Error budgeting and tolerancing of starshades for exoplanet detection

Stuart Shaklan*

C. Noecker, T. Glassman, A. Lo, P. Dumont, J. Kasdin, E. Cady, R. Vanderbei, P. Lawson

SPIE Meeting 7731

July 2, 2010

*Jet Propulsion Laboratory, California Institute of Technology

Goals and Overview

- Group effort, started late summer 2009.
- Develop accurate yet tractable optical models.
  - Evaluate performance in the image plane.
- Identify the main perturbations to be analyzed.
- Establish performance goals and an approach to evaluating performance.
- Present a representative error budget for a strawman mission.

Sample Problem: 61 m Starshade

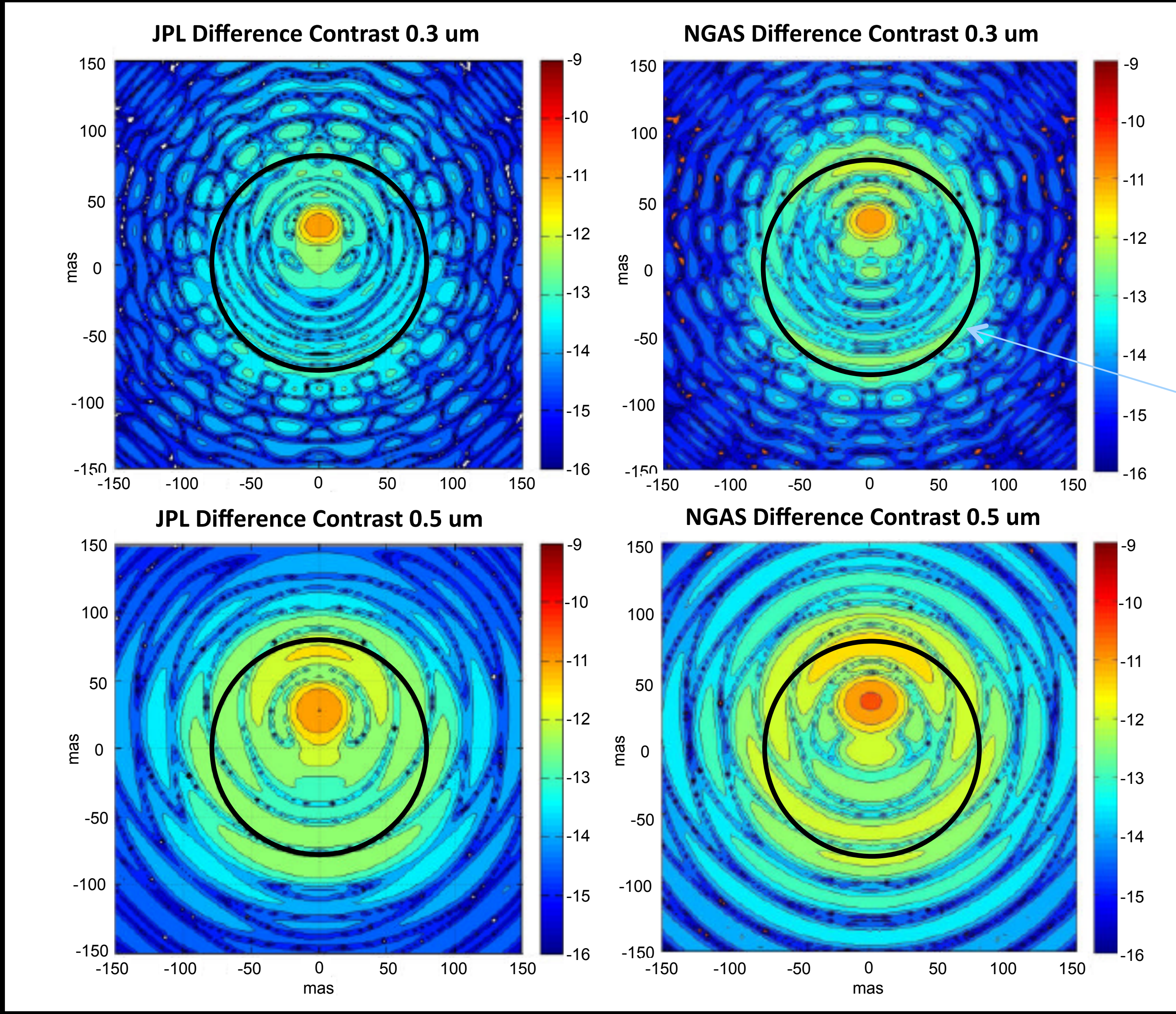
- Hypergaussian design, similar to NGAS baseline but slightly different inner and outer radii (see Glassman et al, this conference).
- Telescope: 4 m diameter, unobscured.
- Bandpass: UV to NIR

Parameter	**Symbol**	**Value**
Solid disk radius	a	12.5 m
Gaussian radius parameter	b	12.5 m
Petal shape parameter	n	6
Occulter separation	F	80,000 km
Number of petals	P	16
Tip cutoff point	r_{tip}	30.6 m

Models

- NGAS: “spectral integration method” (Glassman 2010 in prep)
- JPL: Analytical approximation for nominal field (Vanderbei et al ApJ, 665, 794-798 (2007)) and ensemble of small slits (512 per petal edge) around perturbation area (Dumont et al Proc. SPIE 7440 (2009))
- JPL also used the method of Dubra and Ferrari (Am. J. Phys., 67, 87-92 (1999)) to get nearly identical results.
  - U. Colorado has been using this approach too.

Image Plane Results



0.3 um

0.5 um

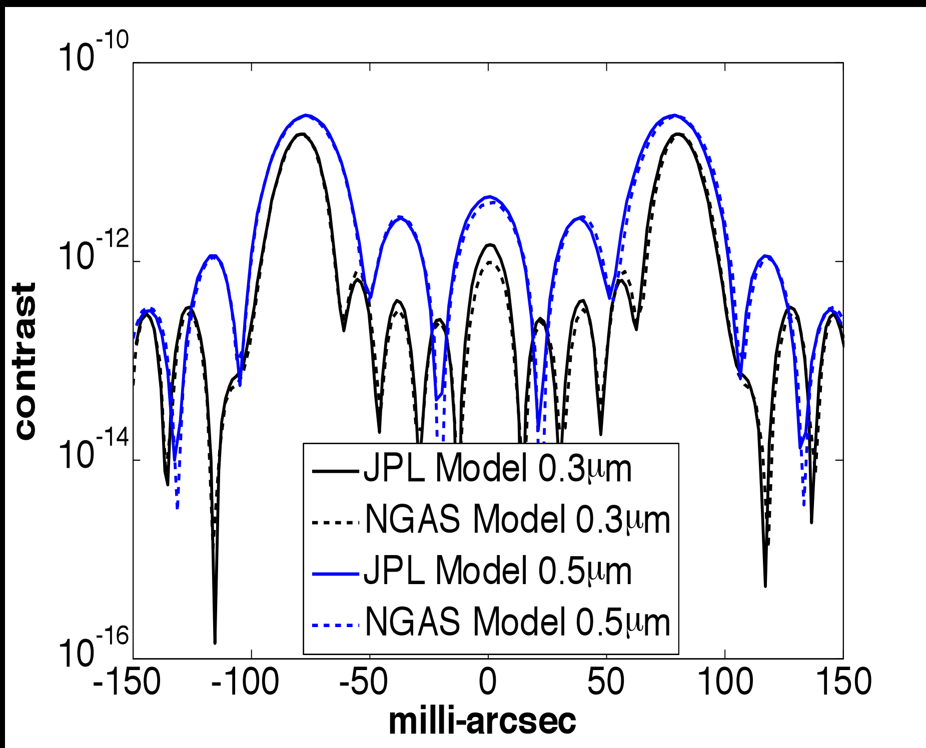
78 mas

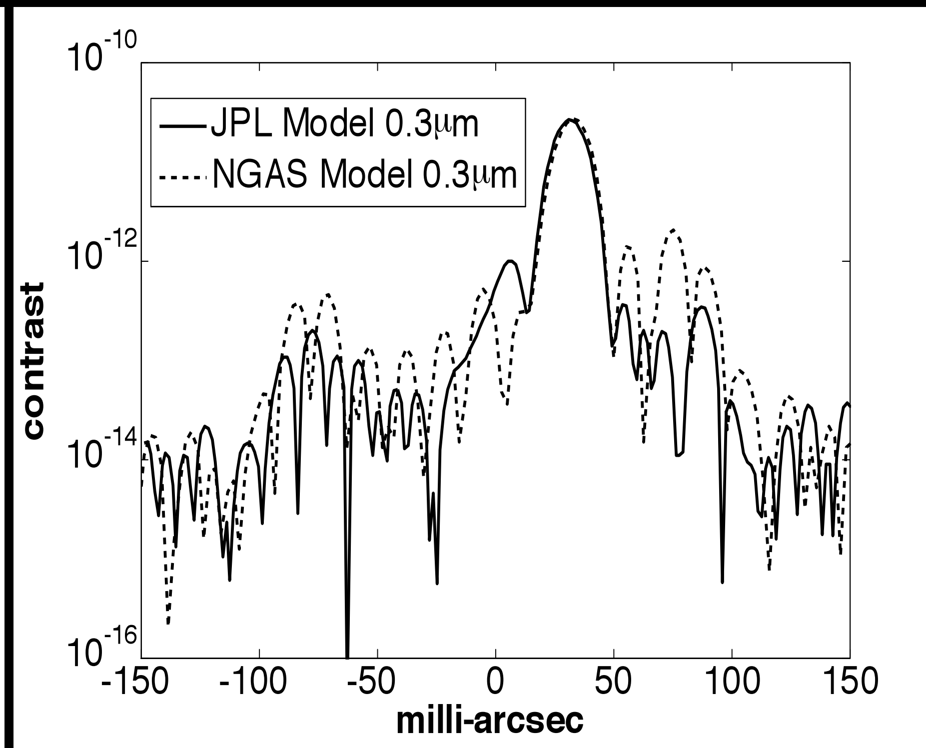
JPL

NGAS

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Model Agreement in the Telescope Image Plane





Perturbation is 10^{-4} proportional width error of a single hypergaussian petal.

Perturbation List I

- Petal Rigid Body Motions
  - 6 DOFs
  - In plane radial and lateral translation, in plane rotation matter most
- Petal Bending
  - Polynomial forms
  - In plane: quadratic, higher
  - Out of plane: quadratic, higher
- Manufacturing errors
  - Petal length (tip clip)
  - Petal width (proportional or uniform)
  - In-plane sine-wave errors on edges
    - Symmetric (Width)
    - Antisymmetric (wiggle): much less important than symetric
  - Petal edge step
  - Out of plane sine-wave errors

Perturbation List II

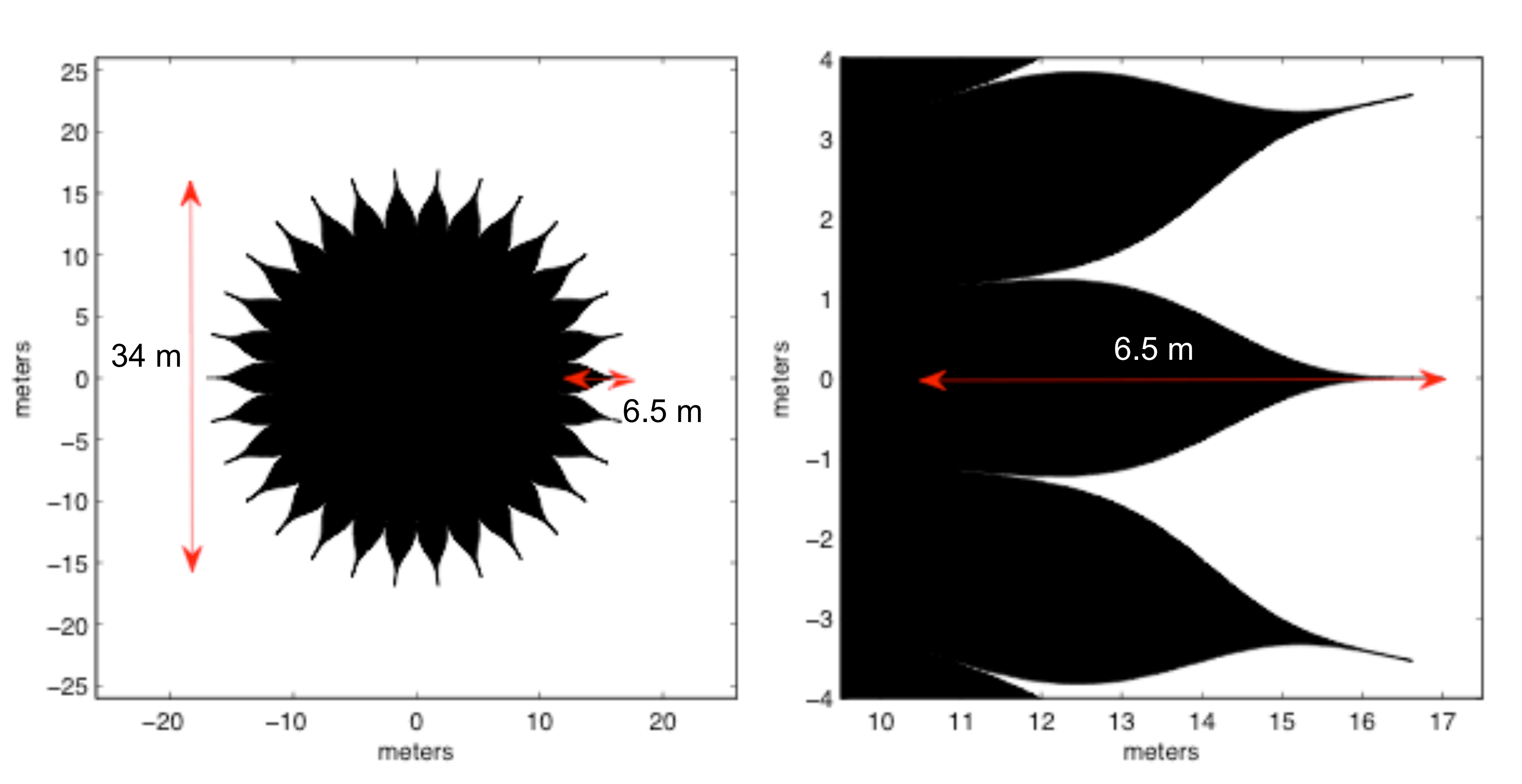
- Global: apply identical errors to all petals
- Modal: bend, twist, deformation of starshade
- Cross-track (telescope/starshade/star alignment)
- Holes (direct leakage of starlight)
- Starshade tilting: Not a perturbation per se, but couples out-of-plane errors to projected shape errors.
- Incoherent scatter (sunlight, mainly)
  - Edges, reflections

Case Study: Optimized Starshade Design

- Analyze sensitivity to perturbations
  - Find amplitude that scatters light at 10^{-12} at the Inner Working Angle (IWA).
- Individual petal and global perturbations
- Set up an error budget that meets science requirements.
  - r.m.s. contrast, mean scatter level

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

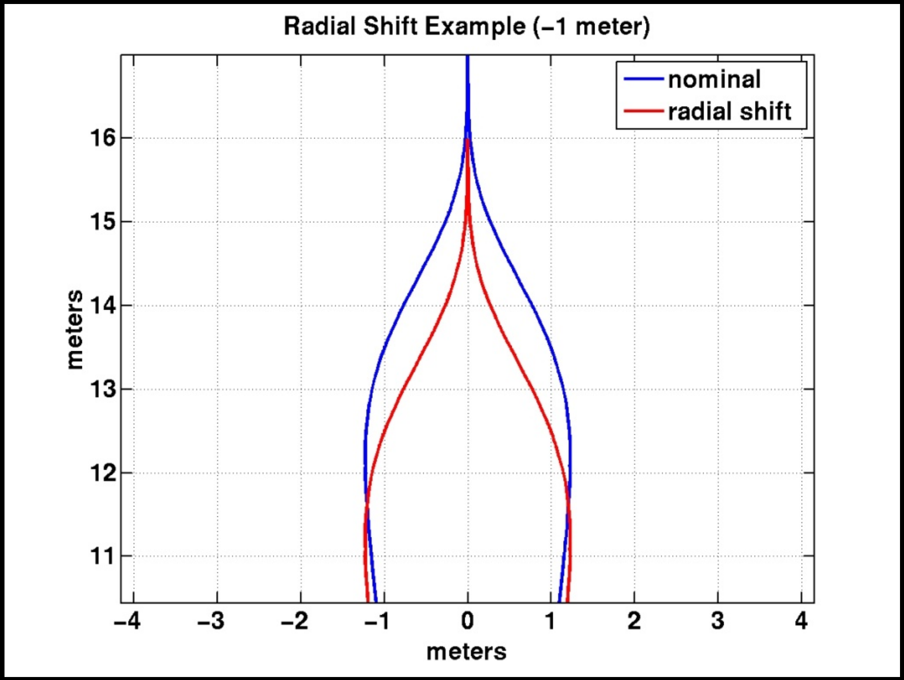
Optimized Occulter Design

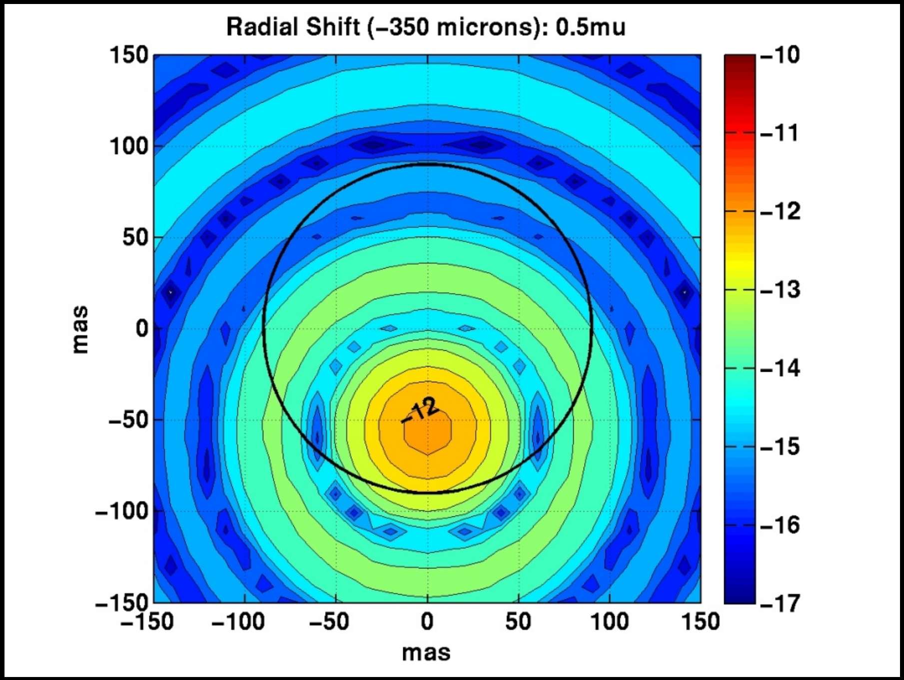


Occulter design for our study. Located ~ 39,000 km from the telescope, the Fresnel number is 29.6 at the shortest wavelength 0.25 um, and 13.5 at the longest wavelength 0.55 um.

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

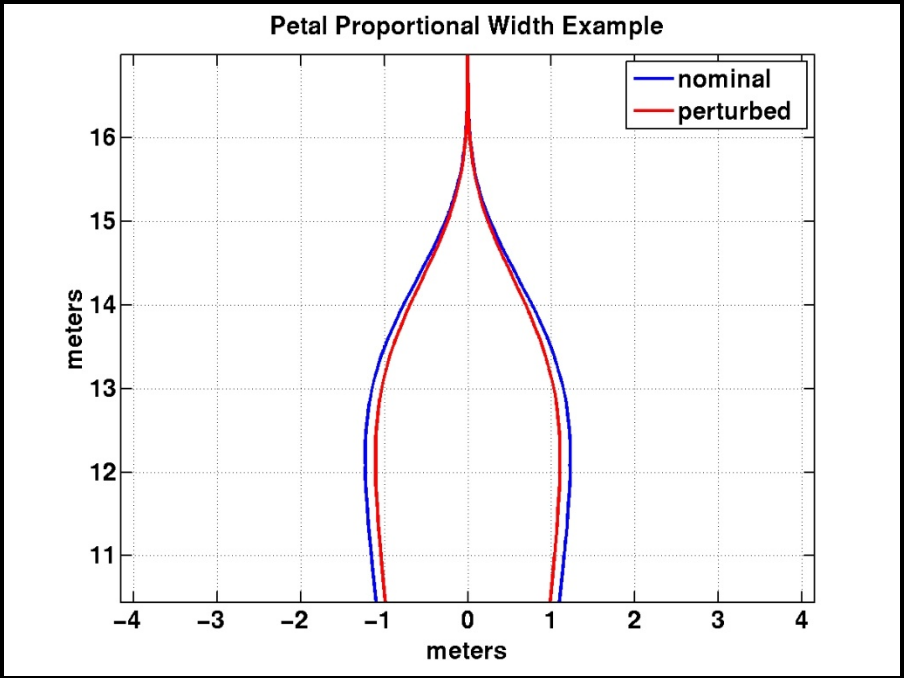
Radial Shift

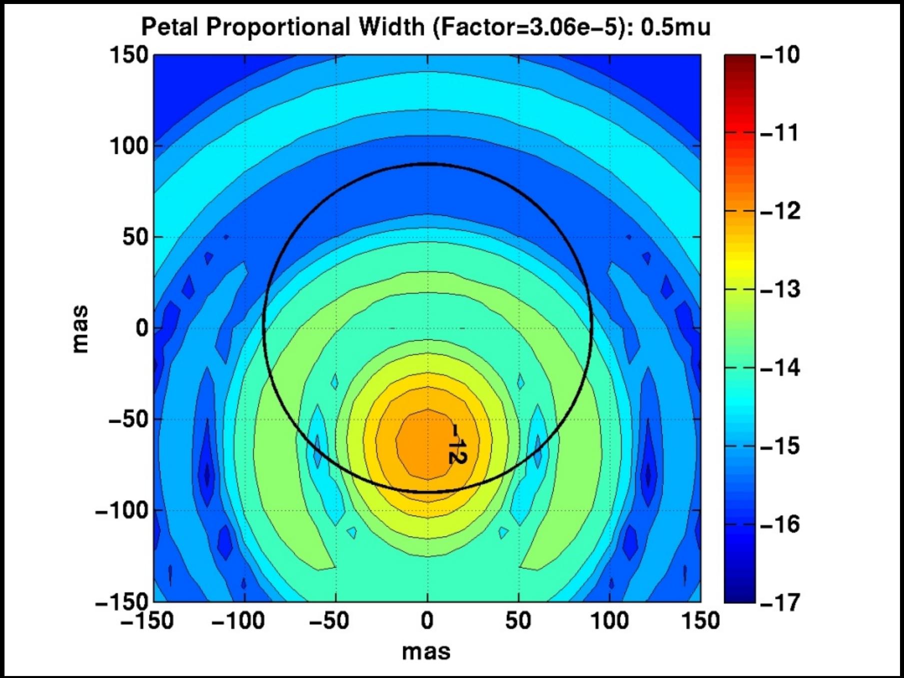




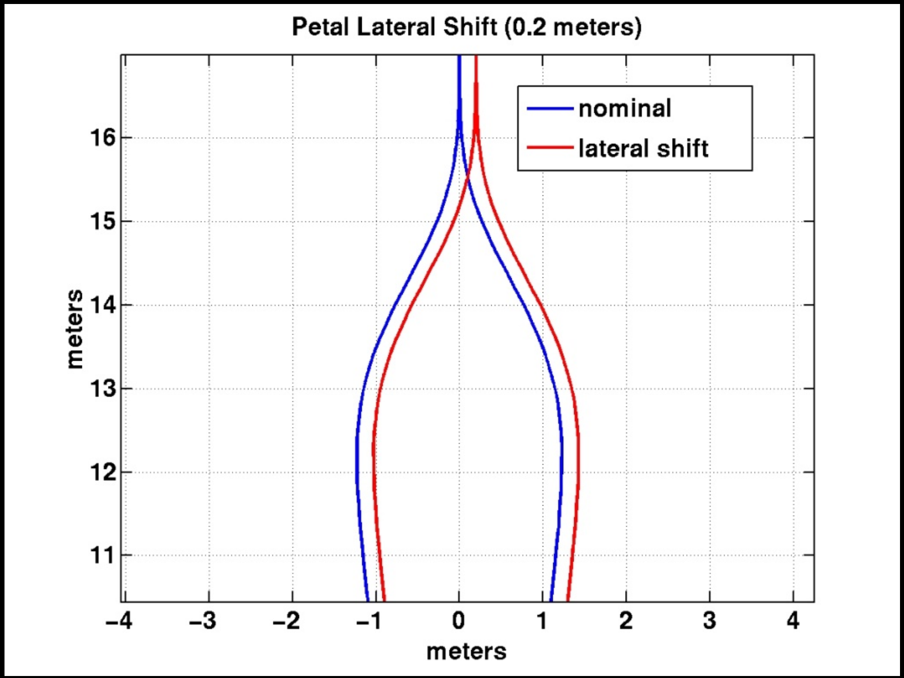
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

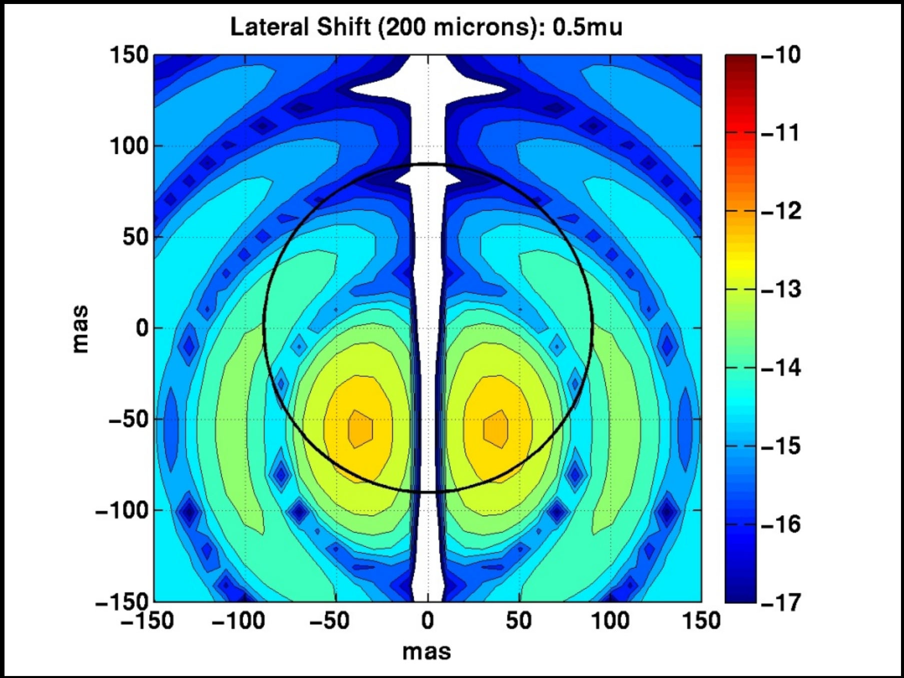
Proportional Width



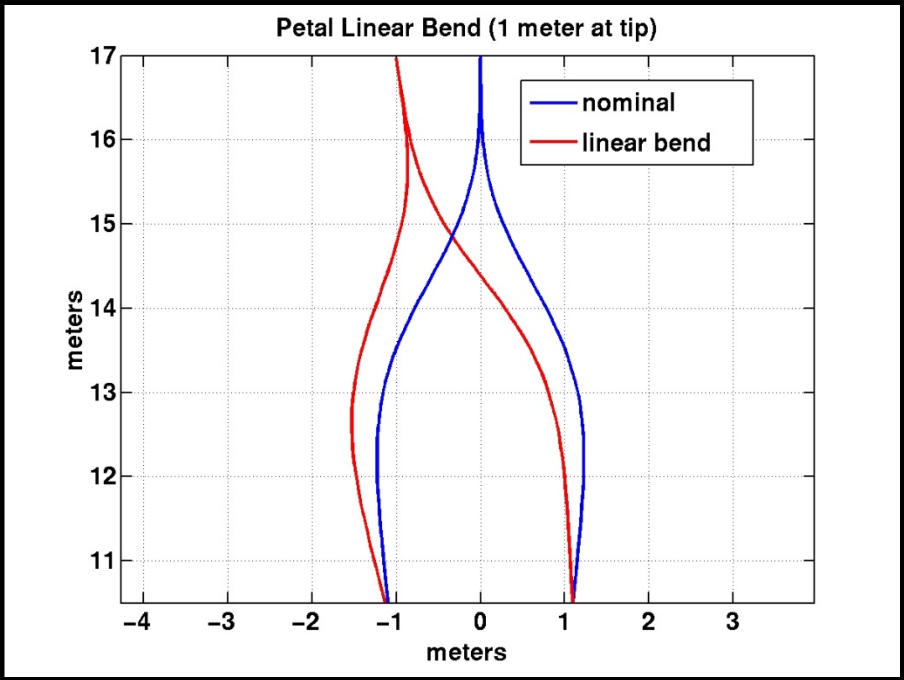


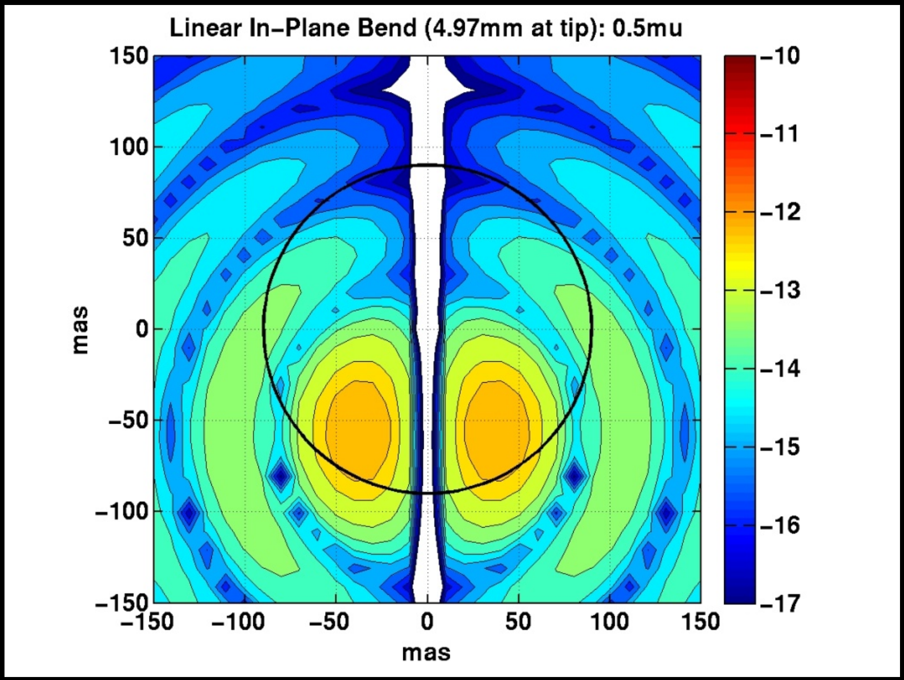
Lateral Shift





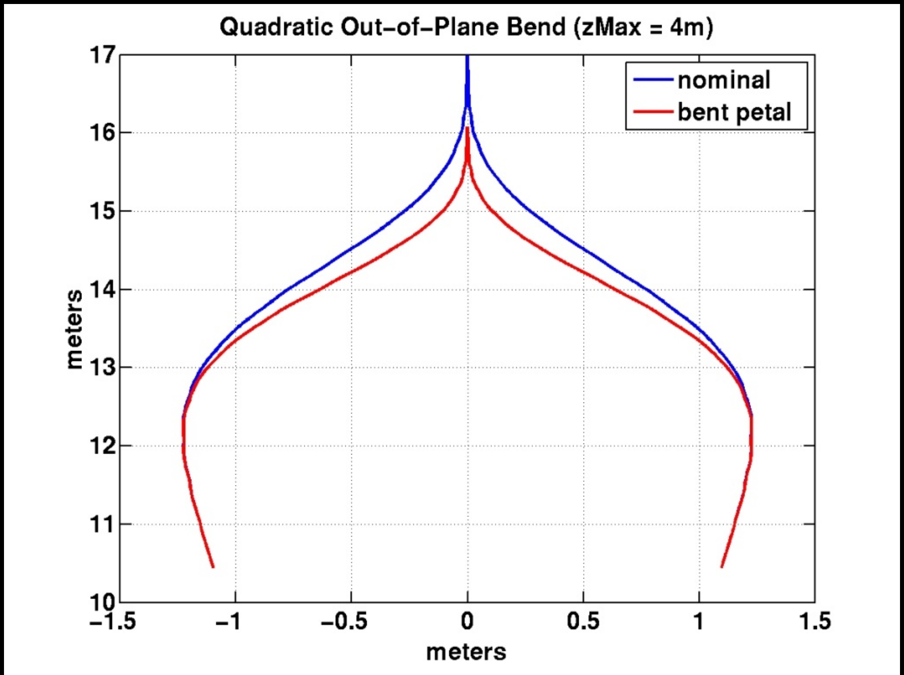
Linear In-Plane Bend

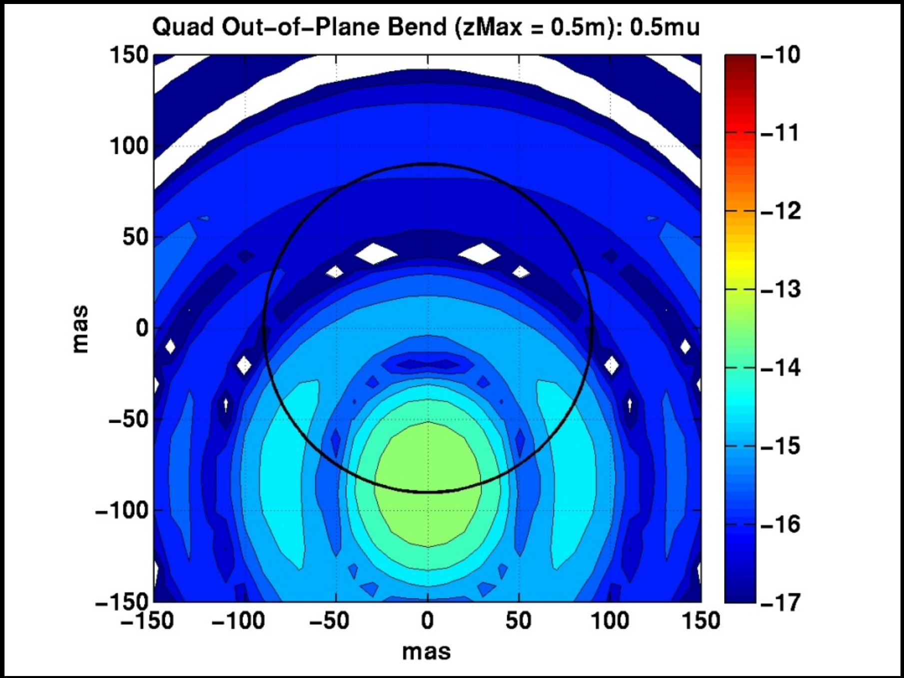




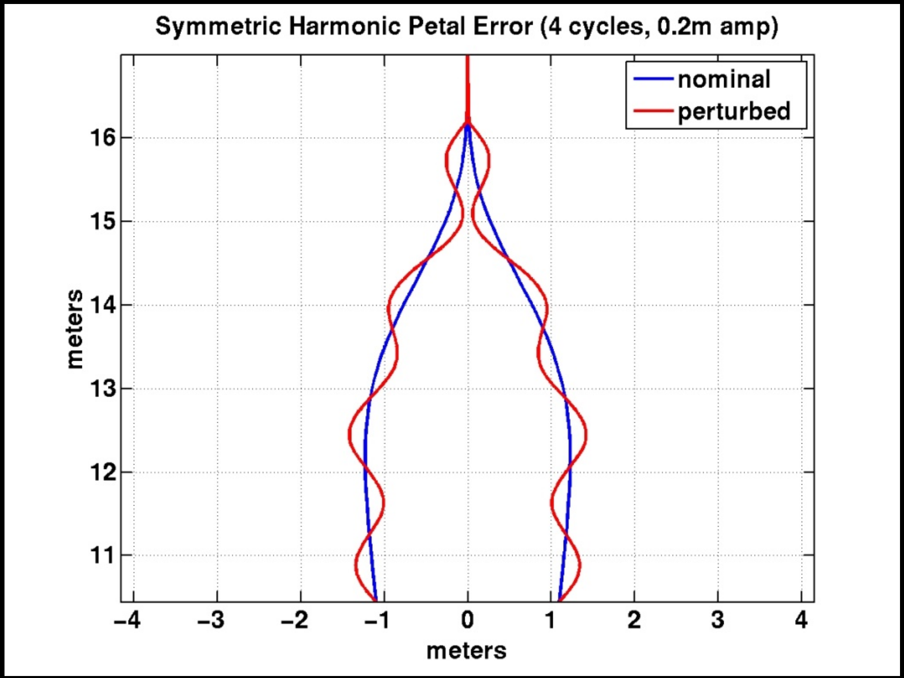
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

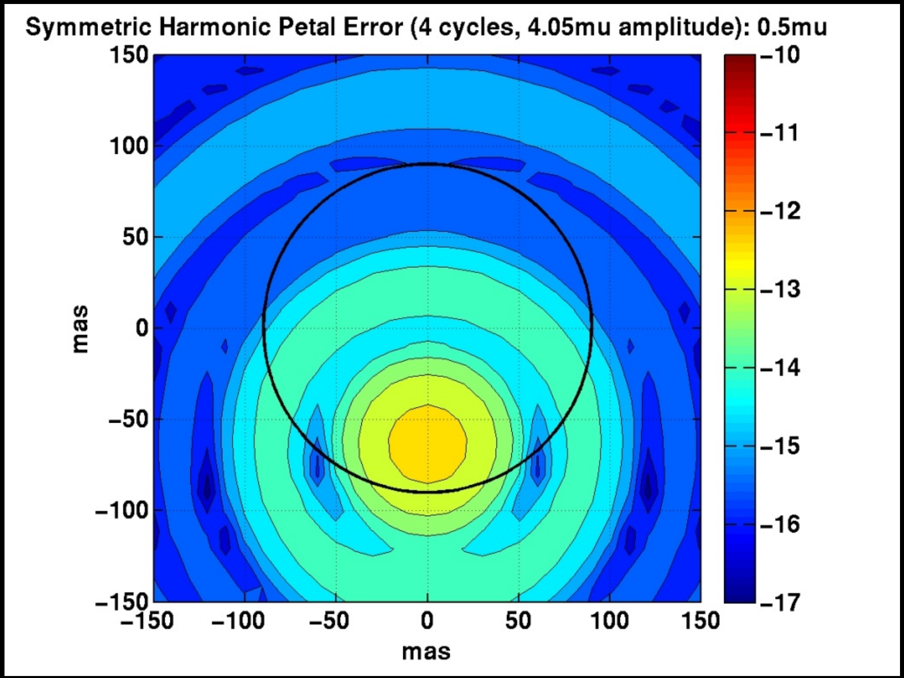
Quadratic In Plane Bend





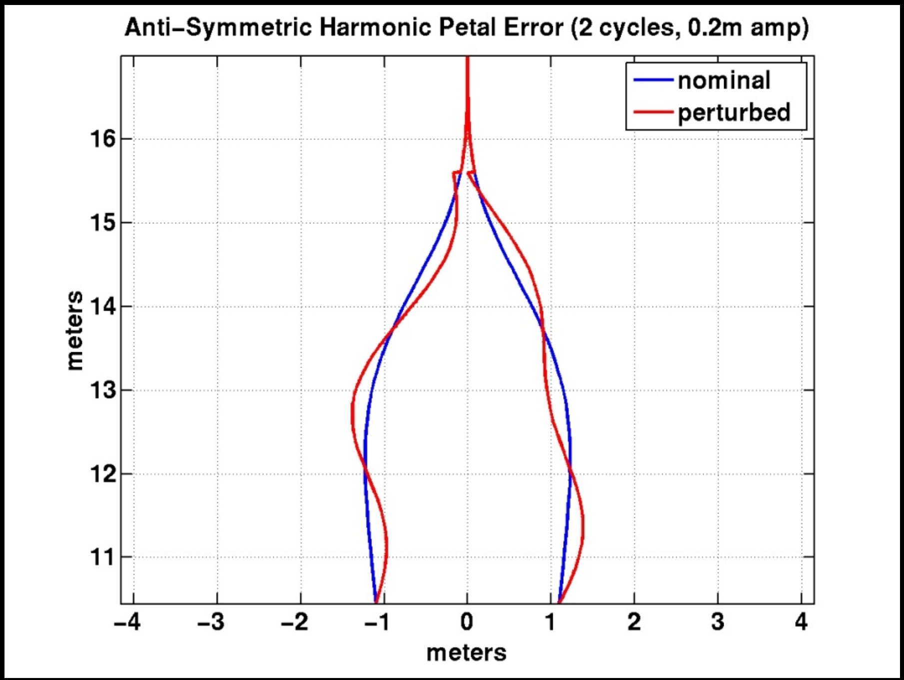
Symmetric Sine Wave

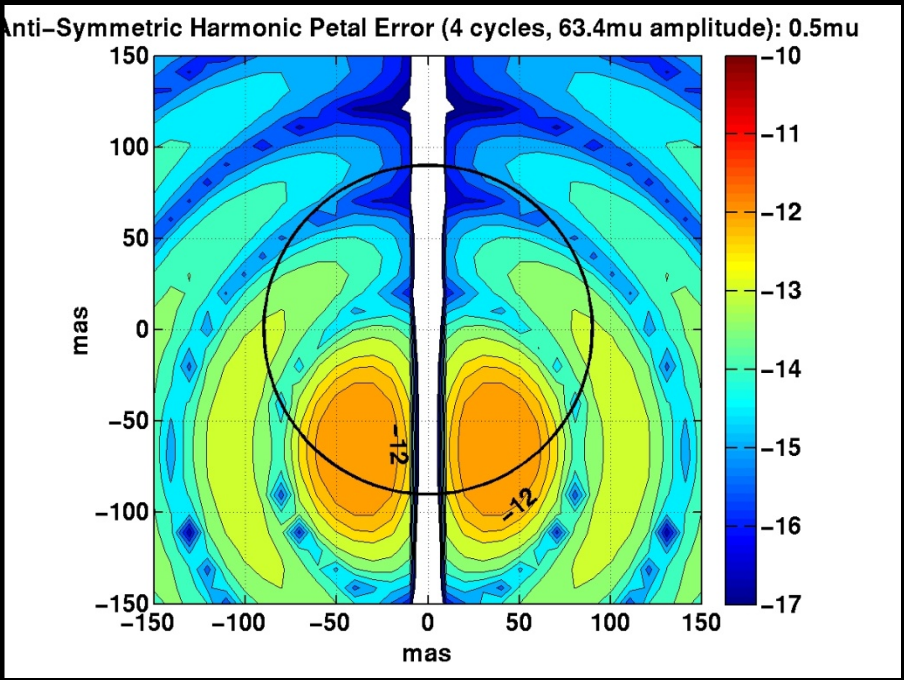




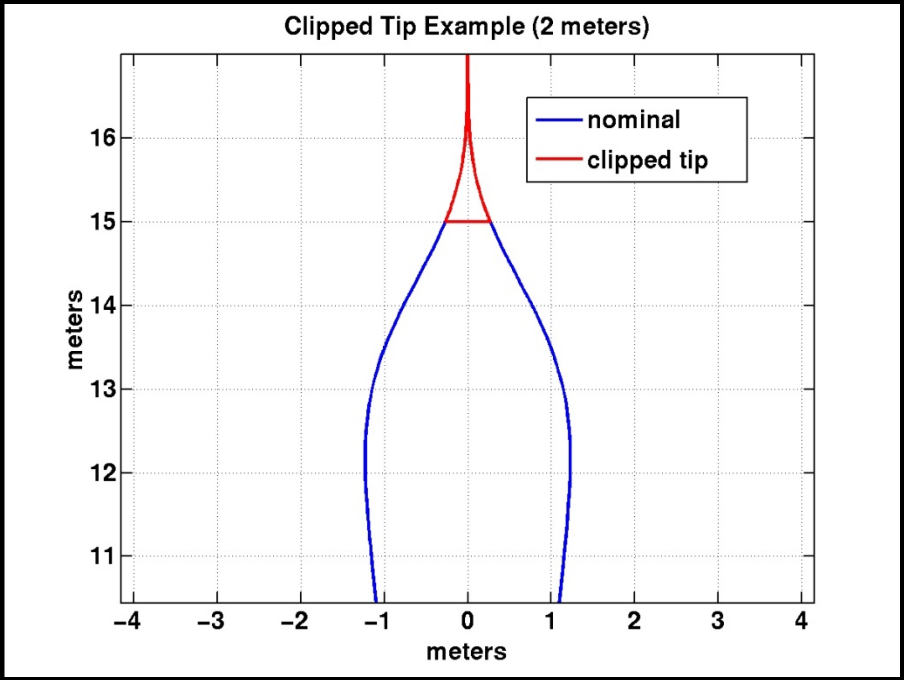
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

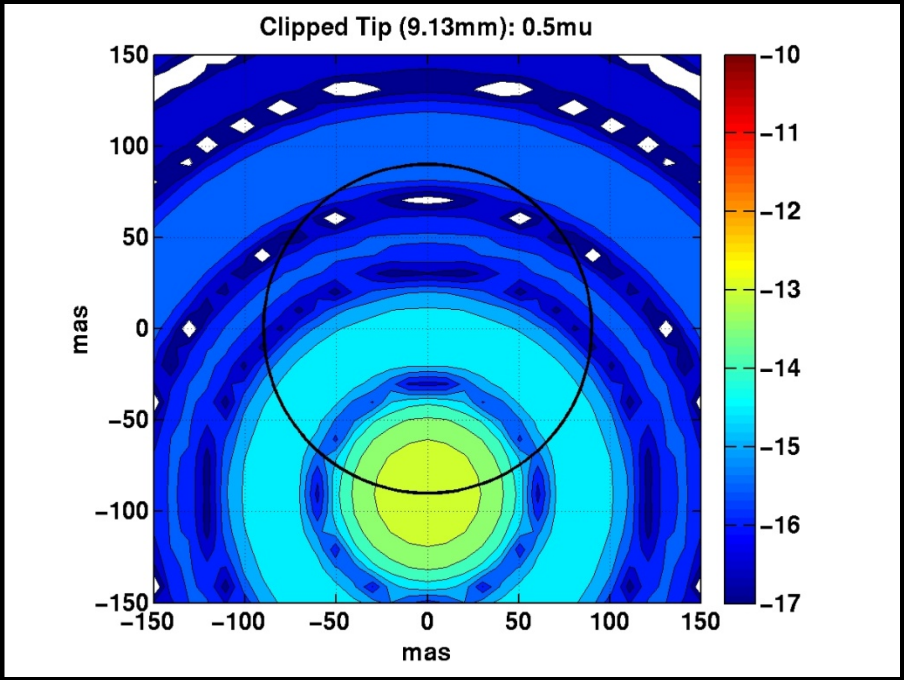
Antisymmetric Sine Wave



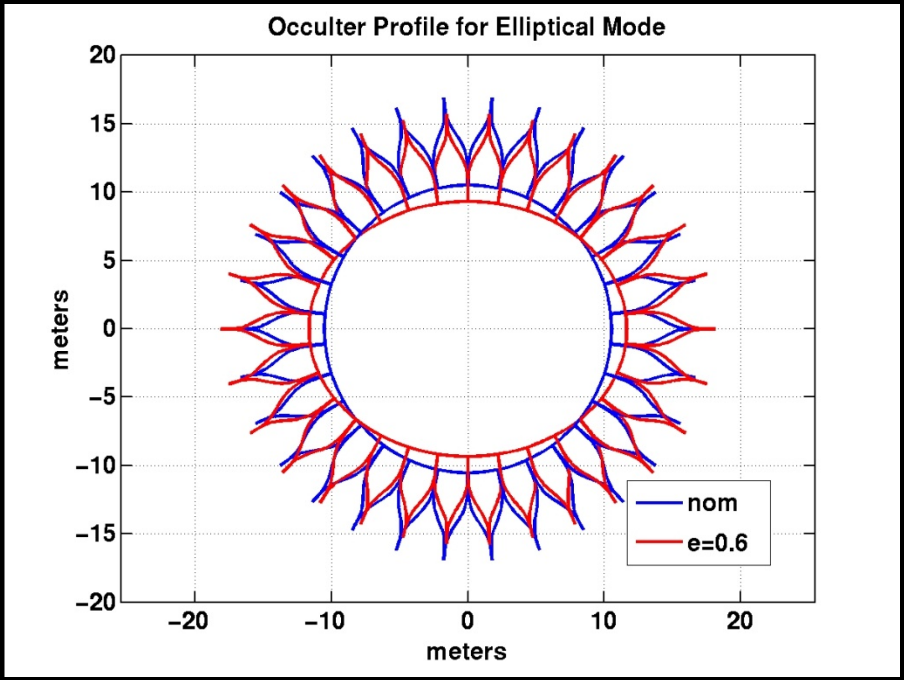


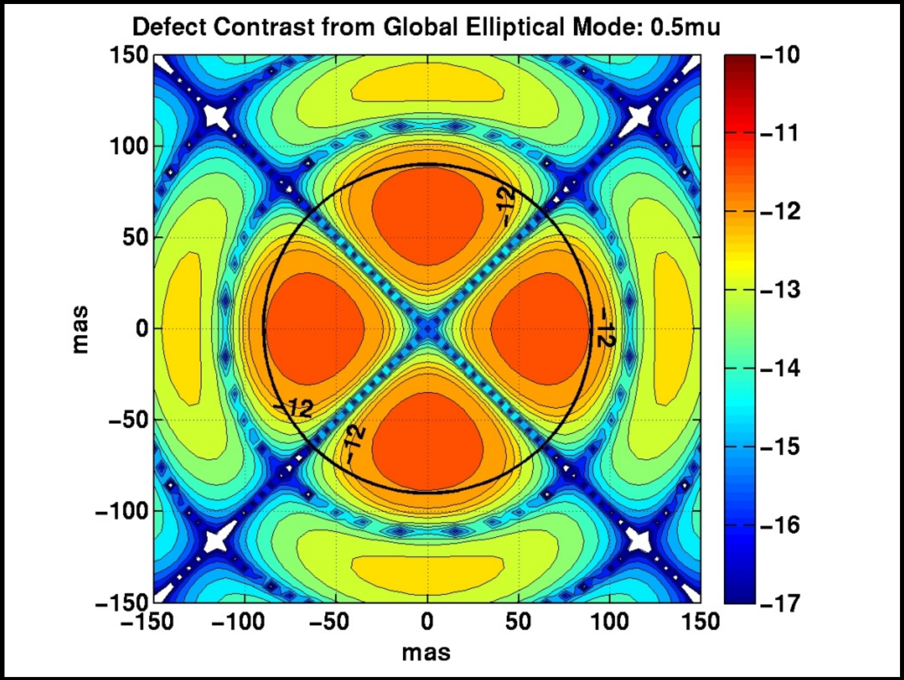
Clipped Petal Clip



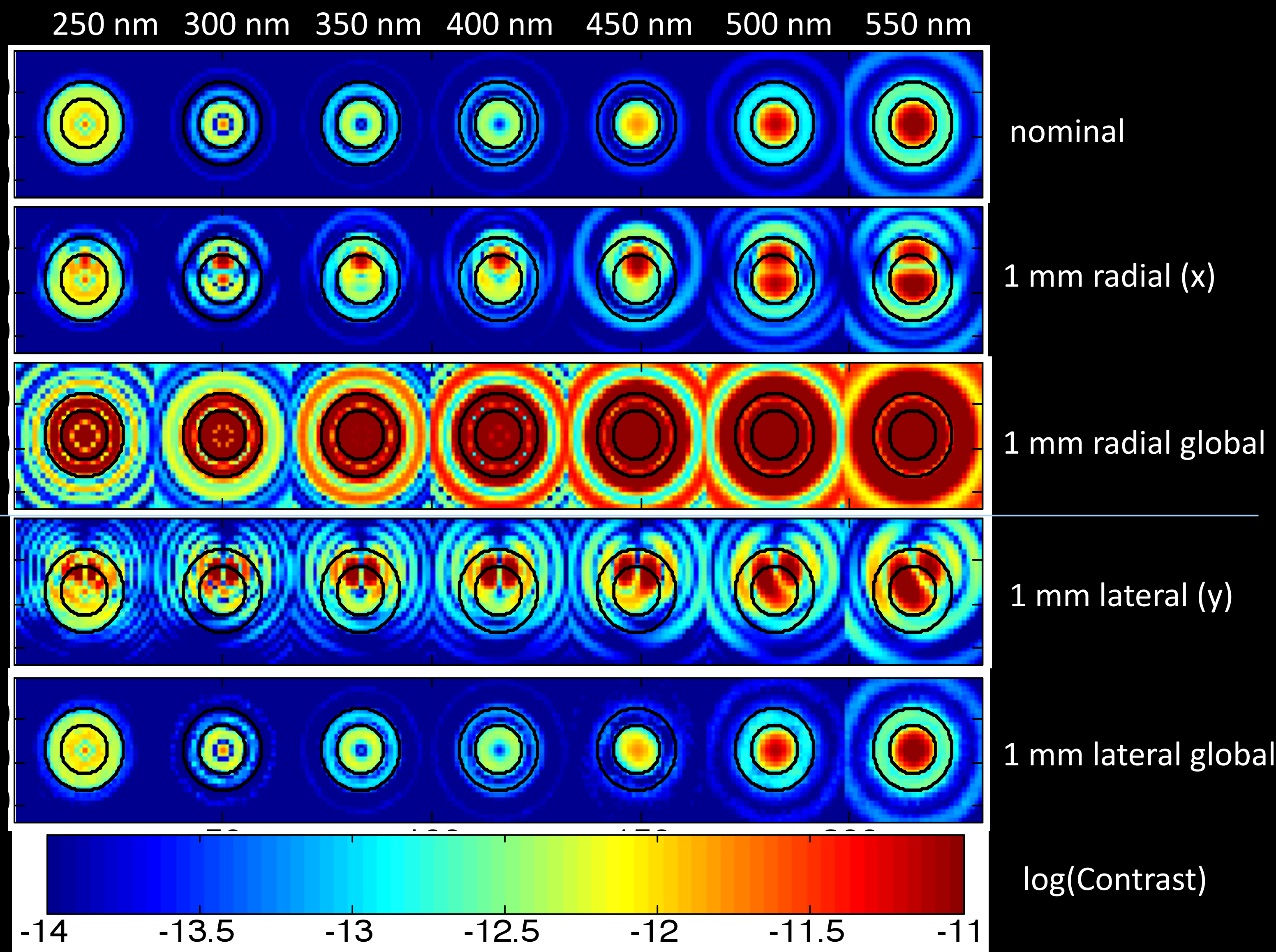


Elliptical In Plane



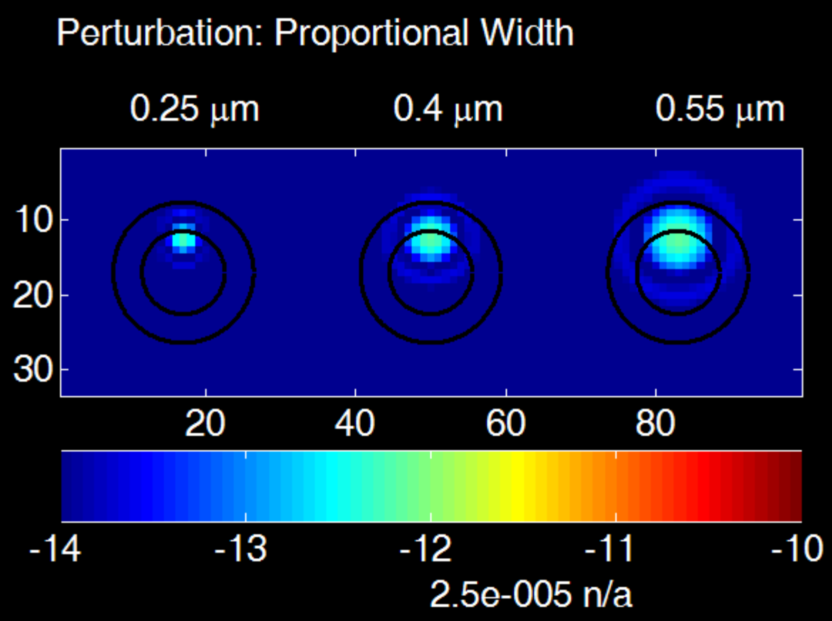


Single-Petal and Global Perturbation

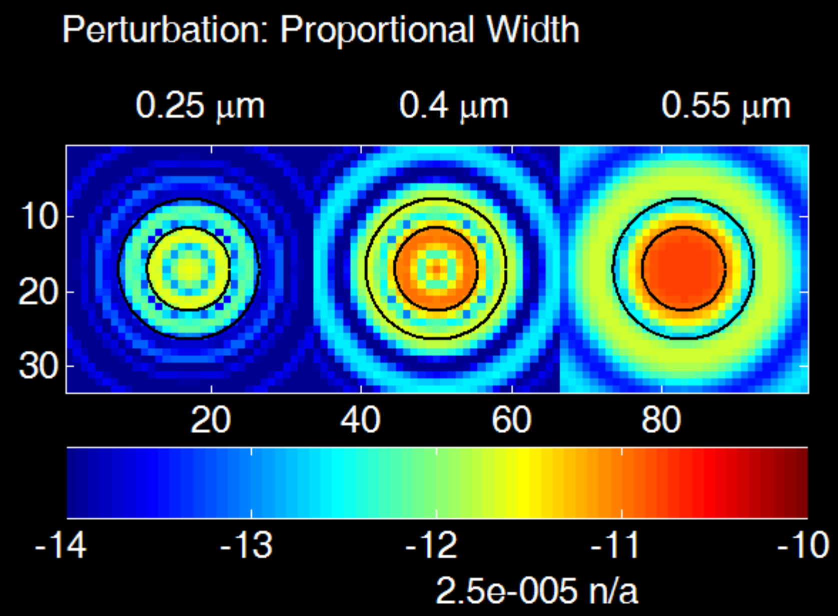


Perturbations vs. Wavelength

Single Petal



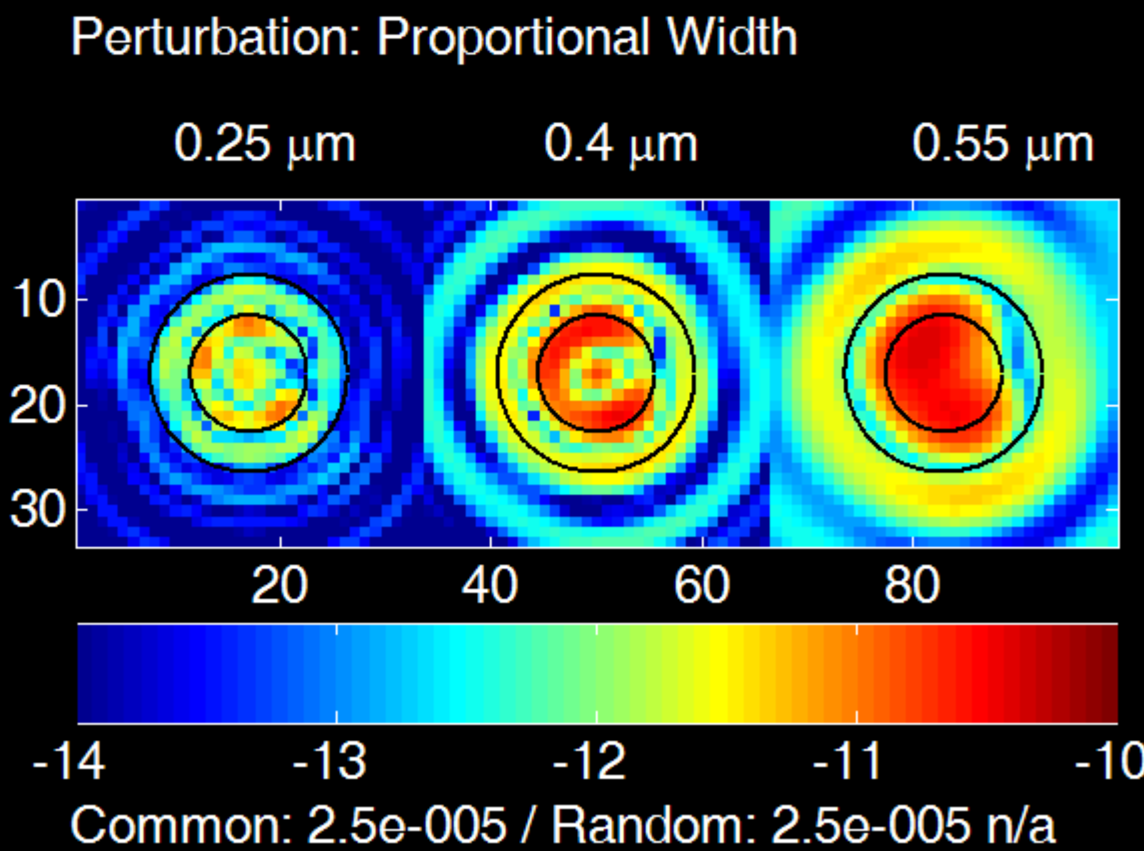
Global



National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

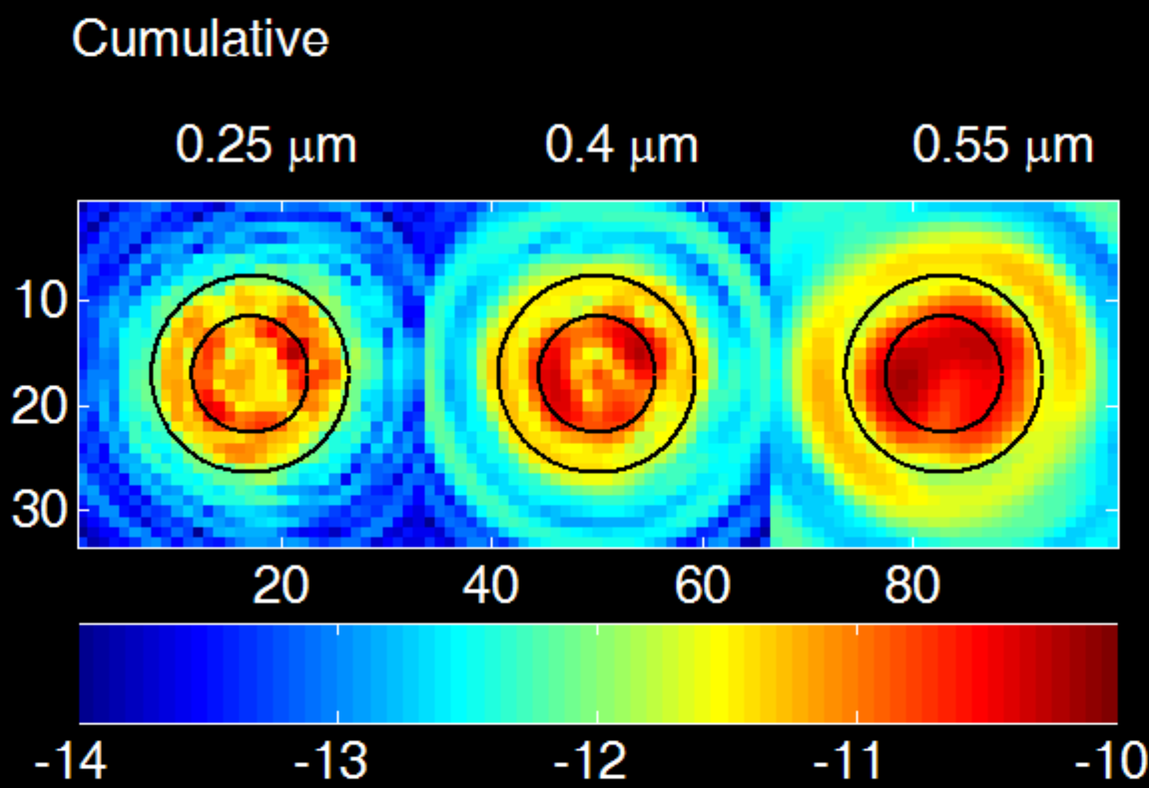
Perturbations vs. Wavelength

Random petal errors + Global



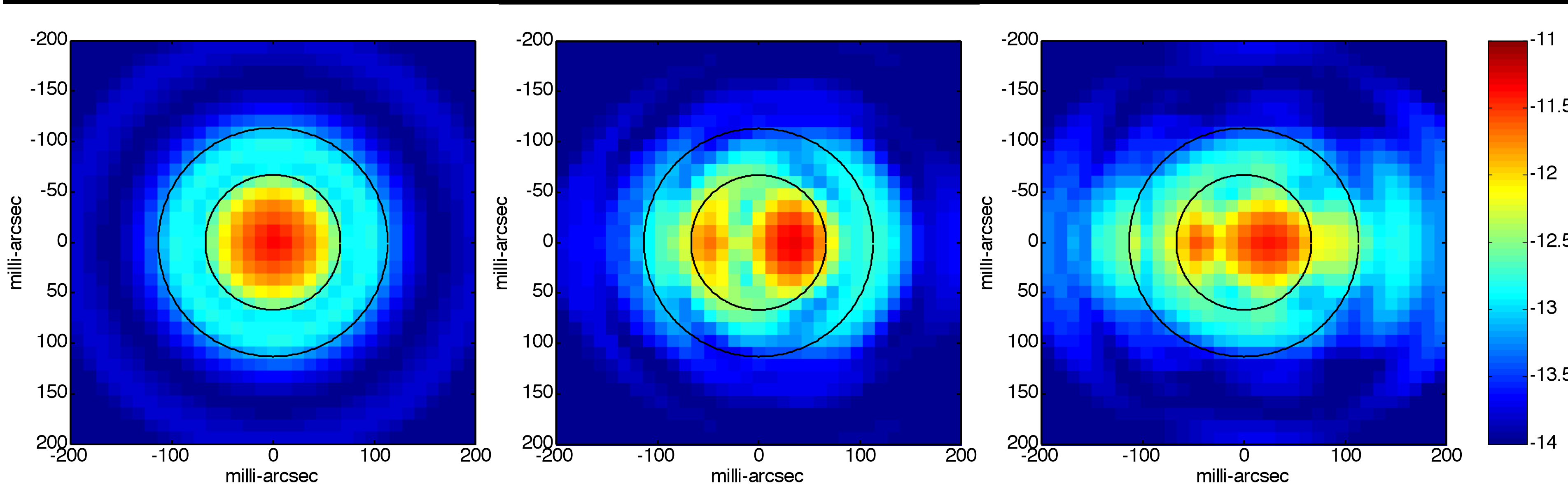
Perturbations vs. Wavelength

Random petal errors + Global



Cross-Track (Guiding) Error

a) on-axis b) 1 m offset c) 1.4 m offset



Log of the contrast for the ideal starshade. The images are a composite of the uniformly weighted bandpass 0.25 – 0.55 um

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Single-Petal Sensitivity for 10^{-12} peak contrast

		90 mas Wa	
No.	Perturbation	Amplitude	Glassman SPIE 2010
1	Proportional width	3.06E-05	5.00E-05
2	Tip clip	9.13	20
3	Radial shift	0.35	
4	Quadratic out-of-plane bend	>500	
5	lateral shift	0.20	
6	In plane rotation	4.97	5
7	In plane quadratic bend	32.32	50
8	Symmetric sine wave 1 cycles	89.67	300
9	Symmetric sine wave 2 cycles	36.82	
10	Symmetric sine wave 4 cycles	4.05	
11	Symmetric sine wave 8 cycles	3.29	3
12	Symmetric sine wave 12 cycles	9.02	
13	Symmetric sine wave 16 cycles	30.71	
14	Symmetric sine wave 20 cycles	59.68	
15	Anti symmetric sine wave 4 cycles	63.40	90

	Glassman	This work
Design	Hypergaussian	Optimized
Band (um)	0.3 - 0.8	0.25 - 0.55
IWA_{tip} (mas)	80	90
Sep (km)	80000	39000
D_{tip} (m)	62.6	34
L_{petal} (m)	17	6.5
N_{petals}	16	30
Dtel (m)	4	2

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Global Sensitivity for 10^{-12} peak contrast

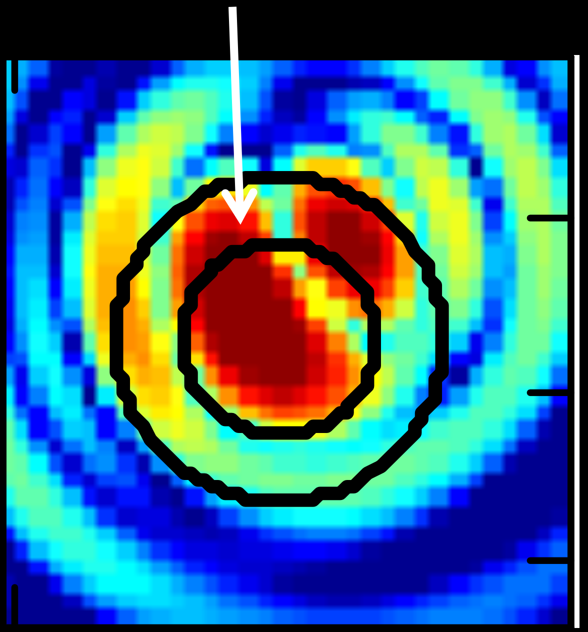
		90 mas		75 mas		
No.	Perturbati on	Amplitude	Wavelength (um)	Amplitude	Wavelength (um)	Units
1	Proporti onal width	1.03E-05	0.50	1.03E-05	0.50	n/a
2	Tip clip	6.67	0.55	6.67	0.55	mm
3	Radial shift	0.11	0.55	0.11	0.55	mm
4	Quadrati c out-of-plane bend	> 500	0.35	>500	0.35	mm
5	lateral shift	12.30	0.55	9.60	0.55	mm
6	In plane rotati on	157.62	0.55	157.62	0.55	mm at ti p
7	In plane quadrati c bend	>500	0.55	>500	0.55	mm at ti p
8	Symmetric sine wave 1 cycle	31.16	0.50	31.16	0.50	um
9	Symmetric sine wave 2 cycles	12.27	0.50	12.27	0.50	um
10	Symmetric sine wave 4 cycles	1.21	0.55	1.21	0.55	um
11	Symmetric sine wave 8 cycles	2.18	0.30	2.18	0.30	um
12	Symmetric sine wave 12 cycles	6.96	0.25	6.96	0.25	um
13	Symmetric sine wave 16 cycles	24.48	0.25	24.48	0.25	um
14	Symmetric sine wave 20 cycles	38.76	0.25	38.76	0.25	um
15	Anti symmetric sine wave 4 cycles	>500	0.55	>500	0.55	um

Science Requirements

- From many studies
- Δmag = 26
- IWA <100
- Mean scatter < 10^{-10}
- SNR > 4
- Requires r.m.s. of speckle noise floor to be < 10^{-11}
- Evaluate in IWA swath, width matched to core of PSF

Error Budget: Combining Terms

IWA swath:
90 +/- 23 mas



$$\sigma^2 = \langle I^2 \rangle - \langle I \rangle^2 - \sigma_b^2$$

σ^2 = variance of speckles with same spatial scale as PSF

σ_b^2 = variance of uniform component of light

It can be shown that:

$$\sigma^2 = 2\langle I_b \rangle \langle I_r \rangle + \sigma_r^2$$

$\langle I_b \rangle$ = mean value of uniform background. Comes from global terms

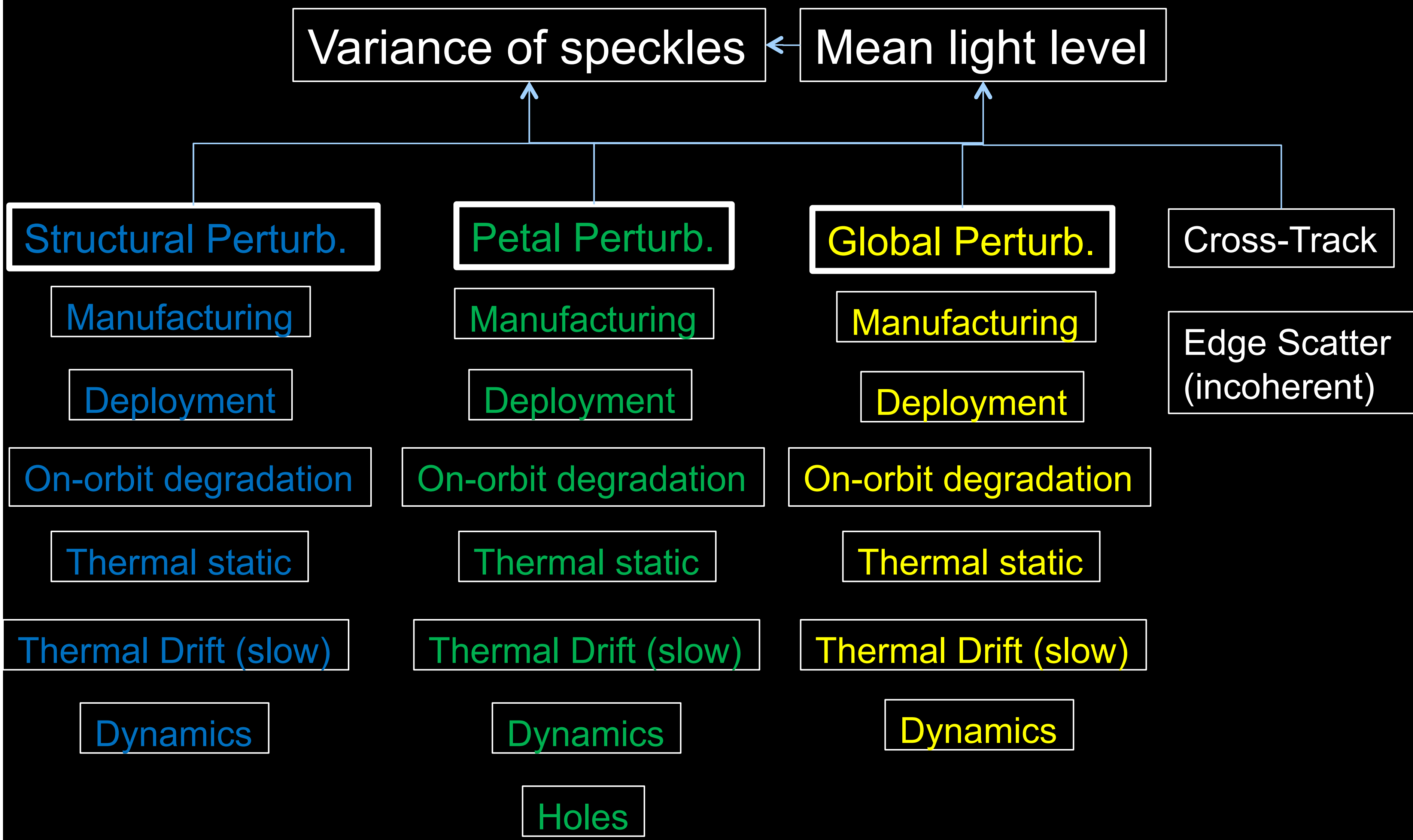
$\langle I_r \rangle$ = mean value of non-uniform light. Comes from single-petal terms

σ_r^2 = variance of non-uniform light. Comes from single-petal terms.

Error Budget Sensitivities

- For each perturbation, for a given amplitude, at each wavelength (7 bands) compute:
  - Single petal: compute I_r and σ_r^2 for a given 1-sigma perturbation amplitude.
    - Multiply by N_{petal} for $\langle I_r \rangle$ and σ_r^2 for all petals
  - Global petal: compute I_b for a given tolerance perturbation amplitude.
- Repeat for all perturbations.
- Structural mode: compute I_r and σ_r^2
- Allocate perturbation amplitudes and tolerances.
- Combine using $\sigma^2 = 2\langle I_b \rangle \langle I_r \rangle + \sigma_r^2$

Starshade Error Budget Structure



Allocation to meet Science Requirements

No.	Perturbation	Single Petal 1-sigma	Global tolerance	Units	r.m.s. contrast	Global Mean Contrast
1	Proportional width	2.50E-05	2.00E-05	n/a	**2.6E-12**	**1.6E-12**
2	Tip clip	4	4	mm	2.6E-13	3.3E-13
3	Radial shift	0.25	0.20	mm	**2.2E-12**	**1.4E-12**
4	Quadratic out-of-plane bend	150	75	mm	2.0E-14	7.0E-15
5	lateral shift	0.15	1	mm	**1.9E-12**	6.5E-15
6	In plane rotation	2	2	mm at tip	9.8E-13	7.4E-17
7	In plane quadratic bend	5	5	mm at tip	1.4E-13	4.8E-17
8	Symmetric sine wave 1 cycles	25	25	um	1.9E-13	1.8E-13
9	Symmetric sine wave 2 cycles	5	10	um	6.1E-14	2.4E-13
10	Symmetric sine wave 4 cycles	3	1	um	**3.1E-12**	4.7E-13
11	Symmetric sine wave 8 cycles	4	2	um	3.7E-14	1.3E-14
12	Symmetric sine wave 12 cycles	5	5	um	1.3E-14	1.9E-14
13	Symmetric sine wave 16 cycles	25	10	um	1.5E-13	3.4E-14
14	Syymmetric sine wave 20 cycles	25	10	um	9.2E-14	2.0E-14
15	Anti symmetric sine wave 4 cycles	10	50	um	3.9E-13	5.6E-16
16	In plane elliptical truss deform.	n/a	0.02	eccentricity	1.6E-16	1.8E-16

Performance

wavelength (um)	0.25	0.3	0.35	0.4	0.45	0.5	0.55
rms	4.9E-12	7.8E-12	5.0E-12	5.3E-12	5.7E-12	8.4E-12	9.4E-12
mean	4.2E-12	5.2E-12	4.9E-12	5.7E-12	6.3E-12	9.8E-12	1.1E-11

Requirement: rms < 10^{-11}, Mean < 10^{-10}

- Verification: to verify that the error budget equation is correct, we performed Monte Carlo simulations.
  - Single petal perturbations normally distributed with r.m.s. from green column (previous slide)
  - Global perturbations = +/- yellow values.
  - Structural perturbation = +/- blue value.
  - All were uncorrelated.
- We recorded rms and mean contrast in the IWA annulus and saw that they matched the results predicted from the combination equation, shown in the above table.

Conclusions

- Satisfying agreement on perturbation sensitivities
  - Three different diffraction algorithms agree well
  - Hypergaussian and optimized petal shapes have very similar sensitivities
  - Agreement on list of perturbations to consider
- We have developed budgets including random and global (identical on all petals) perturbations
  - Based on optical sensitivities
  - Combination of terms verified with Monte-Carlo simulations
  - SNR>4 for optical bands with respect to systematic uncertainties from starshade disturbances
  - Key perturbations: radial petal translation, ripples at 4-8 cycles/petal length
  - Correlation between petals amplifies the performance degradation

Future Work

- Work in progress and for the future
  - Develop budget spreadsheets for communicating/reporting, system engineering
  - Design and evaluate budgets for both characterization and detection
  - Study calibration and neutralization, such as spinning the occulter
  - Consider perturbations with starshade tilted or decentered from the star
  - Refine allocations, compare different designs, revisit design trades
  - Revisit technology readiness assessments