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Wavefront control for high contrast imaging

Exo-planet imaging workshop

Laurent Pueyo

Johns Hopkins University Physics and Astronomy department

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2 Wavefront sensing







Context	Wavefront sensing		Conclusions	References
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Direct Imagir	ng of exo-planets			

At the camera of an instrument dedicated to imaging exo-planets optical artifacts look like planets. We can:

• Remove them "coherently", e.g making PSF core starlight photons interfere destructively with starlight photons scattered by optical errors: *wavefront control.*

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Or Calibrate them using post-processing and use this calibration to reveal planets.

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- Or Calibrate them using *post-processing* and use this calibration to reveal planets.

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Or Calibrate them using *post-processing* and use this calibration to reveal planets.

Purpose of this lecture: wavefront sensing and control

This lecture is **not** and exhaustive review of all the concepts / instruments that have been proposed / are being built to address this problem .

This lecture is an attempt to look at wavefront control from the angle of the post-processing.

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Quasi-sta	tic speckles			

- Mirrors are not perfect.
- In the imperfections scatter light.
- The structure of this scattered light changes with time and wavelength.

From Space: Two Hubble Space Telescope PSFs from two different rolls: the quasi-static speckles change a between the orientations.



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From the ground: Project 1640 PSF.

• Diffuse halo due to the average atmospheric turbulence.



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- Diffuse halo due to the average atmospheric turbulence.
- Quasi-static speckles are present under the halo.



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Quasi-sta	tic speckles			

- Mirrors are not perfect.
- In the imperfections scatter light.
- The structure of this scattered light changes with time and wavelength.

From the ground: Project 1640 PSF.

- Diffuse halo due to the average atmospheric turbulence.
- Quasi-static speckles are present under the halo.
- Speckles vary slowly with time and wavelength.



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Context	Wavefront sensing		Conclusions	References
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Quasi-sta	atic speckles			

Crepp et al. (2010)



Quasi-static speckles decorrelate with time and wavelength.

Impact on post-processing

- In the absence of a good model of the PSF variations, this effect cannot be calibrated.
- Wavelength behavior can somewhat be predicted a priori.
- The time variations can be determined in the statistical sense.

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2 Wavefront sensing











The wavefront actuation is done using a Deformable Mirror.

Cameras do not measure phase delays, they count photons. A wavefront sensor is an optical system which "converts" wavefront into images.

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From Claire Max's lecture.

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From Claire Max's lecture.

Wavefront sensing for Adaptive Optics

- The order of magnitude of the errors is several waves.
- The figure of merit is the Strehl ratio, or how much the PSF is concentrated in its core.
- Non common path error are not critical under such a metric.

Wavefront sensing for High Contrast

- The order of magnitude of the errors is < 1 wave.</p>
- The figure of merit is the actual contrast in the final image.

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AO and	wavefront control			

Courtesy of B. Oppenheimer



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AO and	wavefront control			

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Orders of	magnitude for wave	front sensing and co	ontrol	
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Time constants

- From the ground: Speckles de-correlate in a matter of minutes ~ exposure time, the correction has to to occur during a science observing sequence.
- **9** From space: Speckles de-correlate in a matter of hours ~> exposure time, sensing can occur between observing sequences.

Contrast and non-common path

- **()** From the ground: Self-luminous Jupiters, contrast $\sim 10^7$.
- **② From space:** Reflected light earth-like planets, contrast $\sim 10^{10}$.
- These constraints drive optical design: minimize the number of optics between the wavefront sensor and the science camera.

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Wavefro	nt sensing			



From Claire Max's lecture.

From space:

- Because of the tight contrast constraint the sensing has to be done at the focal plane camera.
- Science exposures can be used for the sensing.

From ground:

- The sensing can happen a little before the science camera, some level of non common path is tolerable.
- Because of the time constants the science exposures cannot be used for the sensing.

Underlying principle: interferences					
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The goal is to measure the wavefront but an images is the square modulus of the field

$$I = |a e^{i\phi}|^2$$

Solution: add a set of known wavefront disturbance in the plane of the camera

$$I_k = |ae^{i\phi} + b_k e^{i\psi_k}|^2$$

$$I_k = a^2 + b_k^2 + 2ab_k \cos(\phi - \psi_k)$$

Inverse problem

Solve for the ϕ , and *a* if needed, based on the know disturbances (b_k, ψ_k) .

Depending on the configuration (b_k, ψ_k) might not be well known: modeling can play a crucial part in the wavefront sensing problem.

Trade-offs

Choosing the a wavefront sensing architecture is a trade-off between: the contrast constraint, time dependence constraint, and sensitivity to modeling.



Wallace et al. (2009); Pueyo et al. (2010)



The light after the coronagraph is interfered with some of the light rejected by the coronagraph. The wavefront is retrieved using phase shifting interferomery. If $(1+r)e^{i\phi}$ is the field **before** the coronagraph. Then the field **after** the coronagraph is $\sim r + i\phi$.



With $b_k e^{i\psi_k} = b_0 e^{i(k-1)\frac{\pi}{2}} k = 1, 2, 3, 4$ then:

$$\begin{array}{rcl} \phi & \sim & I_3 - I_1 \\ r & \sim & I_4 - I_2 \end{array}$$

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Wallace et al. (2009); Pueyo et al. (2010)



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Post Coronagraphic sensing			

Wallace et al. (2009); Pueyo et al. (2010)



The light after the coronagraph is interfered with some of the light rejected by the coronagraph. The wavefront is retrieved using phase shifting interferomery.

Trade Offs

- Sensing can occur during a science exposure: good for a ground based with short time constants.
- Some optics between sensor and science camera: non-common path will limit contrast.
- If the interferometer is properly phased then no model of the system is needed for reconstruction.

Interferometric measurement at the telescope					
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Palomar Hale Telescope



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The light at the science camera is interfered with a tiny amount of light scattered by the DM.

If $Re(E_{Cam}) + iIm(E_{Cam})$ is the field at the science camera.



$$Re(E_{Cam}) \sim l_3 - l_1$$

$$Im(E_{Cam}) \sim l_4 - l_2$$





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$$Re(E_{Cam}) \sim l_3 - l_1$$

$$Im(E_{Cam}) \sim l_4 - l_2$$







Trade Offs

- Sensing cannot occur during a science exposure: good for space based with long time constants.
- No non-common path errors.
- Usually we do not know perfectly DM actuators to focal plane transfer function, sensitive to modeling.

The light at the science camera is interfered with a tiny amount of light scattered by the DM.

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Focal pl	ane sensing in practice			

The field at the camera is a non-linear function of the voltages:

$$E_{cam}(\xi,\eta) = \mathscr{G}(V_1,...,V_n,...,V_N)$$

We discretize the focal plane in and linearize this function:

$$\begin{bmatrix} Re[E_{cam}(\xi_{1},\eta_{1})] & \dots & \frac{\partial Re(\mathscr{G})}{\partial V_{1}}|(\xi_{1},\eta_{1}) & \dots & \frac{\partial Re(\mathscr{G})}{\partial V_{N}}|(\xi_{1},\eta_{1}) \\ \dots & \dots & \dots \\ Re[E_{cam}(\xi_{p},\eta_{q})] \\ \dots \\ Re[E_{cam}(\xi_{1},\eta_{1})] \\ Im[E_{cam}(\xi_{1},\eta_{1})] \\ \dots \\ Im[E_{cam}(\xi_{p},\eta_{q})] \\ \dots \\ Im[E_{cam}(\xi_{M},\eta_{M})] \end{bmatrix} = \begin{bmatrix} \frac{\partial Re(\mathscr{G})}{\partial V_{1}}|(\xi_{1},\eta_{1}) & \dots & \frac{\partial Re(\mathscr{G})}{\partial V_{N}}|(\xi_{p},\eta_{q}) & \dots \\ \dots & \dots & \dots \\ \frac{\partial Re(\mathscr{G})}{\partial V_{1}}|(\xi_{1},\eta_{1}) & \dots & \frac{\partial Re(\mathscr{G})}{\partial V_{N}}|(\xi_{M},\eta_{M}) \\ \frac{\partial Im(\mathscr{G})}{\partial V_{1}}|(\xi_{1},\eta_{1}) & \dots & \frac{\partial Im(\mathscr{G})}{\partial V_{N}}|(\xi_{1},\eta_{1}) \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \frac{\partial Im(\mathscr{G})}{\partial V_{1}}|(\xi_{1},\eta_{1}) & \dots & \frac{\partial Im(\mathscr{G})}{\partial V_{N}}|(\xi_{M},\eta_{M}) \end{bmatrix} \end{bmatrix} \begin{bmatrix} V_{1} \\ \dots \\ V_{n} \\ \dots \\ V_{N} \end{bmatrix}$$

Modelling tools

The modeling tools presented in the previous lecture are critical to compute a "G" matrix that is accurate enough.





Use the Deformable Mirror for diversity

• We apply a set of probe voltages to the Deformable Mirror $\mathbf{V}_k = [V_1^k, ..., V_n^k, ..., V_N^k, k = 1, ..., N$

$$I_k = |E_{abb}|^2 + |G\mathbf{V}_k|^2 + 2 * Re(E_{abb}G\mathbf{V}_k)$$

2 We solve for $(Re[E_{abb}], Im[E_{abb}])$ for each point in the focal plane array.



Guyon et al. (2010)



- A planet is not coherent with the starlight.
- It does not interfere with the light from the probes.
- The signal from a planet does not appear in the coherent estimate.

Use wavefront sensing for detection

- In general any information on the structure of the aberration can be included in a detection algorithm.
- However the speckles are not fully deterministic, so this approach will be limited by photon noise.
- For this reason we try to suppress these speckles coherently.

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2 Wavefront sensing







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From the	ground			

In practice there are wavefront errors in the interferometer, amplitude error and coronagraphic leak can complicate the sensing:



The $\phi \sim l_4 - l_2$ approximation is not valid.



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Model based correction

We measure G using the four step phase shifting interferometer.

- We estimate (*Re*[*E*_{Lyot}], *Im*[*E*_{Lyot}]) using the four step phase shifting interferometer.
- Since the Deformable Mirror can only create an imaginary field we seek to minimize the quadratic cost function:

 $||Im[E_{Lvot} - GV]||^2$

- Since the coronagraph suppresses the low order modes, the linear problem associated with this minimization is ill-posed.
- Using our "favorite regularization" we solve for the Deformable Mirror commands V

P1640, Courtesy of G. Vasisht.



We iterate.

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P1640, Courtesy of G. Vasisht.

Residual speckles are amplitude

- Amplitude errors arise from reflectivity non uniformities and free space propagation of phase errors, Pueyo and Kasdin (2007).
- Deformable Mirror is a phase only actuator.
- Even if the interferometer can measure amplitude the Deformable Mirror cannot "fully" correct such errors.



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P1640, Courtesy of G. Vasisht.

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The field after the coronagraph or at the image plane can be written under the linear approximation as:

$$(1+r)e^{i\phi}\simeq 1+r+i\phi$$

- Amplitude in the plane of the Deformable Mirror creates an hermitian pattern in the image plane.
- Phase in the plane of the Deformable Mirror creates an anti-hermitian pattern in the image plane.
- Phase on the Deformable Mirror can correct for half of the amplitude errors
- This leads to a dark hole only on one side of the optical axis.

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Model based co	orrection				
We calculate G model.	G using a high t	idelity			

- We estimate $(Re[E_{abb}^{\lambda_p}], Im[E_{abb}^{\lambda_p}])$ for a series of wavelength λ_p in the bandpass.
- Since we minimize the cost function only on one side of the image plane:

$$\sum_{p} || \boldsymbol{E}_{abb}^{\lambda_{p}} - \boldsymbol{G}^{\lambda_{p}} \mathbf{V}] ||^{2}$$

- Since the coronagraph suppresses the low order modes, the linear problem associated with this minimization is ill-posed.
- Using our "favorite regularization" we solve for the Deformable Mirror commands V

High Contrast Imaging Testbed, JPL



Borde and Traub (2006); Give'on et al. (2007)

We iterate.



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- Using our "favorite regularization" we solve for the Deformable Mirror commands V
- We iterate.



One of the deformable mirrors is not placed at a conjugate of the pupil.



Weak coupling

The coupling between phase and amplitude is weak, a large phase deformation is needed to create a small amplitude term.



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Two Deformable Mirrors correction

Correction methodology

- At a given iteration we choose a target contrast.
- We estimate $(Re[E_{abb}^{\lambda_p}], Im[E_{abb}^{\lambda_p}])$ for a series of wavelength λ_p in the bandpass.
- Mininize $\sum_k |V_k^{(1)}|^2 + |V_k^{(2)}|^2$ under the constraint $\sum_p I^{\lambda_p} < 10^C$: use the two DMs to correct the amplitude part.
- In this case the intensity in the Dark Hole is still a quadratic form

$$I^{\lambda_{p}} = \sum_{p} (\frac{2\pi\lambda_{0}}{\lambda_{p}})^{2} [\mathbf{V}_{1} \, \mathbf{V}_{2}] \begin{bmatrix} M_{11}^{(\lambda_{p})} & M_{12}^{(\lambda_{p})} \\ M_{12}^{(\lambda_{p})} & M_{22}^{(\lambda_{p})} \end{bmatrix} [X_{1} \, X_{2}]^{T} \\ + 2 \frac{2\pi\lambda_{0}}{\lambda_{p}} [\mathbf{V}_{1} \, \mathbf{V}_{2}] . \Im ([b_{1}^{(\lambda_{p})} \, b_{2}^{(\lambda_{p})}]^{T})$$

Where the *M*'s are the self correlation of the G^{λ_p} 's with themselves and *b*'s are the correlation of the G^{λ_p} 's with $(Re[E_{abb}^{\lambda_p}], Im[E_{abb}^{\lambda_p}])$

• Once the correction has been applied we iterate to a lower contrast target. This ensures convergence.

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Two Deformable Mirrors correcti		rection		

High Contrast Imaging Laboratory, Princeton





Two Deformable Mirrors correction



Convergence is slower because of the extra care with which the weak cou was treated.

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Wavefront correction changes chromaticity

$$\begin{split} \mathcal{I}^{\lambda_{p}} &= \sum_{p} (\frac{2\pi\lambda_{0}}{\lambda_{p}})^{2} [\mathbf{V}_{1} \, \mathbf{V}_{2}] \begin{bmatrix} M_{11}^{(\lambda_{p})} & M_{12}^{(\lambda_{p})} \\ M_{12}^{(\lambda_{p})} & M_{22}^{(\lambda_{p})} \end{bmatrix} [X_{1} \, X_{2}]^{T} \\ &+ 2 \frac{2\pi\lambda_{0}}{\lambda_{p}} [\mathbf{V}_{1} \, \mathbf{V}_{2}] . \Im ([b_{1}^{(\lambda_{p})} \, b_{2}^{(\lambda_{p})}]^{T}) \end{split}$$

Whether one or two Deformable mirrors are used, various weights are given to wavelength in the correction.

The wavefront control makes the speckles smaller but much more chromatic and thus a lot harder to model in order use chromaticity priors for detection.

Wavefront sensing is an "optical solution" of the detection problem

For focal plane wavefront sensing: if the speckles can be perfectly estimated at a given contrast, then a planet can be detected at that contrast. For non focal plane sensing: the estimate can be used to inform the detection algorithm about the potential PSF structures due to aberrations.

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