Anomalous intensity of pinned speckles at high adaptive correction

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Received May 28, 2003

Ground-based optical searches for faint stellar or planetary companions about other stars may be limited by speckle noise, which is the rapid intensity fluctuations that are due to motions of remnant atmospheric speckles. Adaptive optics (AO) can reduce residual wave-front phase errors to low values, substantially reducing the unwanted power in the speckle halo. At high correction, however, the noise in the halo will be dominated by anomalously bright "pinned" speckles that have a number of unusual properties. They can have negative intensities and will appear in spatially antisymmetric patterns; they are spatially pinned to Airy rings and have zero mean in a sufficiently long integration. Some of these properties may be used to reduce the unanticipated effect of pinned speckles on companion searches, depending on details of the AO system. But, in short exposures, pinned speckles dominate speckle noise over much of the inner halo for Strehl ratios S as low as 0.6 and over much of the outer halo too as Strehl and deformable-mirror actuator densities increase. I show that these anomalously bright pinned speckles are not included in the traditional expression for speckle power in an image, $(1 - S)$, on which sensitivity estimates of future high-performance AO systems have been based.

OCIS codes: 010.1080, 030.6140, 030.6610, 070.2580, 350.1270.

Ground-based astronomical adaptive optics (AO) systems have recently demonstrated Strehl ratios (S) of $S \sim 60-70\%$ in the near infrared.[1] Future extremely high correction (perhaps $S \sim 99\%$) is expected to be important to the search for extrasolar planets and for their effective study once they have been found.[2] The sensitivity limits on detection of companions may be set by the speckle noise[3] that characterizes the fast-changing halo of remnant light that surrounds the image of the primary star.

Recently, a new and important technical aspect[4,5] of high-Strehl imaging has become apparent: Not all speckles in a highly corrected image are accounted for by the traditional halo power budget, $(1 - S)$, which has been used to calculate companion-detection sensitivities of future AO systems of high performance.[2] At high Strehl ratio there are two distinct types of speckle, including a class of anomalously bright "pinned" speckles that contribute no net power when they are integrated over the focal plane. These speckles appear in antisymmetric patterns (which explains their lack of net image power) and are spatially pinned to rings of the diffraction-limited point-spread function (PSF), from which they may add or subtract intensity. These anomalous postcorrection speckles are dominant in brightness (intensity) and in their contribution to speckle noise in the regimes that are considered here; at a high enough Strehl ratio they are dominant throughout the speckle halo, except in the immediate vicinity of Airy nulls. The apportionment of image power is discussed below, and it is shown how image power, in particular the traditional halo estimate $(1 - S)$, is an unreliable indicator of speckle noise in the high-Strehl regime at least in short exposures.

Early estimates of speckle noise were based on Poisson statistics and were hence unduly optimistic, as they neglected the essential coherent nature of speckles. In practice, the noise from a speckle is not proportional to the square root of the number of photons that it contains. Instead, because speckles move as coherent entities in and out of a search field of interest, they modulate the field with their full intensity. Racine *et al.*[3] showed that speckle noise is proportional to the intensity of a typical speckle and to the square root of the number of statistically independent realizations of the speckle halo that are coadded.

The following heuristic picture of speckle formation in a large ground-based telescope is crude but in some ways instructive, as it brings out the essential characteristics of the speckle halo in a simple and accessible way. The turbulent atmosphere is characterized by a transverse coherence scale, r_0 (Fried's length), $\sim$10 cm at visible wavelengths in good seeing, over which the integrity of a wave front is roughly preserved.[6] Focal-plane speckles result when a few r_0-sized patches, randomly distributed over the aperture, happen to be mutually coherent. This coherence lasts for roughly the atmospheric coherence time, τ_0, which is $\sim$10 ms at visible wavelengths. Because the r_0-sized seeing cells may be distributed anywhere over the aperture of diameter D, they produce speckles of diffraction-limited spatial scale $\sim\lambda/D$, which is less than $\sim$0.1 arc sec on a 5-m telescope in the near infrared. Speckles move about randomly within an envelope (the speckle halo) of spatial scale $\sim\lambda/r_0$, typically 1 arc sec. A more rigorous picture of speckle formation proceeds by consideration of the Fourier relation between phase shifts in the pupil plane and image intensity in the focal plane, as is now discussed in much more detail.

Bloemhof *et al.*[4] considered a high-Strehl AO system attached to a telescope with aperture function $A = A(\xi, \eta)$, where (ξ, η) are pupil-plane coordinates.

0146-9592/04/020159-03$15.00/0

The unperturbed diffraction-limited PSF for such an aperture is $PSF_0 = |\hat{A}(x, y)|^2$, where the circumflex indicates Fourier transformation. Bloemhof *et al.*[4] introduced small phase perturbations $\Phi = \Phi(\xi, \eta)$ that represent the remnant atmospheric turbulence across the aperture that the AO system could not correct. Then an expansion of the Fourier-optical expression for the image of an unresolved star, $|\widehat{Ae^{i\Phi}}|^2$, leads to the following intensity pattern in the focal plane when only first-order terms in Φ are retained and a clear, unaberrated aperture is assumed:

$$\begin{aligned}\text{Image intensity} &= |\widehat{Ae^{i\Phi}}|^2 \\ &\approx |\hat{A}|^2 - 2\,\mathrm{Im}(\Phi)\hat{A} + |\hat{\Phi}|^2. \qquad (1)\end{aligned}$$

The first term on the right-hand side of Eq. (1) is the diffraction-limited PSF characterizing aperture A that is familiar from Fourier optics. The second and third terms, linear and quadratic in $\hat{\Phi}$, are small compared with the first (the PSF) if the degree of adaptive correction is high. They are the first-order contributors to the remnant halo (speckle) pattern, and they are thus the largest speckle terms at high correction (high Strehl, i.e., small remnant phase Φ). Amplitude $\hat{A}$ has the same zeros as the PSF, $|\hat{A}|^2$, and so imposes that structure on the linear term $-2\,\mathrm{Im}(\hat{\Phi})\hat{A}$, instantaneously dominant at highest Strehl, which it enters multiplicatively. This algebraic demonstration of speckle pinning was supplemented[4] by simulations of weak phase screens, confirming that speckles first appear, for small aberrations, on secondary maxima of the diffraction-limited PSF.

Figure 1 illustrates the linear and quadratic terms in a simulated speckle halo at moderate correction ($S \sim 0.6$) and deformable-mirror actuator density ($D/a \sim 64$; a is the actuator spacing). Symmetries of the two terms[5] are apparent and derive from the fact that remnant phase Φ is real, so speckle amplitude $\hat{\Phi}$ is a Hermitian function. It follows that the quadratic term $|\hat{\Phi}|^2$ is even, or spatially centrosymmetric (cf. Fig. 1, lower left). (A similar symmetry of speckles arising in a highly corrected coronagraph was discussed by Rouan *et al.*[7]; symmetries in AO imaging were discussed by Sivaramakrishnan *et al.*[8] Both references propose interesting observational tactics that exploit speckle symmetry.) The linear term, $-2\,\mathrm{Im}(\hat{\Phi})\hat{A}$, is spatially antisymmetric (cf. Fig. 1, upper right), so its integral over the halo vanishes and it contributes nothing to the instantaneous power in a short-exposure image. It is shown below that the quadratic-term speckles alone account for the traditional expression for power in the speckle halo, $(1 - S)$.

The Strehl ratio S is the fraction of image power contained in a diffraction-limited PSF peak. At high image correction, remnant phase errors are small, and one may relate S to the mean-square wave-front error by the equation $S = 1 - \langle\Phi^2\rangle_{\text{pupil}}$. Remnant phase Φ and speckle amplitude $\hat{\Phi}$ are Fourier conjugates and so by the Rayleigh theorem contain equal power in their respective pupil and image planes: $\iint |\Phi|^2 \mathrm{d}\xi \mathrm{d}\eta = \iint |\hat{\Phi}|^2 \mathrm{d}x \mathrm{d}y$. Substituting gives the power in the speckle halo,

$$\begin{aligned}(1 - S) = \langle\Phi^2\rangle_{\text{pupil}} &= \frac{1}{A_{\text{pupil}}} \iint |\Phi|^2 \mathrm{d}\xi \mathrm{d}\eta \\ &= \iint |\hat{\Phi}|^2 \mathrm{d}x \mathrm{d}y, \qquad (2)\end{aligned}$$

in units in which the PSF contains unit power [i.e., applying the theorem to an unaberrated pupil, $A(\xi, \eta) = 1$, and choosing units such that $\iint |\hat{A}|^2 \mathrm{d}x \mathrm{d}y = 1$ to normalize Eq. (2)]. Here A_{pupil} is the area of the pupil, i.e., of the telescope aperture, and $\Phi^2 = |\Phi|^2$ because the phase is a real function. Equation (2) shows that, at this level of approximation, the integrated power from the unpinned quadratic speckles of Eq. (1), $|\hat{\Phi}|^2$, fully accounts for the traditional speckle power total in a highly corrected image, $(1 - S)$.

Past analyses of highly-corrected imaging[2] used this traditional estimate of the power left in the speckle halo, $(1 - S)$ or its equivalent at high correction, the mean-square aperture phase $\langle\Phi^2\rangle$, to quantify companion-search sensitivity of future AO systems

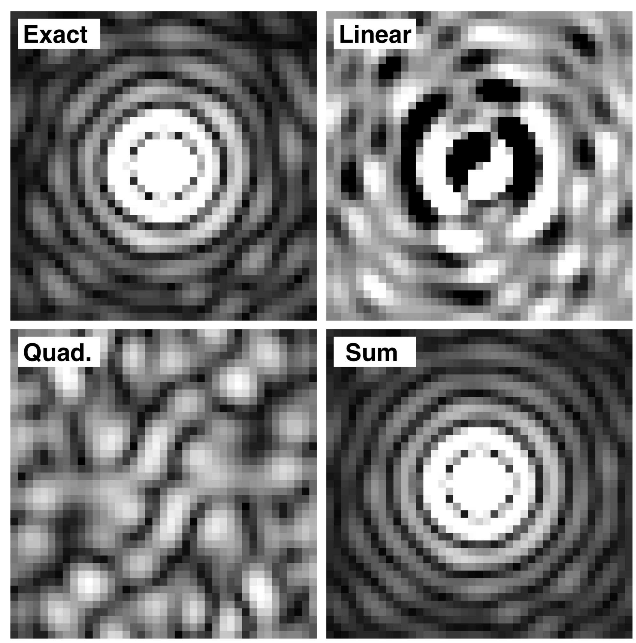

Fig. 1. Simulations of the two kinds of speckle that arise from first-order expansion of the phase exponential appropriate to high-Strehl imaging. A random remnant phase screen with Strehl ratio $S = 0.6$ and deformable-mirror actuator density $D/a = 64$ is assumed. Upper left, exact image, $|\widehat{Ae^{i\Phi}}|^2$. Upper right, antisymmetric pattern of anomalously bright pinned speckles from the linear term, $-2\,\mathrm{Im}(\hat{\Phi})\hat{A}$. (Here the negative peak is shown in black, and zero as gray; in the other parts of this figure, zero is black.) Lower left, symmetric pattern of unpinned speckles from the quadratic term $|\hat{\Phi}|^2$. Lower right, sum of PSF, linear, and quadratic terms. Both linear- and quadratic-term speckles may be identified in the exact and sum images. The quadratic speckles are free to roam anywhere within a halo of diameter $\sim\lambda/a$, but the anomalously bright linear-term speckles are pinned to Airy maxima and have nulls on every Airy null.

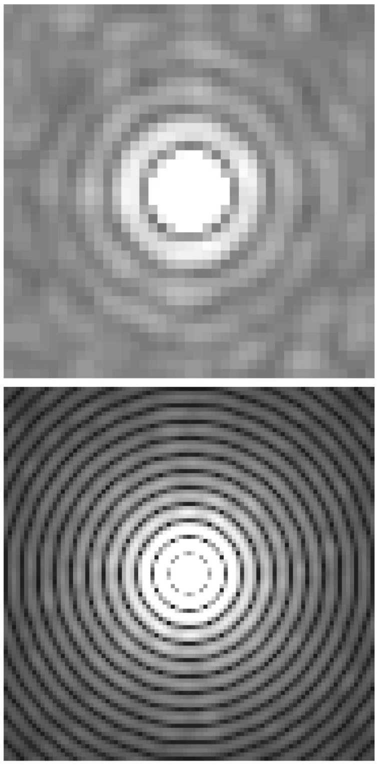

Fig. 2. Simulated distribution over the speckle halo of rms fluctuations in image intensity over many realizations of the speckle halo. The largest fluctuations follow the rings of the PSF because the anomalous (linear-term) speckles occur only there. Top, current AO ($S = 0.60$, $D/a = 16$); bottom (on a different spatial scale), advanced future AO ($S = 0.99$, $D/a = 100$). This figure illustrates that the anomalous linear-term speckles, although they contribute no instantaneous speckle power, can be the dominant source of speckle noise, particularly at large Strehl ratio and deformable-mirror actuator density.[5,9]

at high Strehl ratios. But this metric dramatically underestimates the typical speckle intensity and hence the speckle noise that is due to local fluctuations of the anomalously bright pinned speckles that arise in antisymmetric patterns from the linear (second) term in Eq. (1); these speckles have large individual intensities but zero power when they are integrated over the halo. The strength of these anomalously bright pinned speckles also depends on the density of deformable-mirror actuators, D/a, used by the AO system.[5] Quantitatively, at $S = 0.99$ and $D/a = 100$, pinned postcorrection speckles, $-2\,\mathrm{Im}(\hat{\Phi})\hat{A}$, dominate local speckle intensity and noise over short integrations by an order of magnitude on the 10th Airy ring[5] (Fig. 2) but contribute nothing to the total halo power.

The unpinned quadratic-term speckles $|\hat{\Phi}|^2$ fully account for the net halo power estimate, $(1 - S)$, that traditionally quantifies how well speckles have been suppressed. They (along with, possibly, photon noise from the PSF, depending on details such as the flux of the primary star) will provide an ultimate noise floor for AO observations after observations of Airy nulls or long time integrations have adequately suppressed the anomalously bright, zero-mean, spatially pinned speckles.[5] But the operation of future AO systems should be considered in detail to ensure that these anomalously bright pinned speckles are adequately attenuated, particularly in view of concerns that speckle correlation times may be longer than previously assumed.[10] On short time scales the integrated remnant power in the speckle halo $(1 - S)$ does not account for the brightest remnant speckles and so provides a poor guide to estimating speckle noise.

The research described in this Letter was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The author's e-mail address is eeb@huey.jpl.nasa.gov.

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