Inverse Problem Approaches to Exoplanet Signal Extraction

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• Inversion landscape: ingredients & tools for inversion

• Planet detection from the ground with 1 or 2 $\lambda_i$:
  ANDROMEDA: Spectral & angular differential image processing

• Planet detection from the ground with many $\lambda_i$ (IFS):
  myopic inversion
Ingredients for inversion

- **Data y**: raw ore

- **A « good » data formation model** (‘direct’ or ‘forward’ model):
  - Suggested by Physics, very instrument dependent: \( y = \text{model}(x; \theta) \)
    - Exoplanet imaging:
      - \( x \) = parameters of interest e.g., planet position(s) + flux/spectra
      - \( \theta \) = other unknowns = nuisance parameters (instrument aberrations, etc)
    - May be very different from a good model for simulation:
      - Few parameters \( \theta \) (e.g.: not \( 10^3 \) realizations of N phase screens in various planes)
      - Parameters that can be calibrated or estimated along with \( x \)
  
- **Pre-processing**:
  - Basic: massage raw data to fit model
  - Aggressive: possibly, **change what is defined as the data**.
    - E.g.: Darwin [Thiébaut-Mugnier, IAU200, 2005] to eliminate nuisance parameters

- Prior information on noise

- Possibly, prior information on unknowns (scene, aberrations,…): Bayes.

- Estimator & algorithm
Estimators for inversion

- Function of number of unknowns, problem complexity (myopic or not, …)

- Model fitting:
  Max. **Likelihood** : \( p(y|x) \). Includes model + noise statistics.
  Eg.: image registration [Gratadour A&A 05]

- Simple inversion, well-calibrated instrument:
  **posterior likelihood** \( p(x|y) \propto p(x,y) = p(y|x) \cdot p(x) \)
  Eg.: conventional deconvolution; nulling interferometry [Mugnier 05]

- Myopic inversion:
  - Joint estimation: \( p(x, \theta, y) = p(y | x, \theta) \cdot p(x | \theta) \cdot p(\theta) \)
    Eg.: phase diversity (x=phase, \( \theta \)=object),
    deconvolution (Mistral: x=object, \( \theta \)=PSF)
  - Marginalized inversion
    \( p(x,y) = \int p(x,\theta,y) \cdot d\theta \)
    Eg.: phase diversity [Blanc JOSAA 03], retinal imaging [Blanco OpEx 11].

- Note: detection also based on posterior likelihood: \( p(x|y) / p(x=0|y) \)
Approaches for planet detection with 1 or 2 $\lambda$

• Context:
  • SPHERE/IRDIS, Infra-Red Dual-beam Imaging and Spectroscopy camera
  • Stabilized pupil: speckles ~fixed, planet rotating with field

• Joint estimation of star residuals and planet(s):
  • Ideally: have a ‘compact’ data model($x; \theta$) with few $\theta$ (aberrations,...) and invert it.
  • Currently: $\theta =$ star residuals (pixel values), to be estimated with $x$. $\theta(t)$: independent (too many) / fixed / correlated
  • ML method, assumed fixed residuals (MOODS): [Smith, IEEE 09]
  • Subtraction of star residuals via image differences + ML method: ANDROMEDA [Mugnier-Cornia JOSA 09], [Cornia SPIE 10]
Combination of spectral and angular information

SDI: Spectral Differential Imaging (simultaneous images)\[ i'(t) = i_{\lambda_1}(t) - i_{\lambda_2}(t) \]

ADI: Angular Differential Imaging (field rotation)\[ \Delta_k = i(t_k) - i(t_k + \delta t_k) \]

Pseudo-data $\Delta_k = \text{differences between images: star signal rejected}$
Pre-processing in practice

- Computation of differential images: LS minimization of image difference [Cornia SPIE 10],

- Elimination of some instrumental artefacts: high-pass filtering of data & planet PSF [Eggenberger Lyot 10].
Minimum angular separation: $\delta_{\text{min}}$

Minimum displacement of the planet allowed for ang. subtract.

- small $\delta_{\text{min}}$: good suppression of speckles
- large $\delta_{\text{min}}$: preserve planet signal in the difference

Optimal $\delta_{\text{min}}$ depends on instrument stability and obs. conditions
Data formation model

- Data $\Delta(r, k) = \text{image differences: star signal is assumed to be eliminated}$
- Unknowns: flux $a$ and initial position $r_0$ of the planet
- Data model:
  \[
  \Delta(r, k) = a \cdot p(r, k; r_0) + n(r, k)
  \]
  \[\text{[r = position in the image, k = time-related index]}\]
- $p$: planet "signature" = difference between 2 planetary PSFs at different instants computable as a function of $k$ and $r_0$

Examples of "signatures" $p(r, k; r_0)$ for 3 different values of $k$

- Noise model $n(r, k)$: gaussian white, of inhomogeneous variance $\sigma^2(r, k)$ (good approximation of photon+detector noise at high flux)
Principle of estimation method

- Search for \((a, r_0)\) that maximizes the log-likelihood, à la [Thiébaut-Mugnier 2005]

\[
L(a, r_0) = -\frac{1}{2} \sum_{r,k} \left[ \frac{\Delta(r, k) - a \cdot p(r, k; r_0)}{\sigma^2(r, k)} \right]^2
\]

- The optimal flux is analytical for a given \(r_0\): \(\hat{a}(r_0)\)

- Reduced log-likelihood:

\[
L'(r_0) = L(\hat{a}(r_0), r_0)
\]

- maximum in \(L'(r_0)\) = most probable position of the planet

Positivity constraint
\(\rightarrow\) less false alarms

Mugnier, Cornia et al., JOSA A 2009
Summary of Andromeda

- Inputs: images, field rotation angles, minimum separation for differences
- Noise variance: estimated from data
- Assumptions: white Gaussian noise (room for improvement: not really white)
- Andromeda = ML estimator on differential images under white Gaussian assumption $\Leftrightarrow$ Optimal linear estimator (a.k.a. Hotelling observer…) even if noise is non Gaussian

- 2 outputs for 2 tasks:
  - Detect / find planet position: reduced likelihood $L'(r_0)$
  - Estimate flux: flux map $\hat{a}(r_0)$ + error bars on $\hat{a}$, $\sigma_{\hat{a}}(r_0)$
    - SNR defined as $\text{SNR}(r_0) \triangleq \frac{\hat{a}(r_0)}{\sigma_{\hat{a}}(r_0)}$
    - Interpretation: $L'(r_0) \propto [\text{SNR}(r_0)]^2$
Validation on simulated realistic images

Simulation tool: CAOS-SPHERE environment, specification of test case by D. Mouillet

Simulation conditions:
1. varying aberrations and turbulence strength
2. 12 planets on 4 rows, separations 0.2", 0.5", 1"
3. star/planet contrast:
   - $10^5$ for the single-band images (ADI)
   - $10^6$ for the dual band images (SDI+ADI)
4. $\lambda = 1.593$ μm and 1.667 μm
5. G0 star @ 10 pc
6. 4h observation time
7. seeing: 0.85" ± 0.15"
Results of tests on detection: single band images, angular differences only

contrast: $10^5$
- 3D variance map
- $\delta_{\text{min}} = 0.5 \lambda/D$

All planets are detected (even at 0.2")
and flux is well estimated

Cornia et al., SPIE 2010
Results of tests on detection: double band images, spectral+angular differences

**contrast: $10^6$**
- 2D variance map
- $\delta_{\text{min}} = 1 \lambda/D$

All planets are detected (even at 0.2")

and flux is well estimated
Experimental data: observations with NACO

NACO-Large Program (P.I. J-L Beuzit)

Set of images taken in February 2010

Conditions of observation:

- derotated pupil mode
- saturated images, no coronagraph
- $\lambda = 1.65 \, \mu m$
- $V = 9.25$
- 1.8 h observation time
- 319 images (elem. exp. time 6.8 s)
- seeing: $0.81'' \pm 0.14''$

First detection results disappointing:

large scale inhomogeneities (very low spatial frequencies)

$\Rightarrow$ reduced by high-pass filtering both images and PSF
Results on NACO target (ADI only)

Reconstruction parameters: $\delta_{\text{min}} = 0.5\lambda/D$, 2D variance map (bad pixels)

SNR map

Cornia et al., SPIE 2010

SNR map with 1/16 freqs removed (in images and PSF)

$=>$ Inhomogeneities eliminated, detection improved
Performance of ANDROMEDA for detection

NaCo data+ fake companions added to the data by Gaël Chauvin

[ Eggenberger et al., Lyot 2010 ]

SNR map

Flux map

1/4 of the spatial frequencies filtered
Performance of ANDROMEDA for estimation

Separation: 2.1"

No filtering – 1/4 of the frequencies filtered

flux well estimated (expected value within 3 sigma)
Conclusions

• Method for exoplanet detection and flux estimation with ADI only or SDI+ADI

• Maximum likelihood framework. SNR map ‘concentrates’ all planet photons (~deconvolution)

• Validation on simulated SPHERE data:
  • ANDROMEDA meets SPHERE requirements (detection at a contrast $10^6$ and separation 0.2")
  • Flux estimation precision limited by speckle and photon noise, then calibration errors – no ‘’ flux loss’’

• Application to experimental NACO data:
  high-pass filtering helps eliminate large-scale inhomogeneities

• Included in the SPHERE pipeline

• Perspectives:
  Fine astrometry, better pre-processing(?), non-white (or even non Gaussian?) noise.
Inverse problem approach for exoplanet detection with multi-spectral data

- Marie Ygouf’s PhD (ONERA [Châtillon] and IPAG [Grenoble])
- Information redundancy, assuming achromatic aberrations:
  - To 1st order, linear scaling of speckles
    => Spectral ‘deconvolution’ [Sparks and Ford, 2002]
    = Empirical low-order spectral function fit of the speckle field:
      √ Approximation
      √ A planet perturbs the speckle field suppression
      √ No prior information is used
  - Alternatively, inverse problem approach:
    ‘Give me a model and I’ll invert the Earth’ (Archimedes)

Simulated image = Speckle field + Object image
Long exposure coronagraphic imaging model

- Perfect coronagraph: subtracts the coherent energy of the incoming wave (projection on flat wave)
- Exact model \( h_c = f(\phi_{up}, \phi_{down}, D_\phi) \) [Sauvage et al., JOSA A 2010]

  - \( \phi_{down} \) assumed to be calibrated,
  - \( D_\phi \) can be estimated from telemetry (WFS measurements + DM voltages)
  - \( \phi_{up} \) varies fast and impacts the image

\[ \Rightarrow \text{Myopic deconvolution: } i(x,y, \lambda) \rightarrow o(x,y, \lambda) \text{ and } \phi_{up} = \frac{2\pi}{\lambda} \delta_{up} \]
Direct model with possibly extended object

\[ i_\lambda = f_\lambda^* \cdot h_\lambda^c + o_\lambda \ast h_\lambda^{nc} + n \]

- **Unknowns:**
  - \( \{ f_\lambda^* \} \) Star flux
  - \( \delta_u(\rho_x, \rho_y) \) Upstream static aberrations
  - \( o_\lambda(\alpha_x, \alpha_y) \) Object map: possibly extended objects
Principle of Bayesian inversion

• Direct model:

\[ i_\lambda = f_\lambda^* \cdot h_\lambda^c + o_\lambda \ast h_\lambda^{nc} + n \]

\[ h_\lambda^c(\delta_u, \delta_d, D_{\delta_r}) \]

• Inversion: minimisation of the joint MAP metric:

\[ J\left(\{o_\lambda\}, \{f_\lambda^*\}, \delta_u\right) = \sum_\lambda \sum_{x,y} \frac{1}{2\sigma_n^2(x, y)} \left| i_\lambda - f_\lambda^* \cdot h_\lambda^c(\delta_u) - o_\lambda \ast h_\lambda^{nc} \right|^2(x, y) + R_{x,y,\lambda}(o, \phi) \]
Alternate estimation of aberrations and object

- Aberrations / Speckle field estimation with a known object
  = phase retrieval / wavelength diversity
  [Burke&Devaney, 2010]

\[
\left\{ O_\lambda \right\}^0 = 0, \left\{ f_\lambda^* \right\}^0 = 0, \delta_u^0
\]

- Object estimation with known aberrations
  = non-myopic multispectral deconvolution

\[
\Rightarrow \left\{ \hat{f}_\lambda^* \right\}, \hat{\delta}_u
\]

\[
\Rightarrow \left\{ \hat{o}_\lambda \right\}
\]
Simulation conditions

- Typical of a SPHERE-like instrument
  - Simulated images: 128 x 128 px. \( \sim 2 \times 10^4 \) object unknowns
  - \( \delta_u: \sim 30 \text{ nm (unknown)} \), \( \delta_d: \sim 100 \text{ nm (known)} \)
  - \( \delta_r: \sim 60 \text{ nm (known)} \)
  - Object map / planets (unknowns): \( 10^{-5}, 10^{-6} \) & \( 10^{-7} \), 8.5 & 17 \( \lambda/D \)
  - Star: \( \sim \text{Mag. 6 on SPHERE-VLT with a 30 min integration time} \)
  - Spectral bandwidth: [950;1647 nm]

![Simulated image] = ![Speckle field] + ![Simulated object image]
Validation by simulations


Simulated data cube

Speckle field estimation $f^* \cdot h^c$

Object map estimations $o \ast h^{nc}$

950 nm

950, 1650 nm