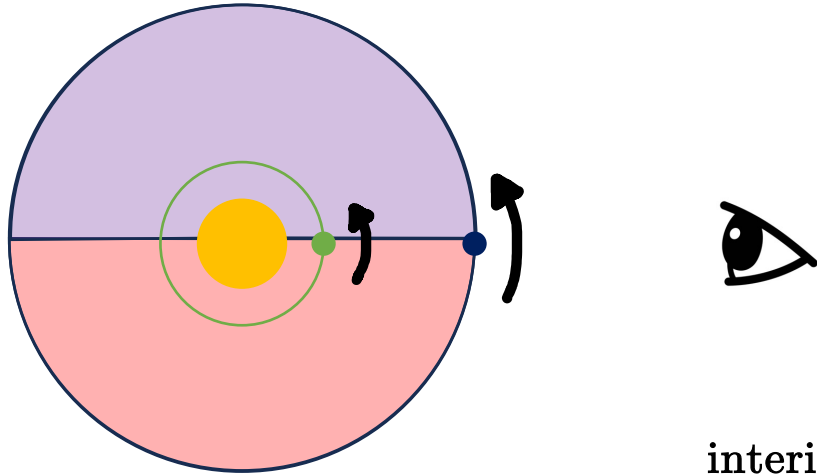
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interior planet TTVs



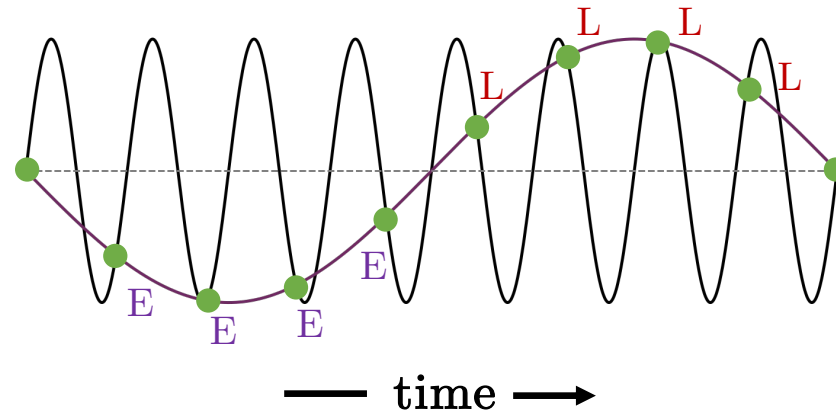
E = early TTV  
L = late TTV

## conjunction induced TTVs



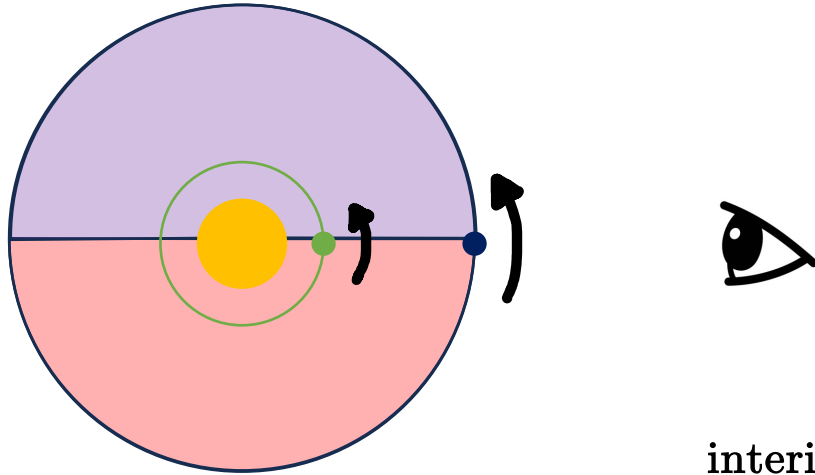
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interior planet TTVs



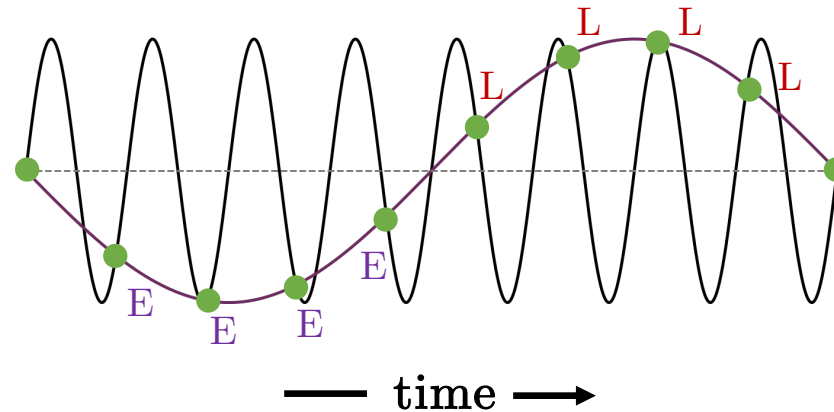
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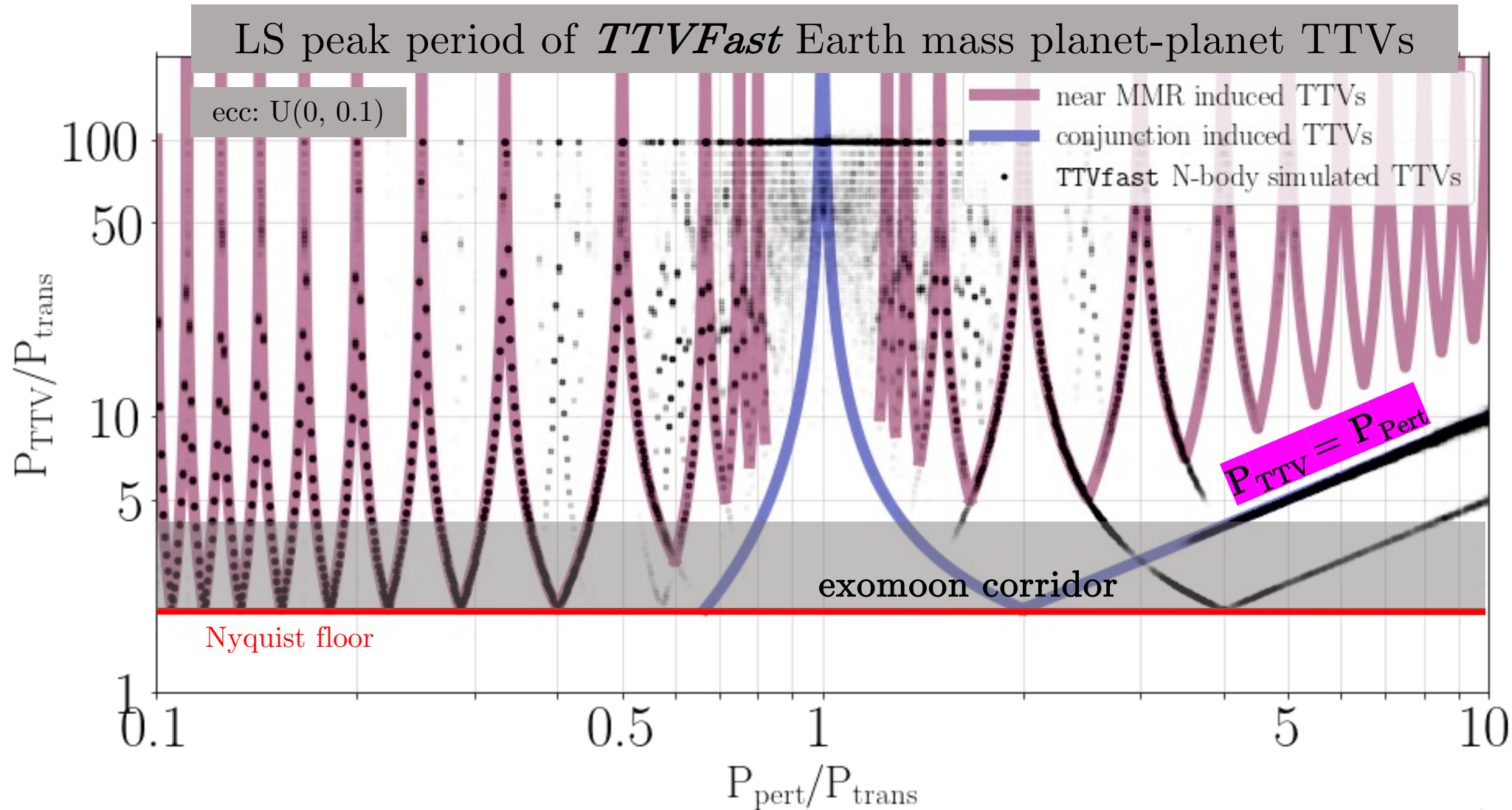
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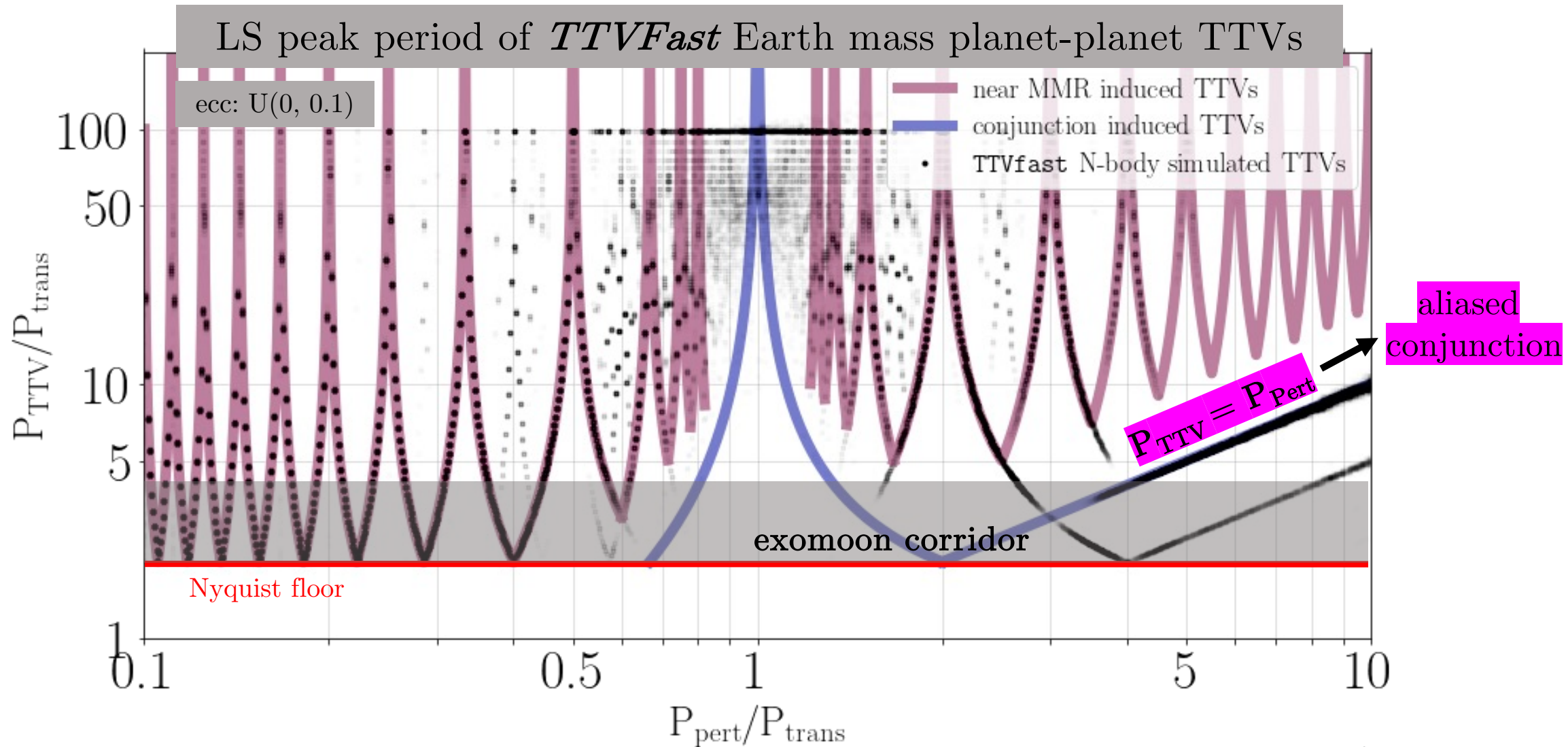
interior planet TTVs

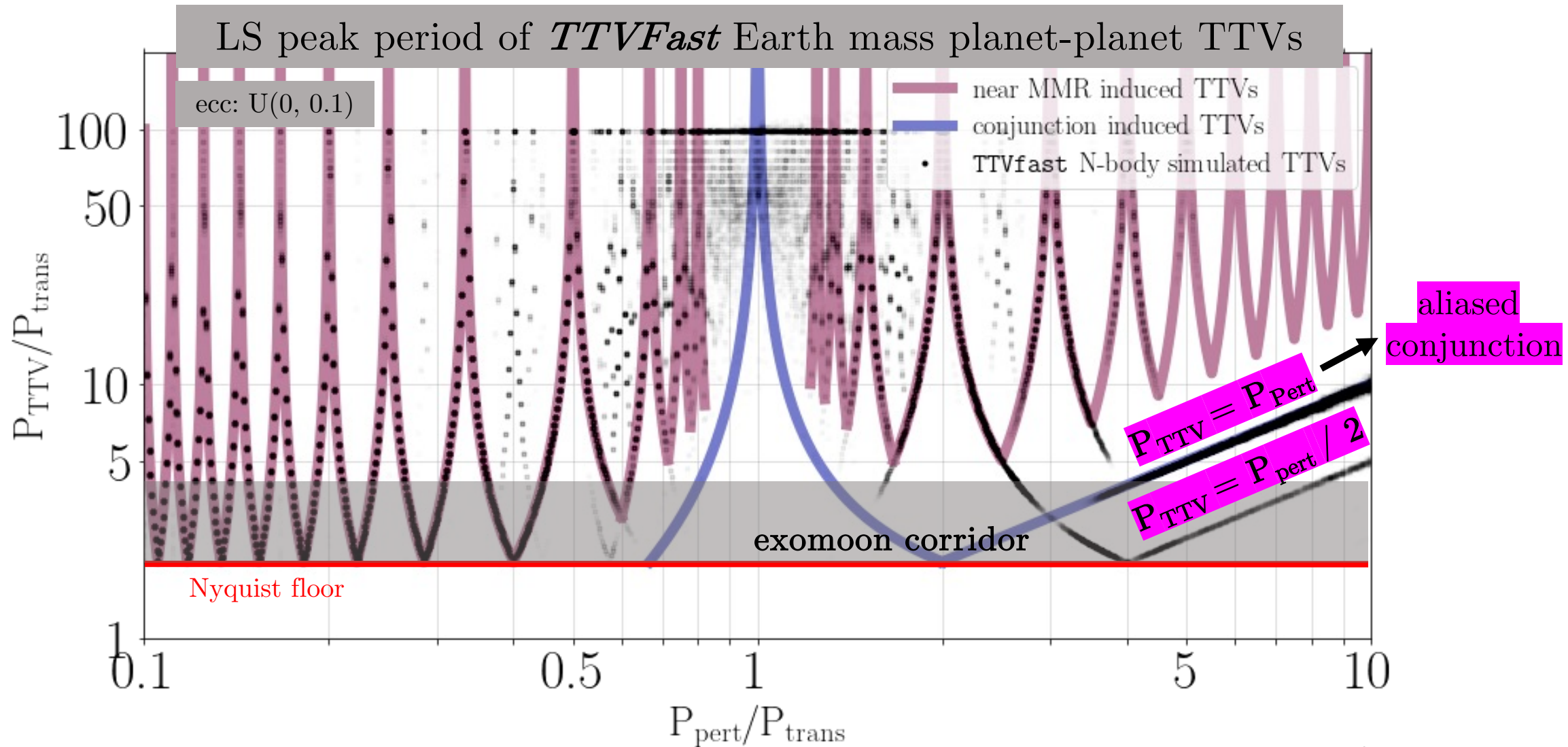


$$P_{TTV \text{ alias}} = P_{\text{exterior planet}}$$



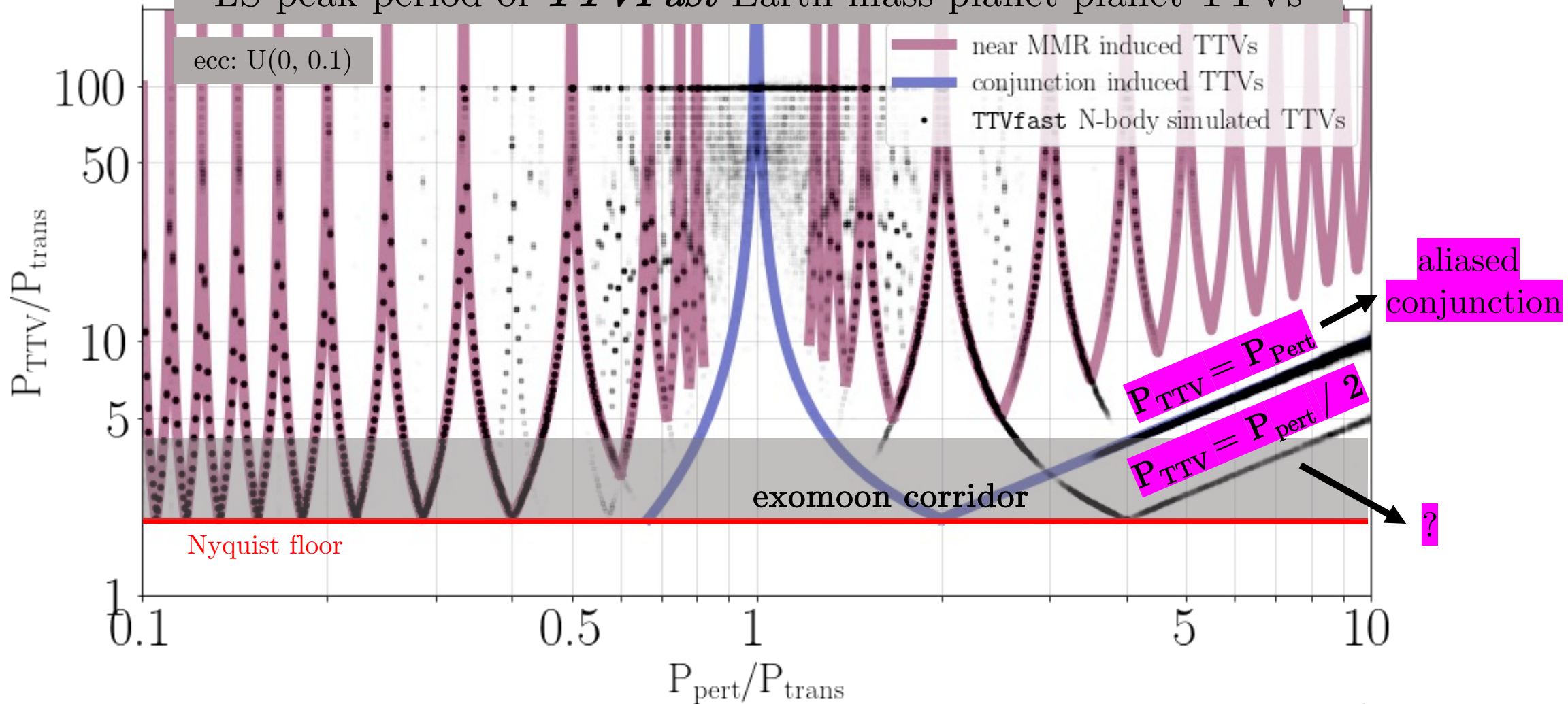








# LS peak period of *TTVfast* Earth mass planet-planet TTVs



# planet-planet TTVs characteristic periods

conjunction induced TTVs

$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}.$$

# planet-planet TTVs characteristic periods

near MMR induced TTVs

$$P_{sup} = \frac{1}{|j/P_1 - k/P_2|}$$



~~conjunction induced TTVs~~

~~$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$~~

# planet-planet TTVs characteristic periods

near MMR induced TTVs

$$P_{sup} = \frac{1}{|j/P_1 - k/P_2|}$$



$$P_{sup} = \frac{1/j}{|1/P_1 - 1/[(\frac{j}{k}) P_2]|}$$

~~conjunction induced TTVs~~

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~~conjunction induced TTVs~~

~~$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$~~

super-period for 1:k MMR  
with  $P_1$  and  $P_2$   
=  
synodic period  
with periods  $P_1$  and  $P_2/k$ .



# planet-planet TTVs characteristic periods

near MMR induced TTVs

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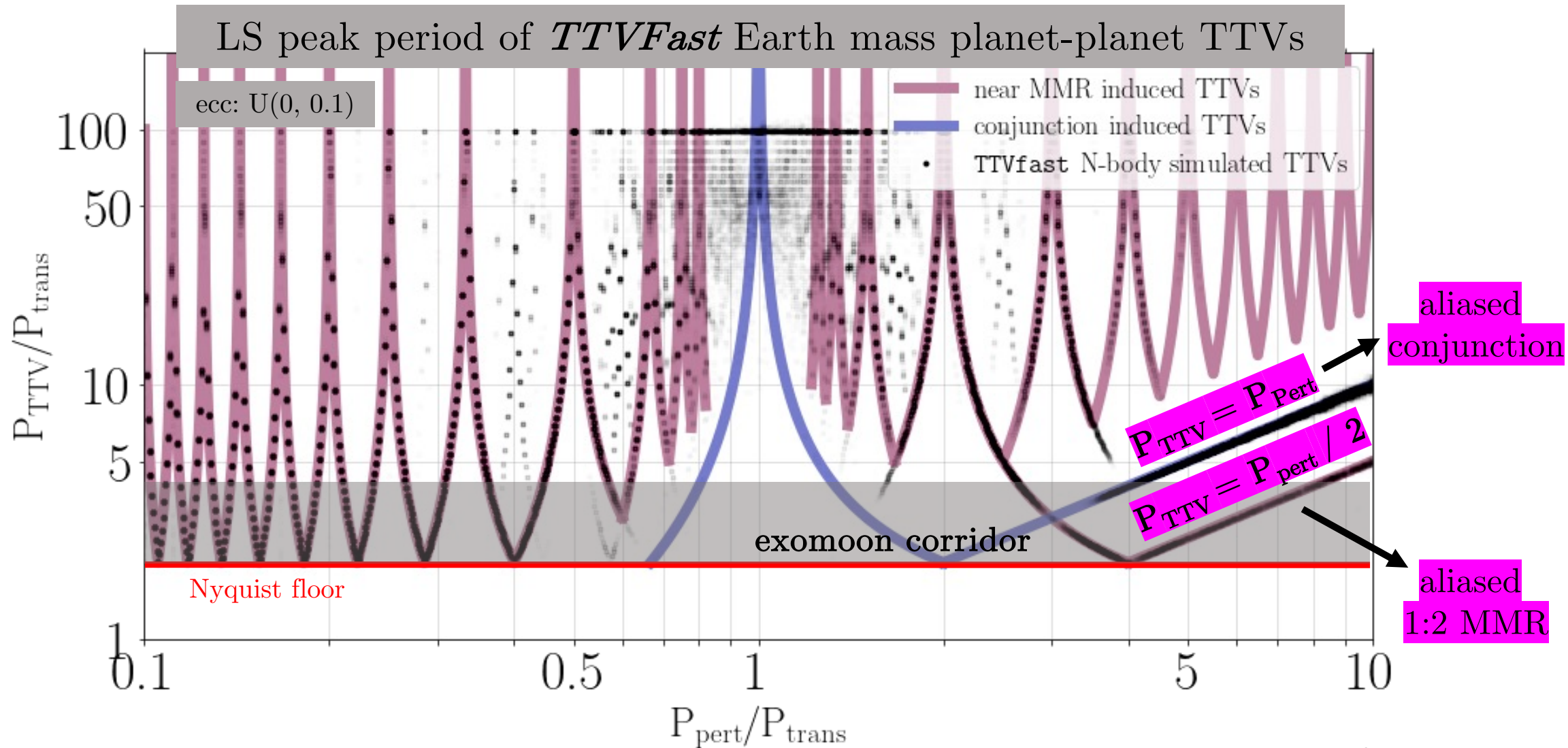
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~~conjunction induced TTVs~~

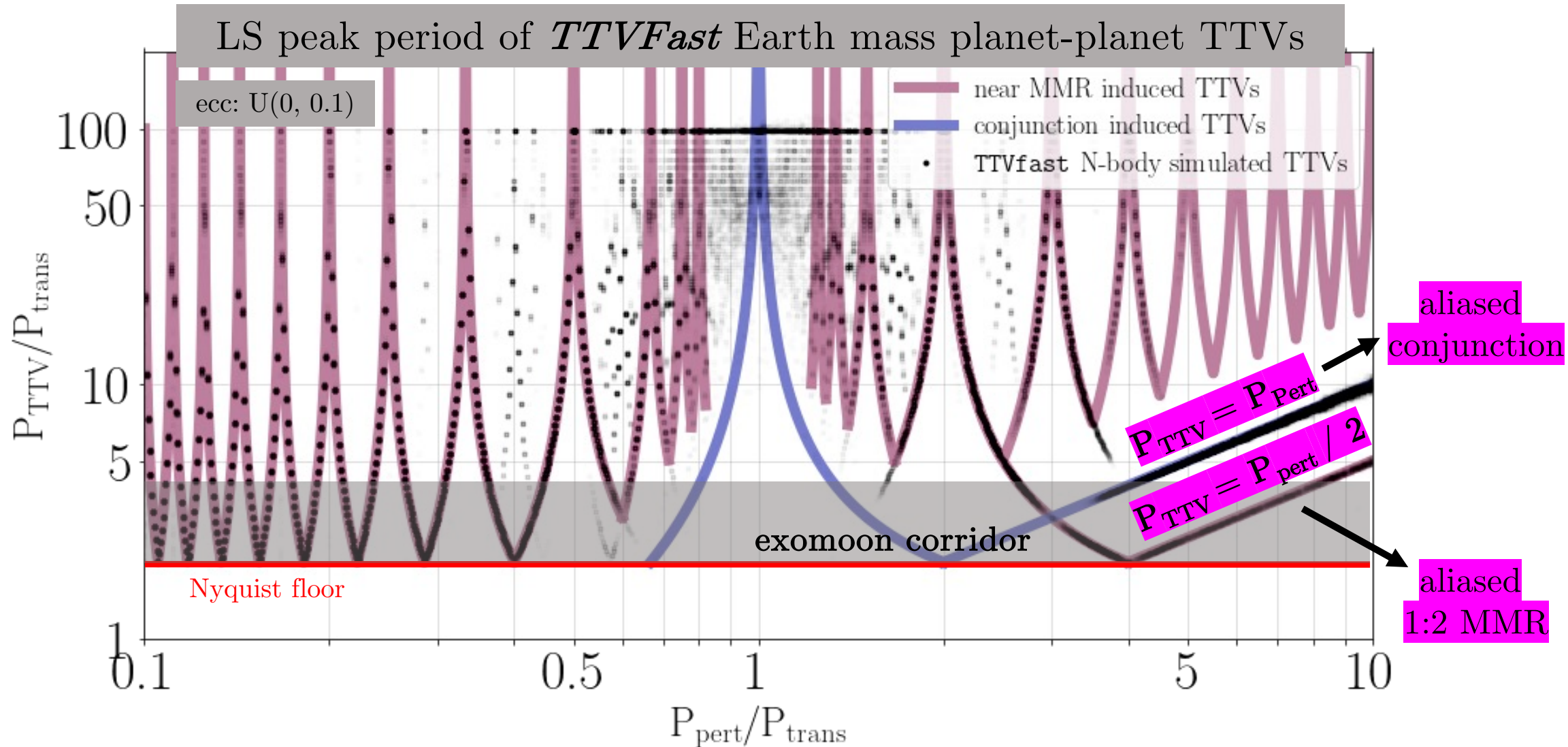
~~$$P_{syn} = \frac{1}{|1/P_1 - 1/P_2|}$$~~

super-period for 1:k MMR  
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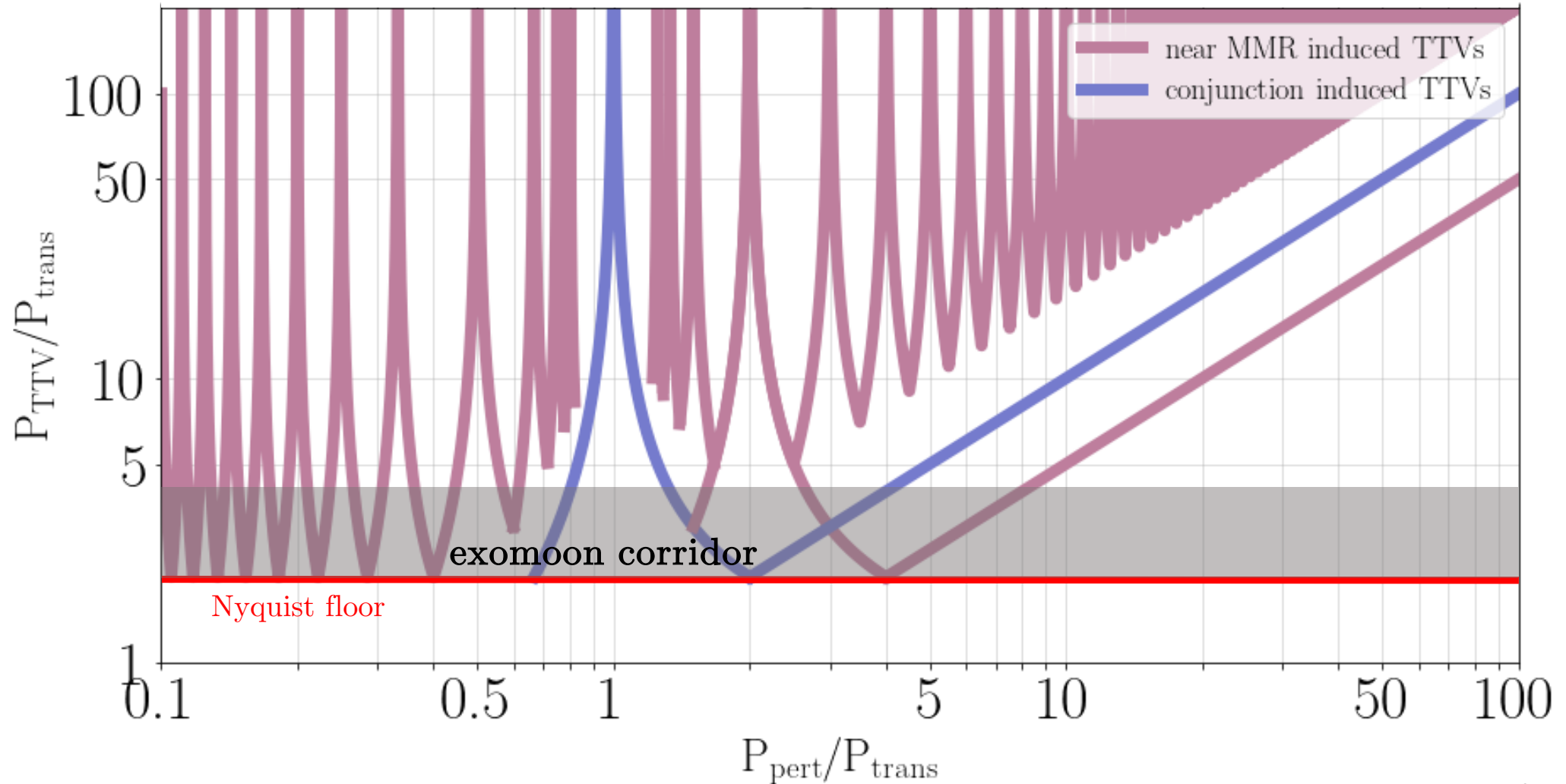
therefore, a 1:2 MMR TTV has an aliased period of  $P_2/2$



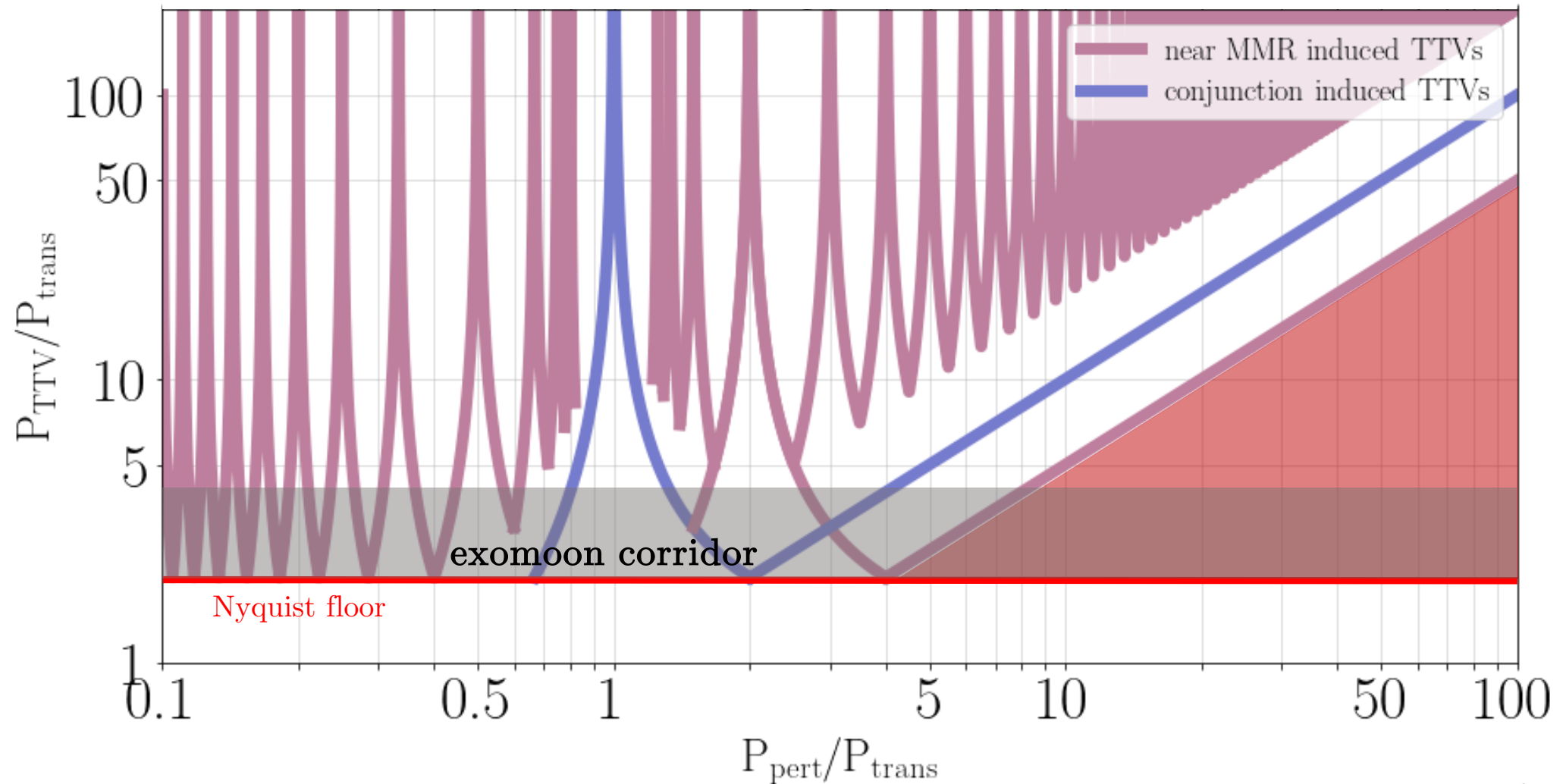
Let's change our x-axis to include period ratios up to 100x...



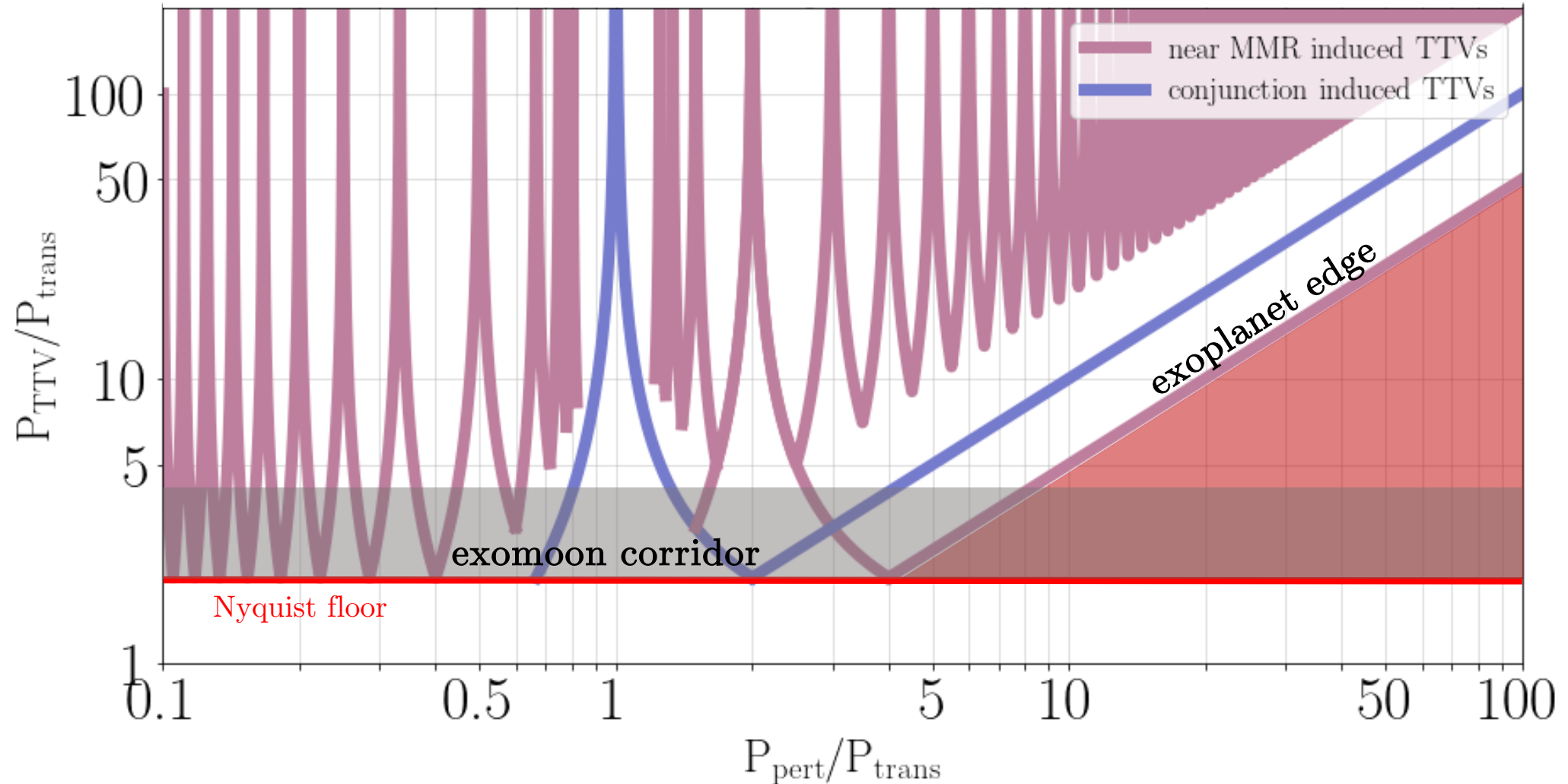
Let's change our x-axis to include period ratios up to 100x...



planet-planet TTVs are not expected below this edge!

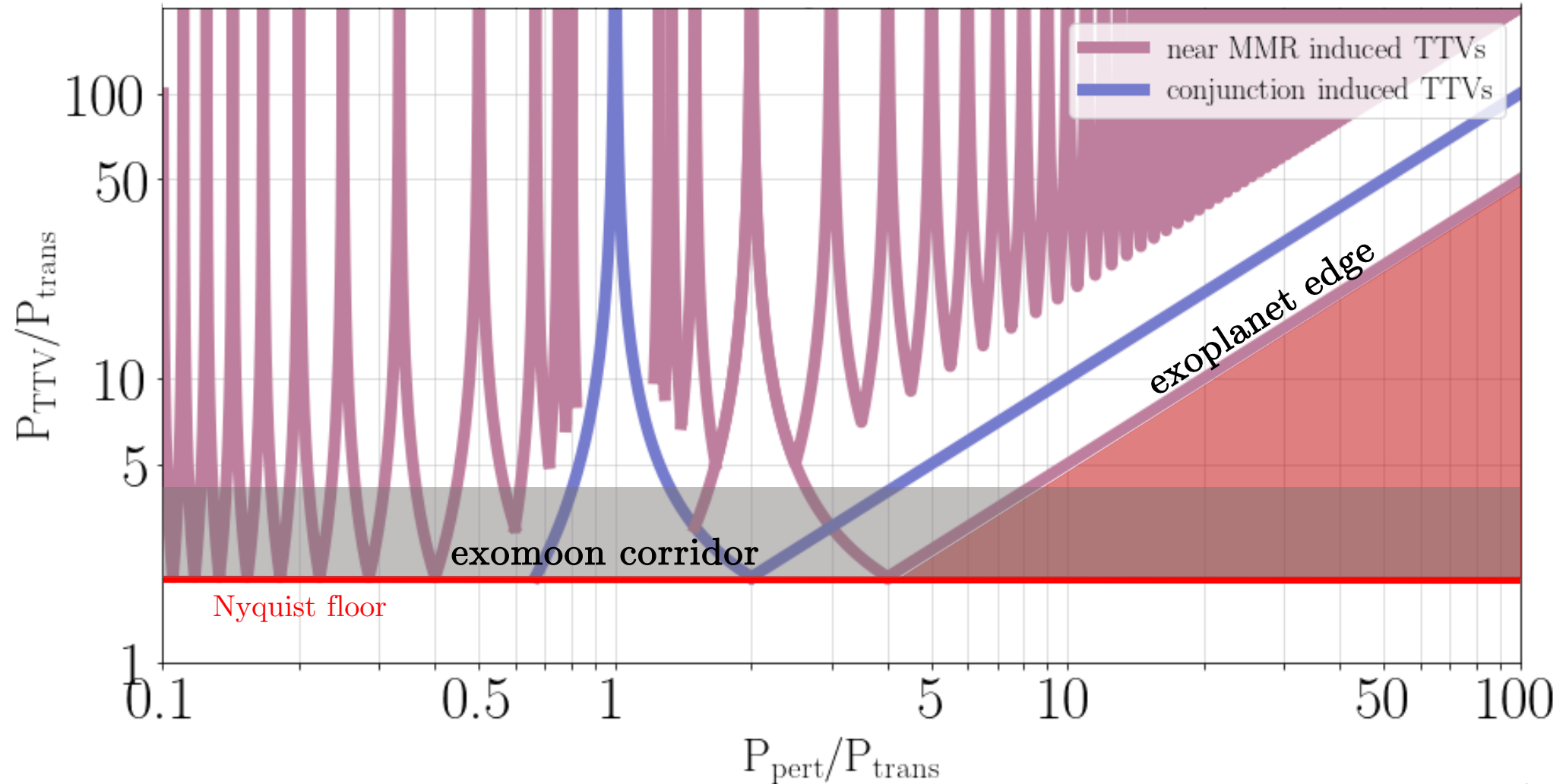


planet-planet TTVs are not expected below this edge!

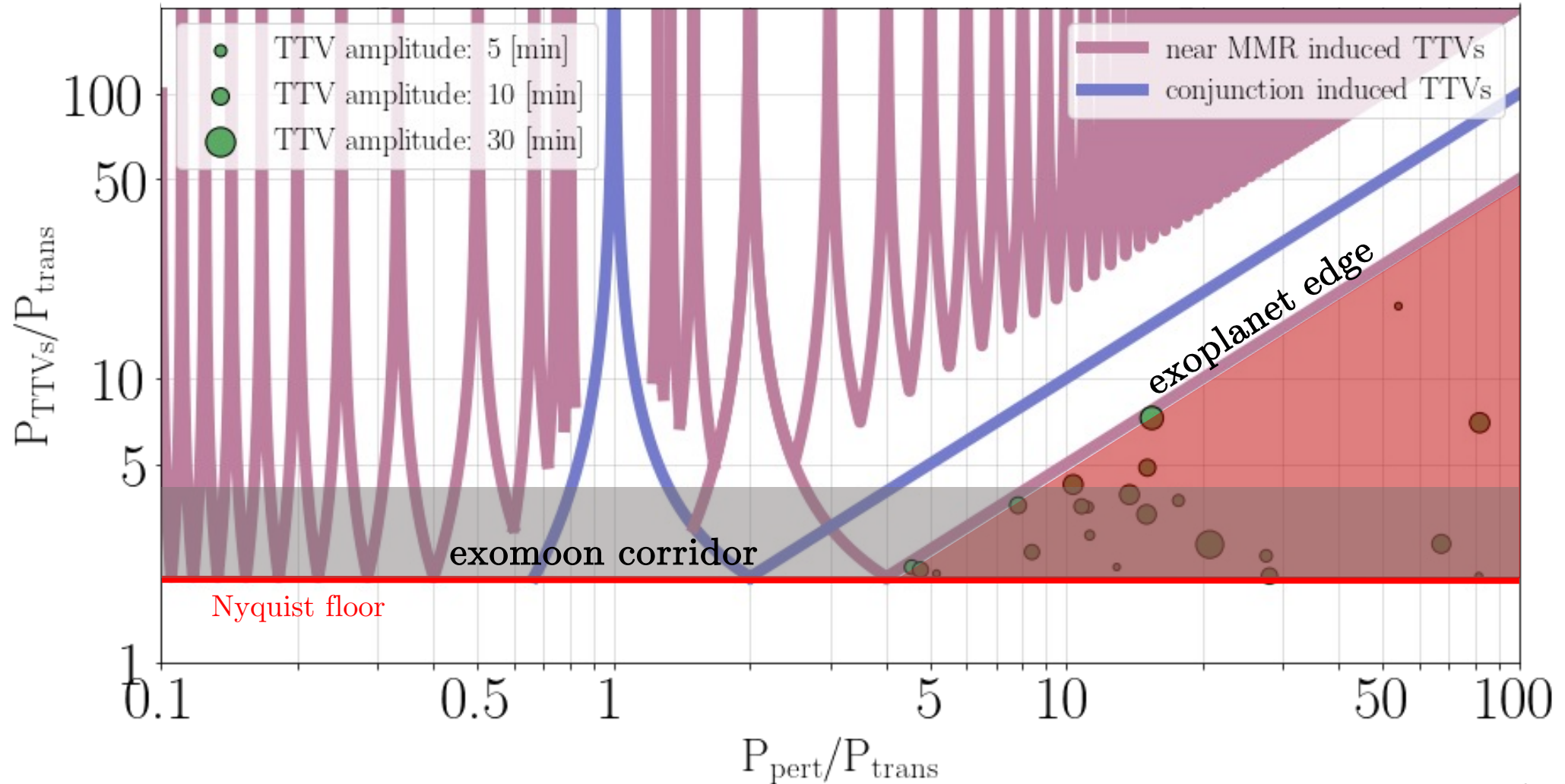




Do any two-planets systems appear below the exoplanet edge?



# 22 two-planet systems in Holczer+ 2016 are anomalous





22 two-planet systems in Holczer+ 2016 are anomalous

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- Possible exomoon corridor systems...

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- Would've been eliminated from Kipping and Yahalomi, 2023

22 two-planet systems in Holczer+ 2016 are anomalous

- Possible exomoon corridor systems...
  - Would've been eliminated from Kipping and Yahalomi, 2023
  - We are currently working on more rigorous analysis of these 22 systems to determine which systems to follow-up.

What could cause anomalous TTV periods?

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- Another (non-transiting) planet?

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- A moon?

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Stay tuned (and I am certainly excited to talk about ideas!)

# Takeaways

1. TTVs are ubiquitous and we must develop more efficient ways to analyze TTV datasets on a population level.

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1. TTVs are ubiquitous and we must develop more efficient ways to analyze TTV datasets on a population level.
2. We are developing analytic methods to differentiate between TTVs cause by moons, planets, and stellar-activity.
  - Analysis of TTV period space is particularly promising.

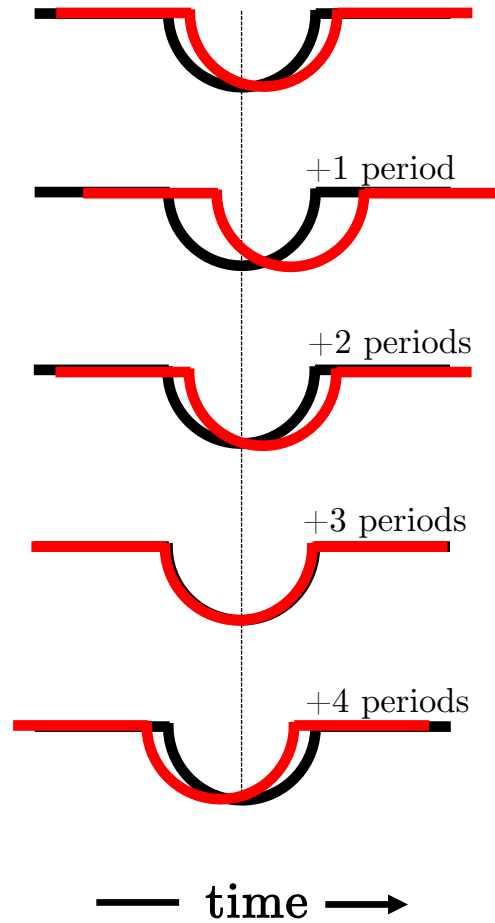
## Takeaways

1. TTVs are ubiquitous and we must develop more efficient ways to analyze TTV datasets on a population level.
2. We are developing analytic methods to differentiate between TTVs cause by moons, planets, and stellar-activity.
  - Analysis of TTV period space is particularly promising.
3. Using the **exomoon corridor** and **exoplanet edge** we can identify targets of interest and follow them up with ground-based facilities. This effort is underway and will continue.

# roadmap

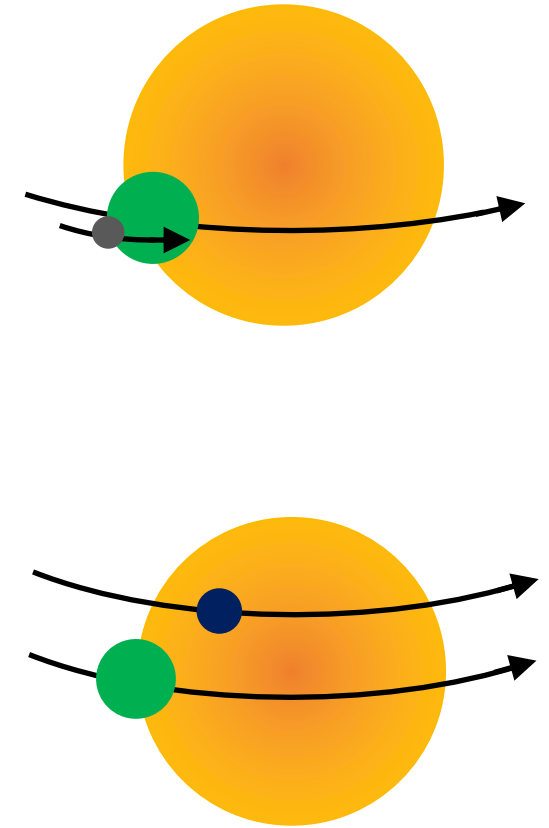
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[Options](#)



exomoon  
corridor

exoplanet  
edge



# From Wobbles to Worlds: Exploring the Orbital Landscape of Exoplanet TTVs

Daniel A. Yahalomi<sup>1</sup>  
David Kipping<sup>1</sup>, Eric Agol<sup>2</sup>, & David Nesvorný<sup>3</sup>

1: Columbia University, 2: University of Washington, 3: SwRI Boulder

ExoExplorers Science Series  
April 12, 2024

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