Linear Lyot Coronagraph Theory

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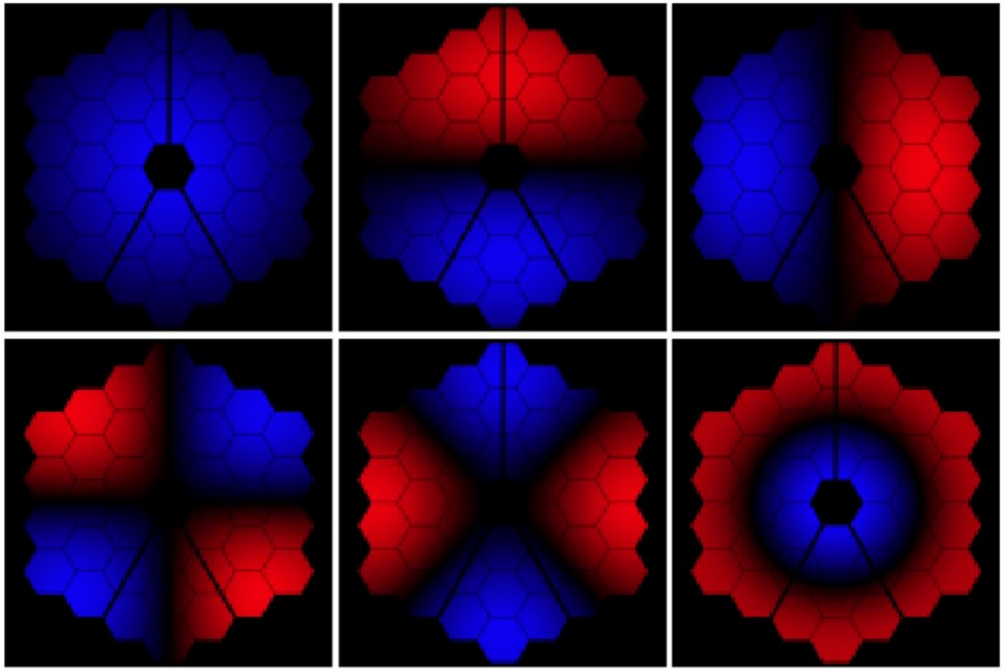
- Searching for PIAACMC Phase focal plane masks (PFPM) is nonlinear, time-consuming, and difficult.
- Would benefit from a simpler approach that can be used for building intuition and actual solutions.

- Develop a field-oriented approach using complex focal plane masks (CFPM) and linear operators.

- After we have a CFPM solution as a function of wavelength, implement it as an array of phase-only pixels (PoPs).
  - Complex dithering for narrowband solution
  - Use full-wavelength dithering to approximate required chromaticity.

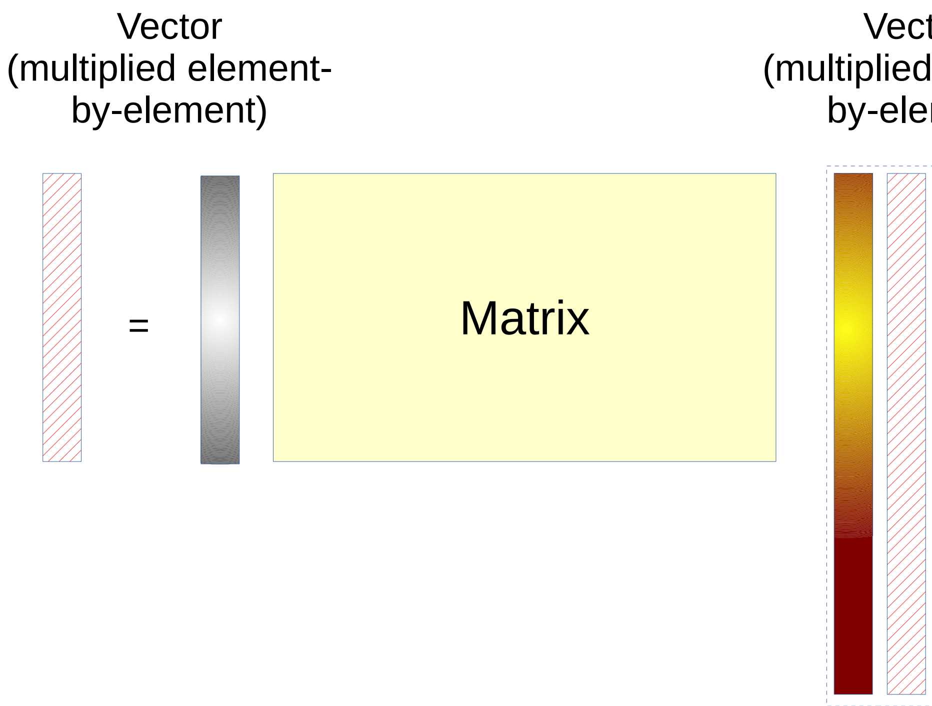
Approach: Hilbert Space & Linear Algebra

- List (unfold) pixel values as vectors. (Examples: pupil field, pupil apodization, focal plane mask, etc.)
- Treat these lists of values as if they were points in a high-dimensional Euclidean space. Do typical geometry with them.
- Use norms to measure “distance.”
- Use inner products to define “angles.”
- Optical operations become matrices.
- Natural vector bases appear that can be replaced into the original image (e.g. pupil plane) to give “modes.”



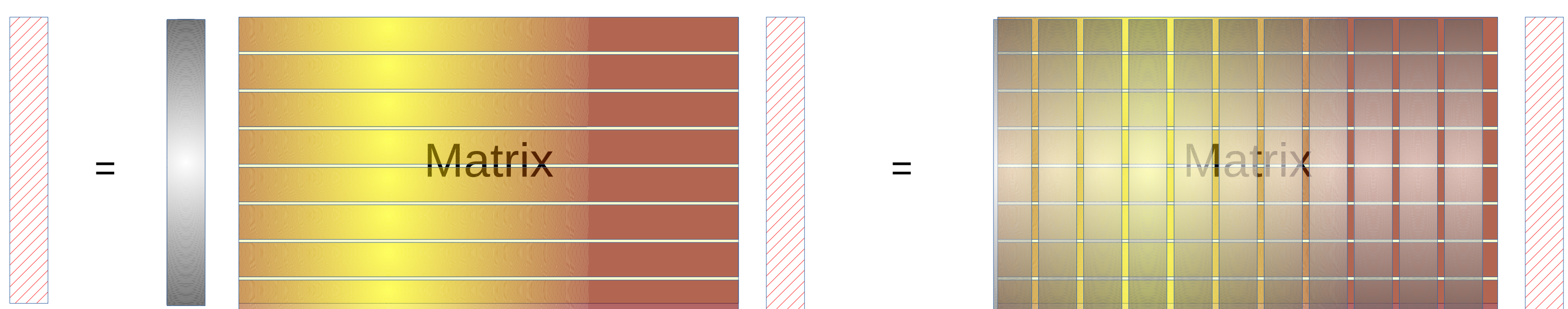
- We will mostly talk about complex fields and transmittance, rather than phase.
- We will need to design and build *complex* focal plane masks (phase and amplitude), but in effect we already are.

Grouping Matrices and Vectors



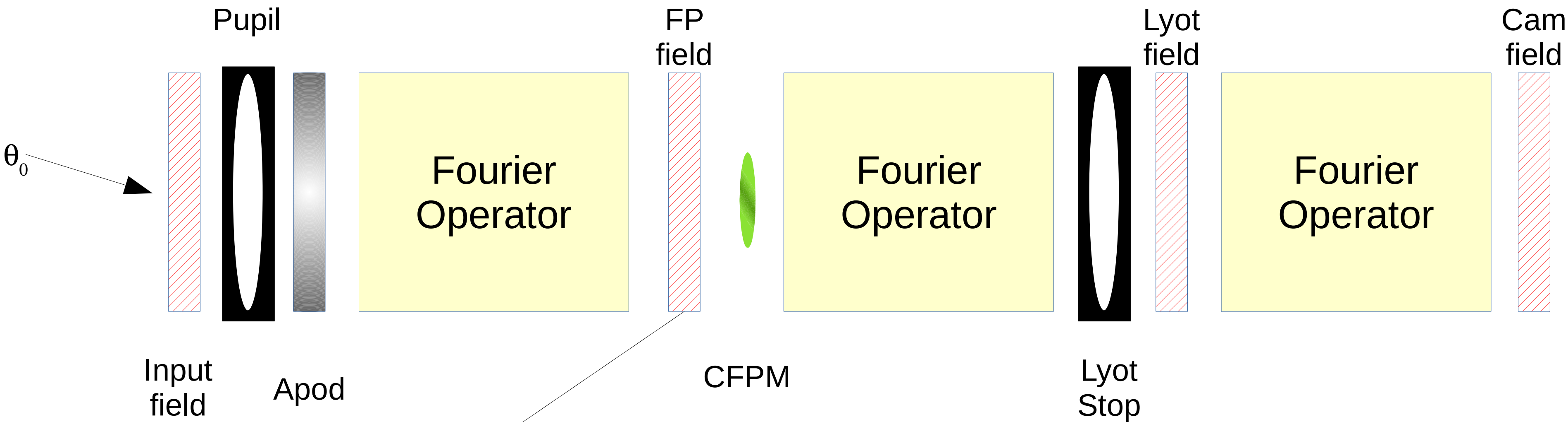
Multiplied vectors on the right can be grouped into the rows.

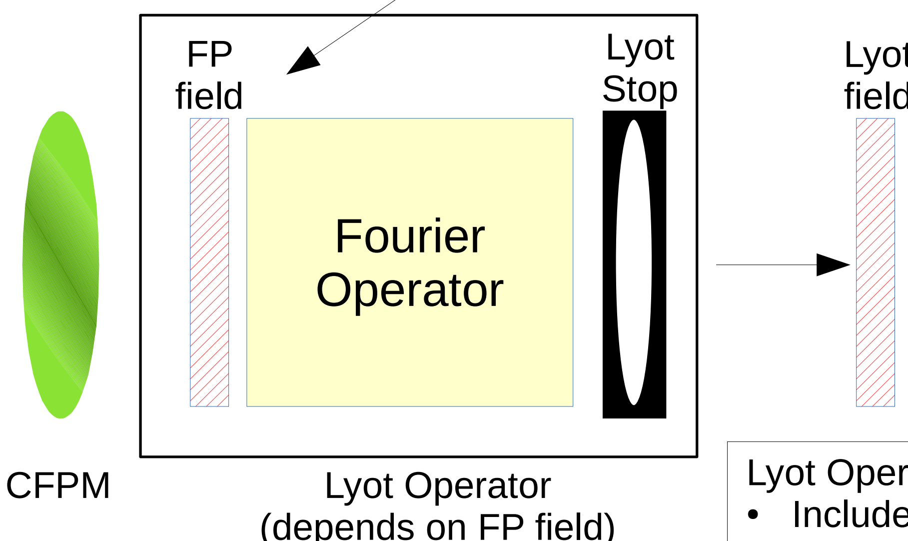
Multiplied vectors on the left can be grouped into the columns.



By including the vectors in the matrices, we can make new matrix operators.

Coronagraph using Linear Algebra





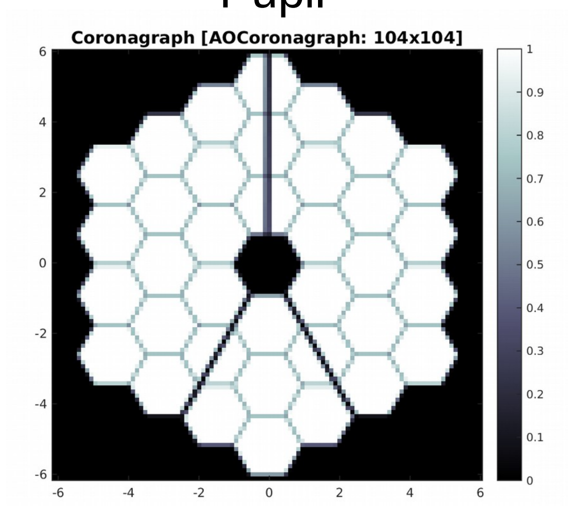
Lyot Operator:

- Includes FP field
- Has modes
- Has "null space"
- Use SVD to study.

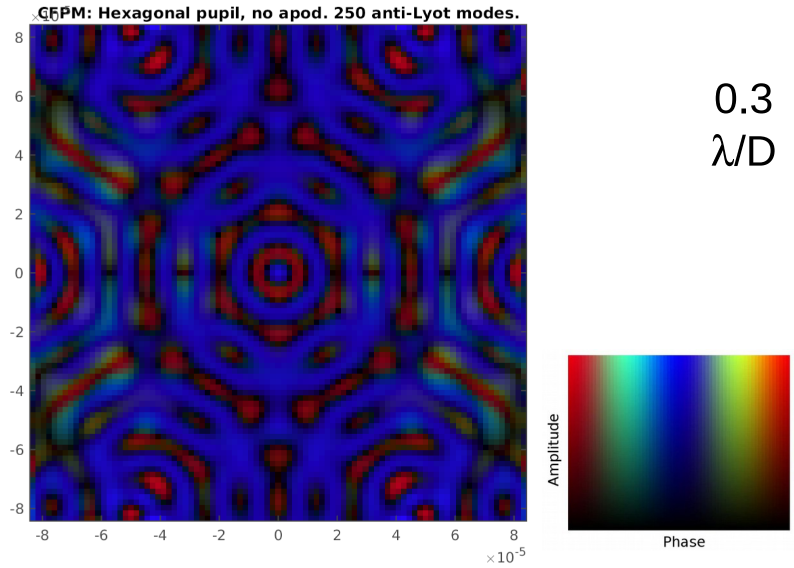
- Lyot Operator takes the CFPM as input and gives the field after the Lyot stop.
- Lyot Op includes the focal plane field and is specialized to that input direction.
- Complete suppression occurs when the CFPM lies in the Null Space of the Lyot Operator.
- Find the null space using SVD.
- Project out non-nullspace CFPM modes for on-axis and small off-axis waves.

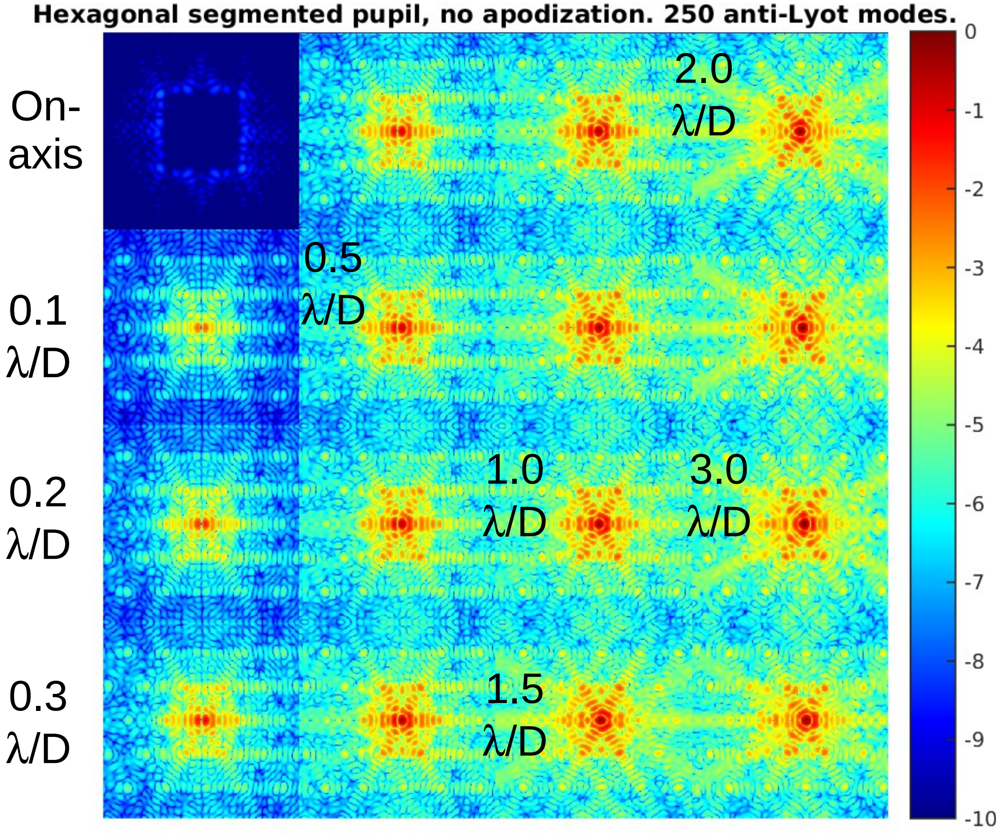
Hexagonal Pupil Example

Pupil



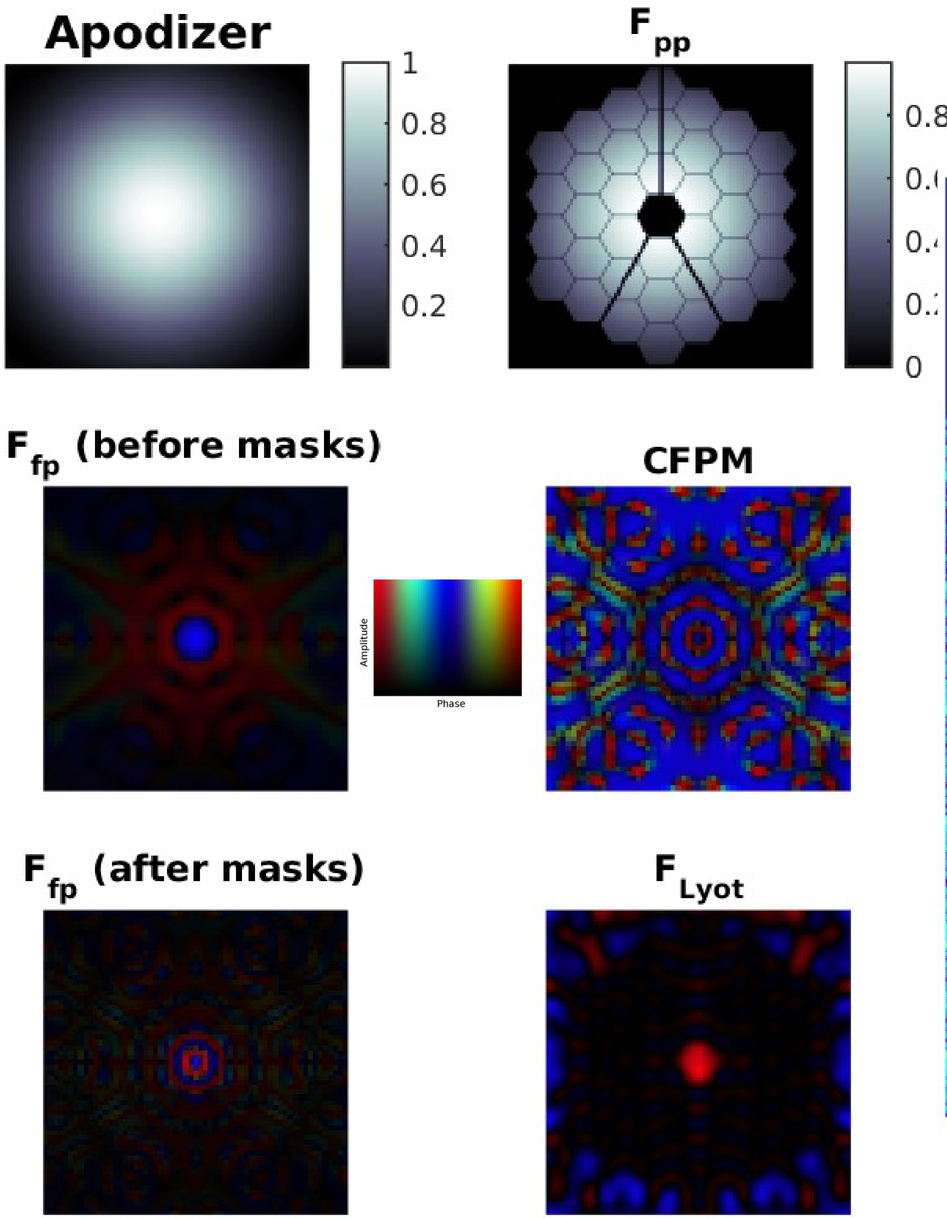
CFPM

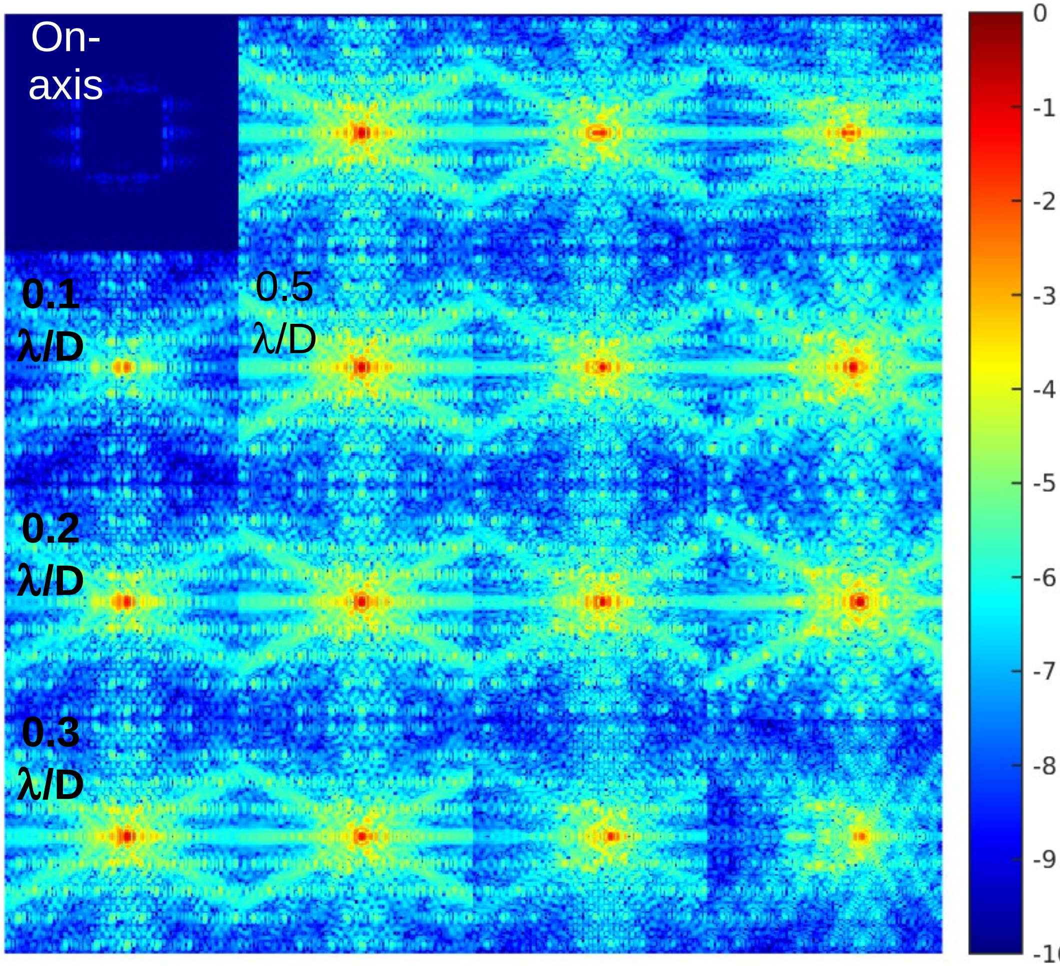




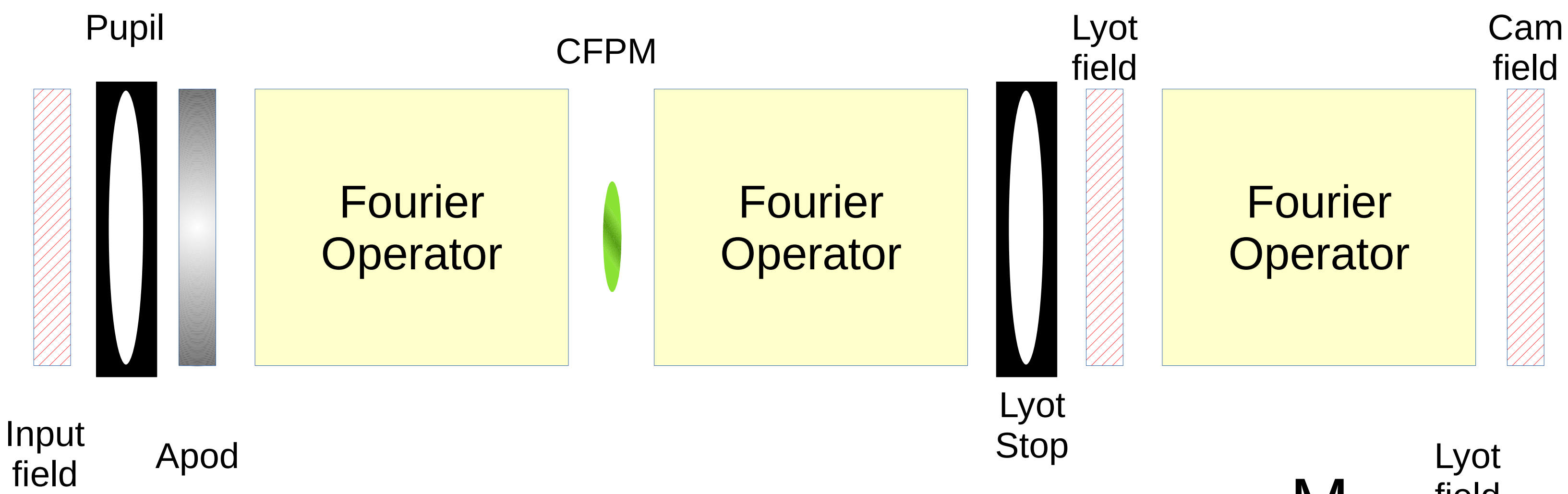
- Segmented Hexagonal Pupil
- No apodization
- CFPM pixels selected at λ/3D
- 250 anti-Lyot modes
- Lyot stop is copy of entrance pupil

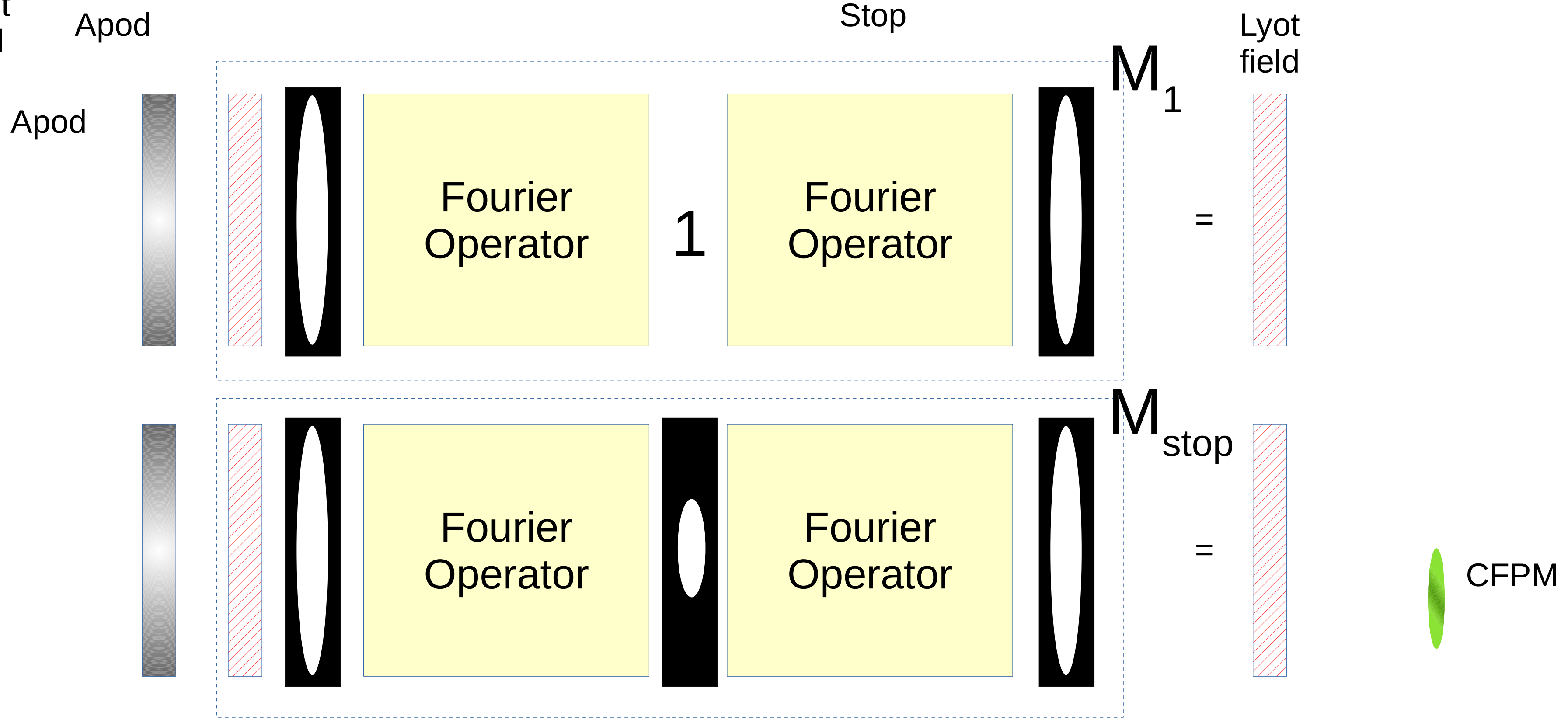
Apodized Pupil Example





APCMLC Using Linear Algebra





APCMLC Using Linear Algebra

- We want to find an apodization ($\underline{\alpha}$) such that the resulting complex field in the Lyot plane seen through the unstopped and stopped focal planes are scaled versions of each other.

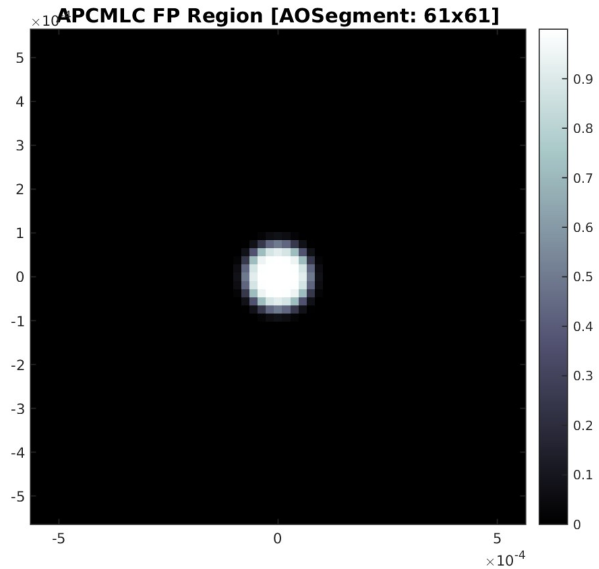
$$\mathbf{M}_1\underline{\alpha} = q\ \mathbf{M}_{stop}\underline{\alpha}$$

- This is a generalized eigenvalue problem. Finding a usable $\underline{\alpha}$ determines the scale factor q.
- Combining these two paths allows us to find a matched apodization and complex mask that completely suppress the on-axis starlight and is completely transparent away from the star.

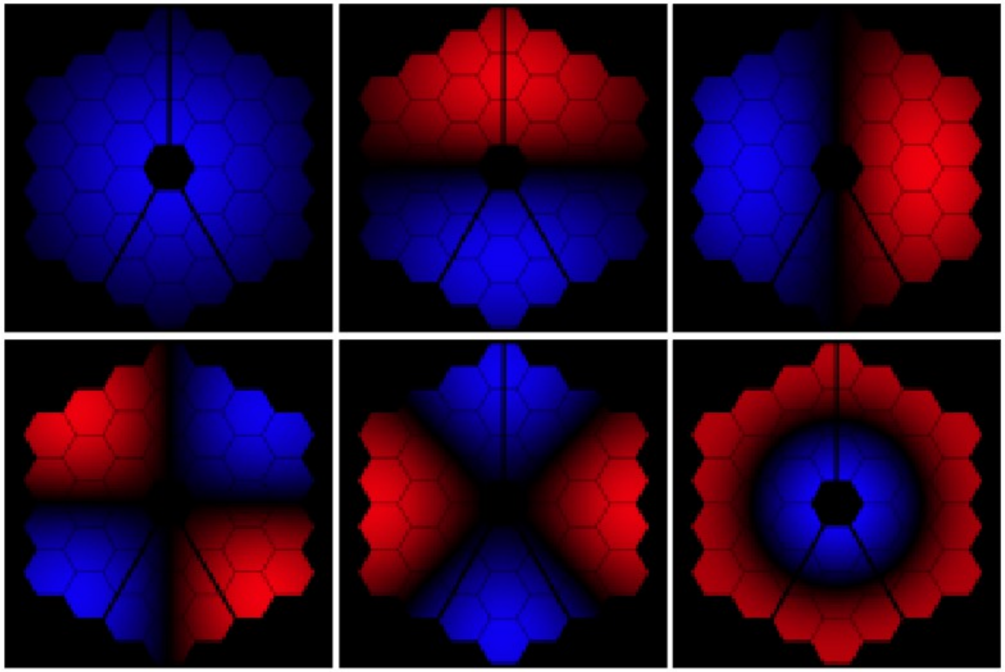
This solution is built to be in the null space of our Lyot operator and is the starting point for the projection techniques.

Eigenvector APCMLC Example

Select a FP stop...



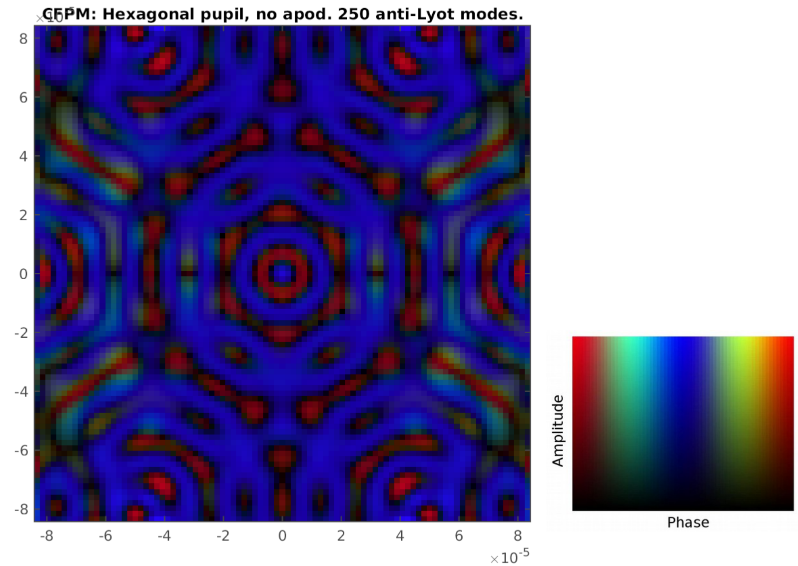
Compute eigenvectors



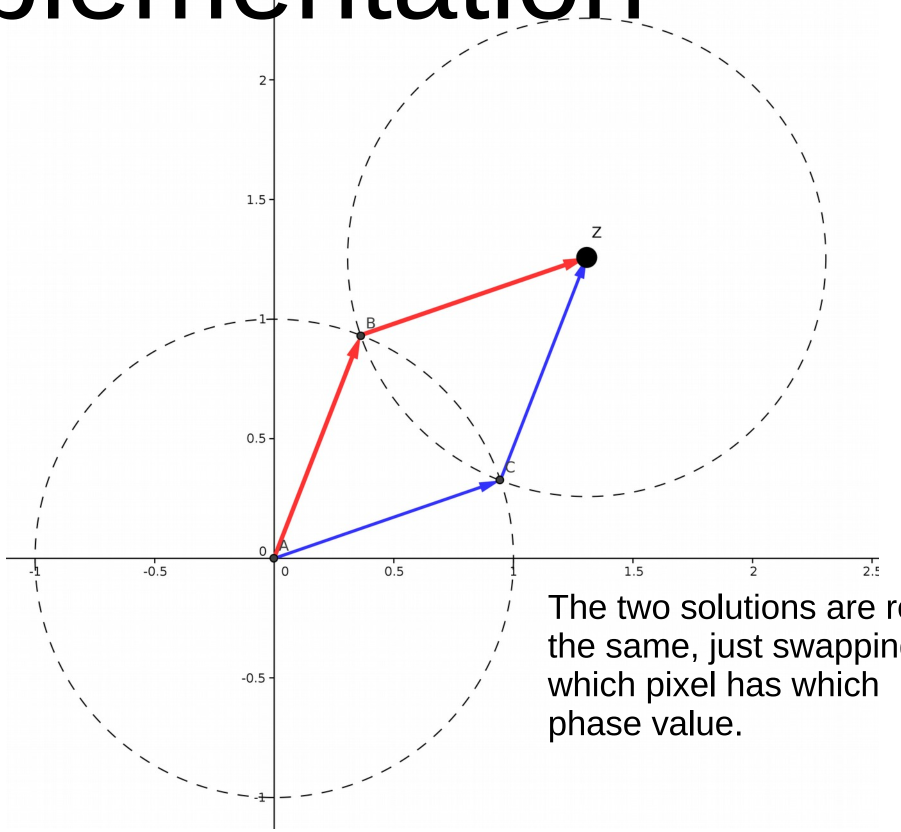
- Eigenvector determines the eigenvalue (i.e. determine q).
- Combine transparent FP with -q times FP stop.
- Gives a complex focal plane mask that eliminates on-axis starlight.
- Use projection techniques to get more features.

CFPM Implementation

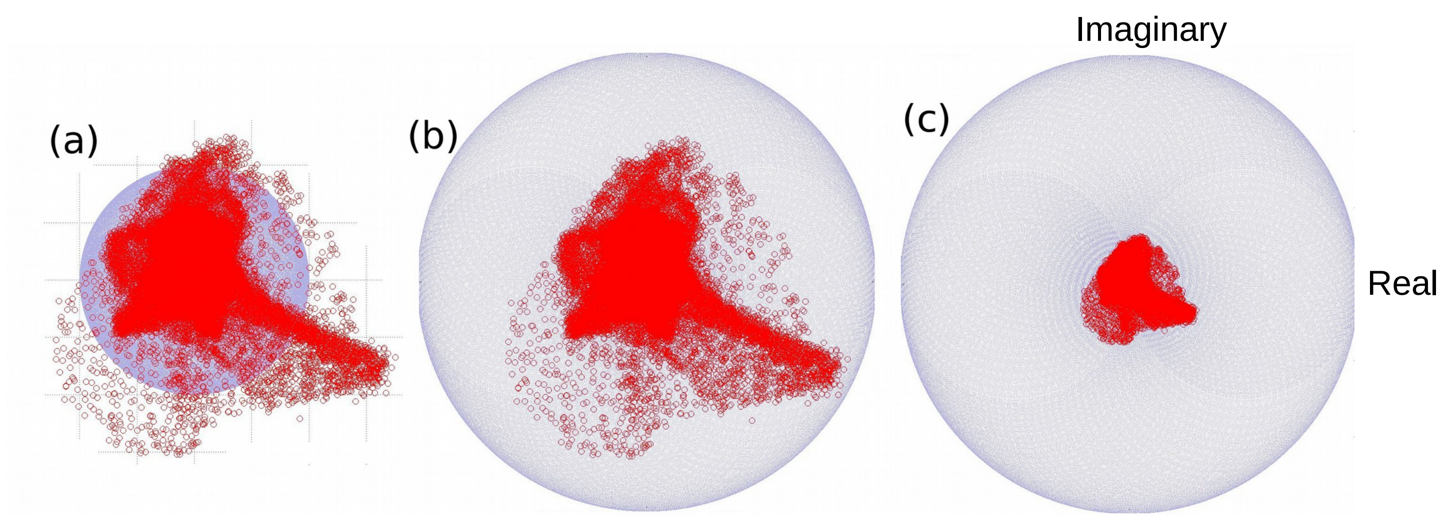
CFPM



Sum of 2 unit phasors
addresses dense complex range



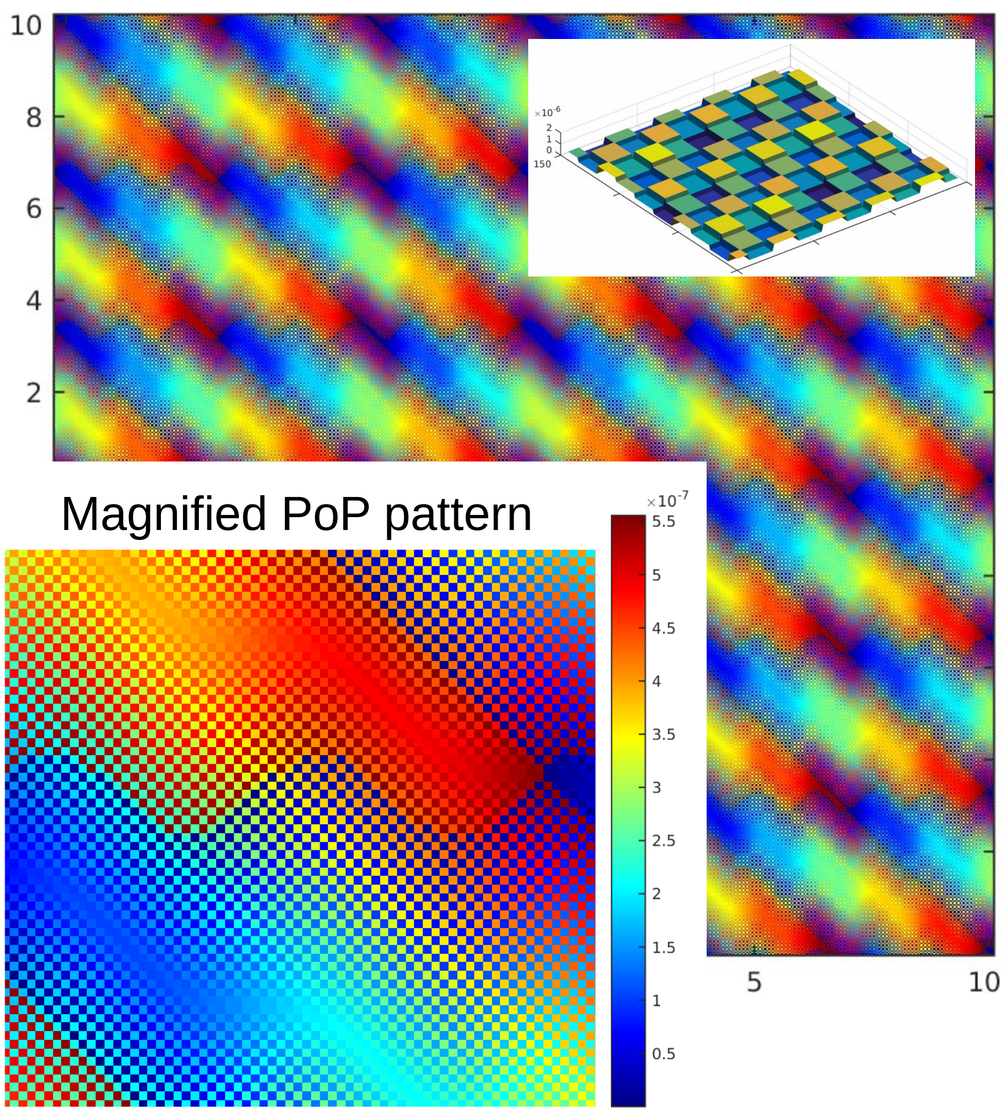
The two solutions are really the same, just swapping which pixel has which phase value.



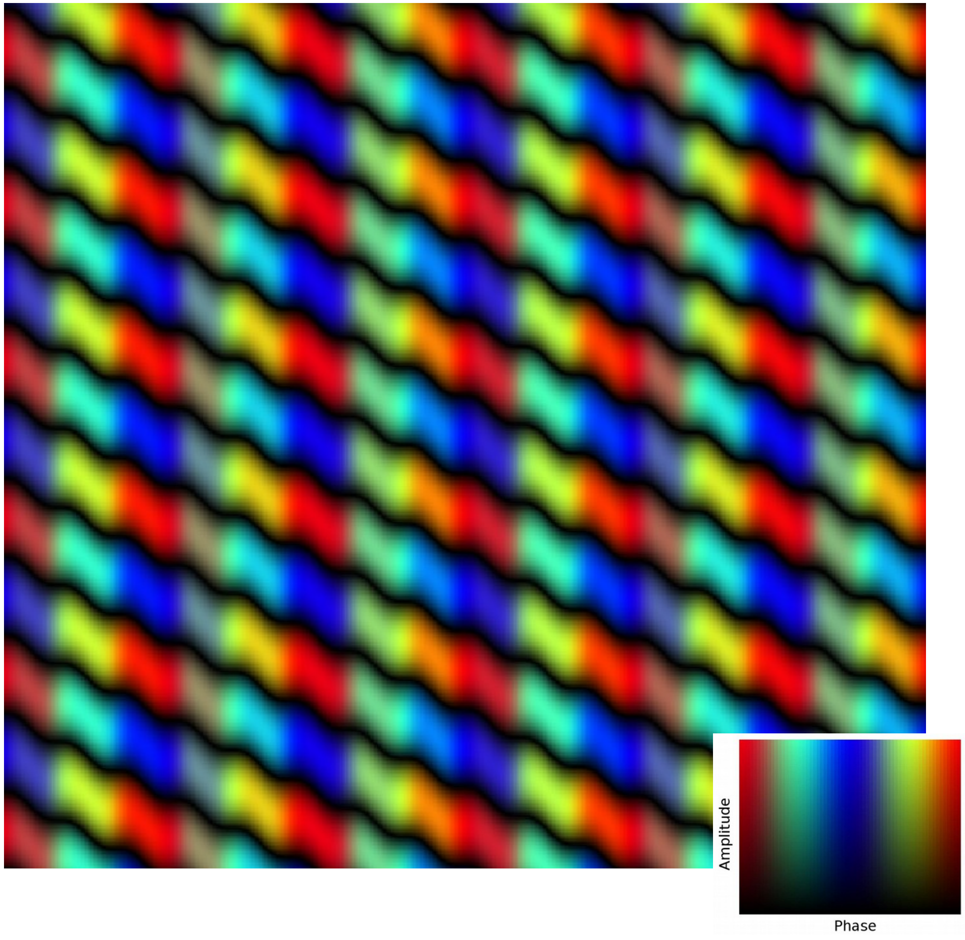
A complex map can be created out of an array of pixel pairs. How it maps onto the possible complex numbers depends on an overall transmission scale factor.

Arbitrary CFPMs using PoPs (phase-only-pixels)

Dithered PoP pattern

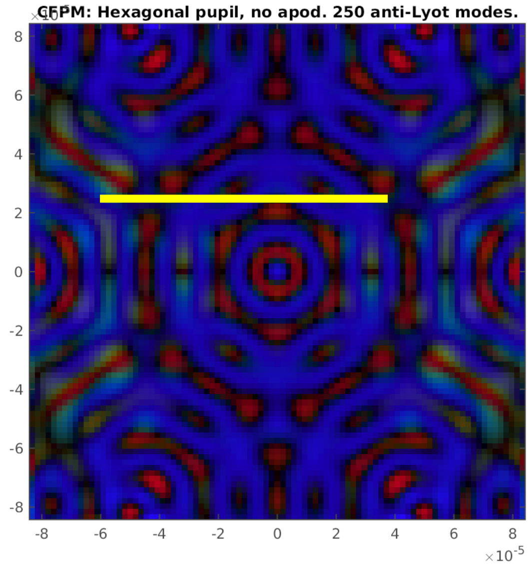


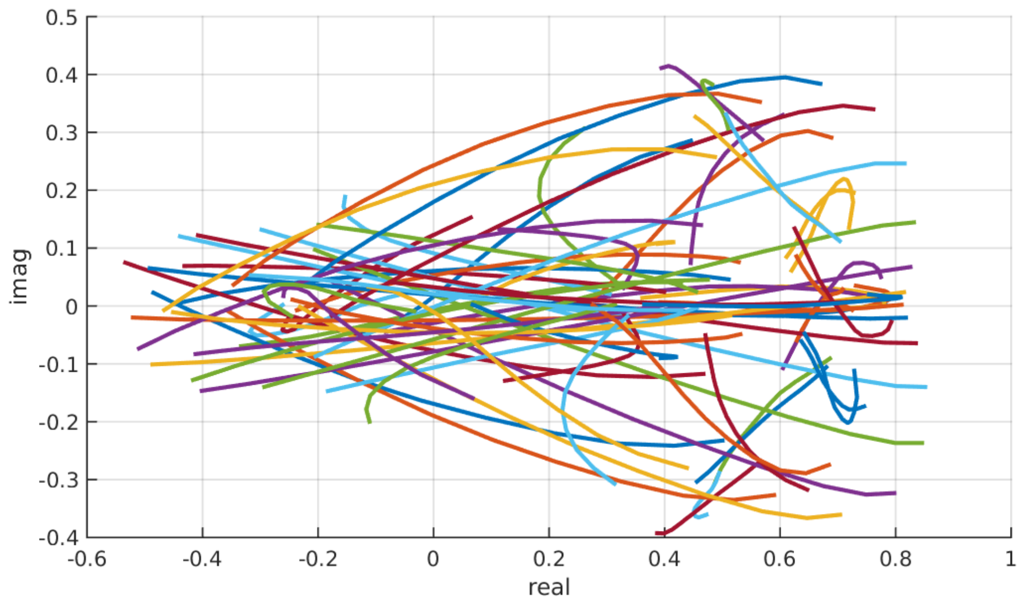
Phasors smoothed over 2x2 region



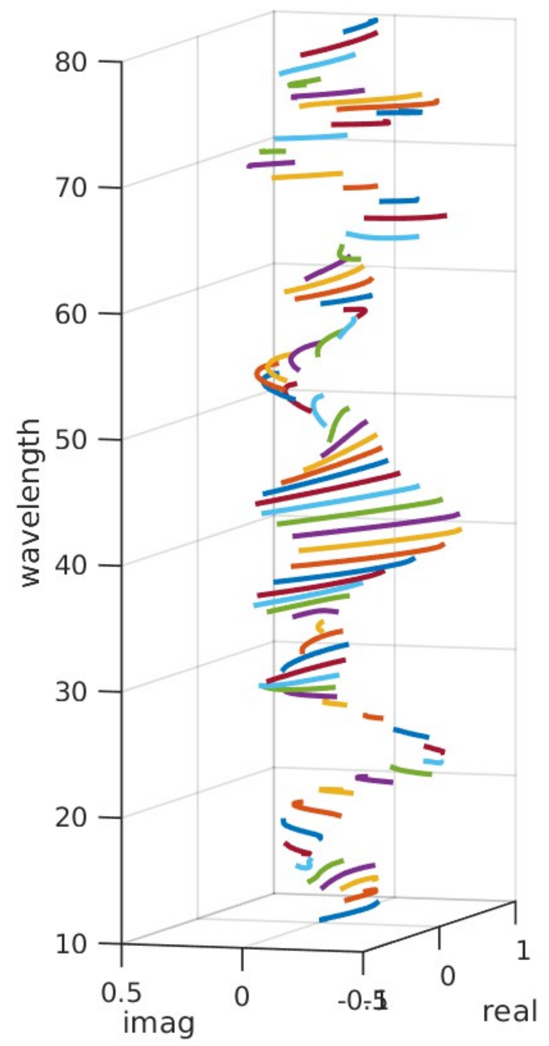
CFPM Chromaticity

CFPM

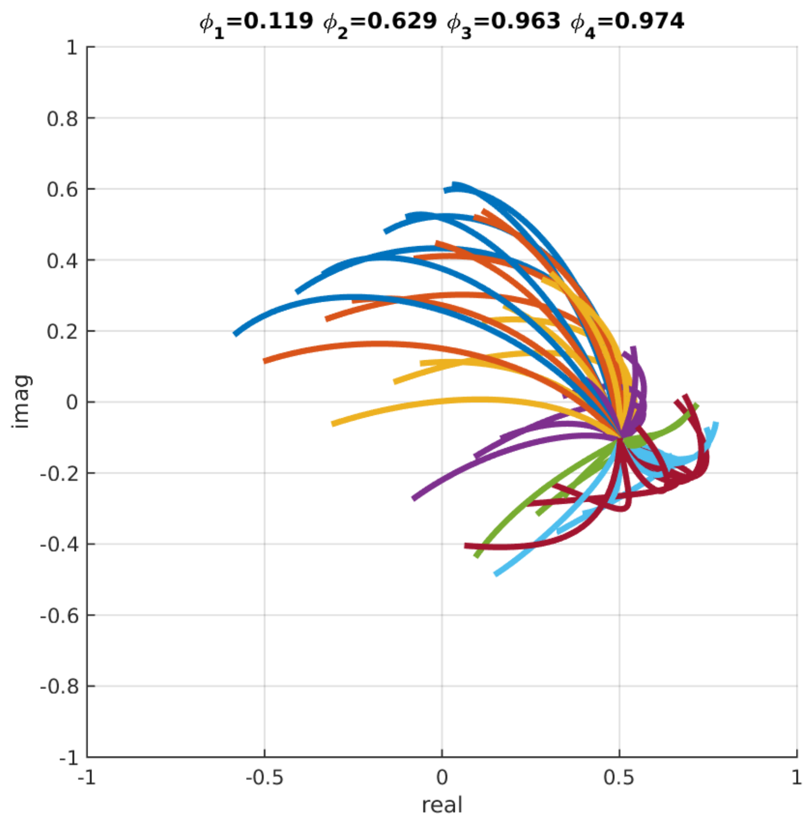


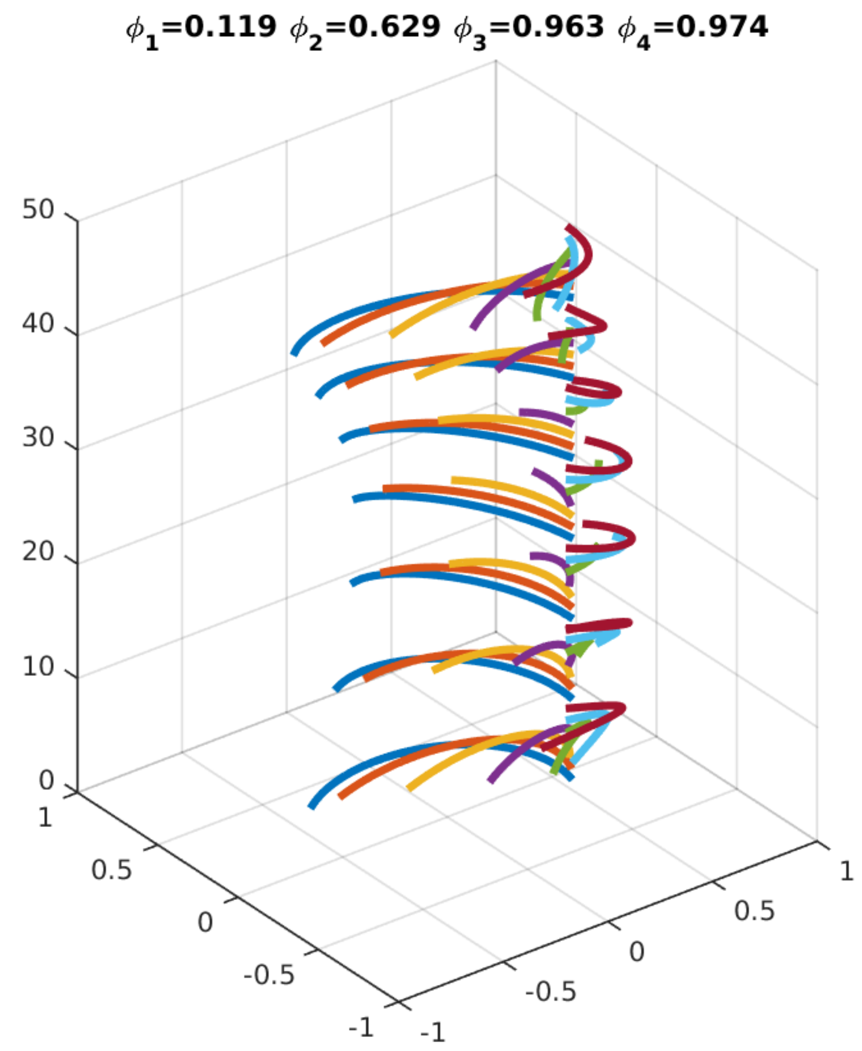


Sample CFPM pixels vs λ



Sample 4-pop wavelength responses





Sum of 2x2 PoP phasors.
Adding different λ_0 offsets does not change complex resultant at λ_0, but does change the overall wavelength dependence.