Inverse Problem Approaches to Exoplanet Signal Extraction

Laurent Mugnier

ONERA / Optics Dept.

ONERA
THE FRENCH AEROSPACE LAB

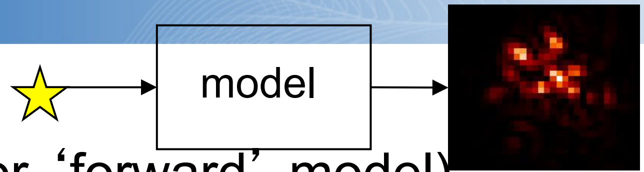
retour sur innovation

Outline

- Inversion landscape: ingredients & tools for inversion
- Planet detection from the ground with 1 or 2 λ_i :
  ANDROMEDA: Spectral & angular differential image processing
- Planet detection from the ground with many λ_i (IFS):
  myopic inversion

Ingredients for inversion

- Data y: raw ore 

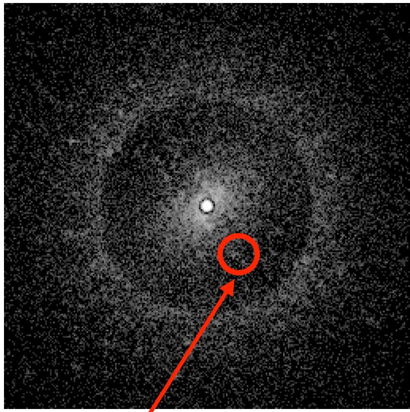


- **A « good » data formation model** ('direct' or 'forward' model).
  - Suggested by Physics, very instrument dependent: **y = model(x; θ)**
    Exoplanet imaging:
    - ✓ x = parameters of interest e.g., planet position(s) + flux/spectra
    - ✓ θ =other unknowns = nuisance parameters (instrument aberrations, etc)
  - **May be very different from a good model for simulation** :
    - ✓ Few parameters θ (e.g.: *not* 10^3 realizations of N phase screens in various planes)
    - ✓ Parameters that can be calibrated or estimated along with x
  - Pre-processing:
    - ✓ Basic: massage raw data to fit model
    - ✓ Aggressive: possibly, ***change what is defined as the data***.
      E.g.: Darwin [Thiébaut-Mugnier, IAU200, 2005] to eliminate nuisance parameters
- Prior information on noise
- Possibly, prior information on unknowns (scene, aberrations,…): Bayes.
- Estimator & algorithm

Estimators for inversion

- Function of number of unknowns, problem complexity (myopic or not, …)
- Model fitting:
  Max. **Likelihood** : p(y|x). Includes model + noise statistics.
  Eg.: image registration [Gratadour A&A 05]
- Simple inversion, well-calibrated instrument:
  posterior likelihood p(x|y) α p(x,y) = p(y|x) . p(x)
  Eg.: conventional deconvolution; nulling interferometry [Mugnier 05]
- Myopic inversion:
  - Joint estimation: $p(x, \theta, y) = p(y \mid x, \theta) \cdot p(x \mid \theta) \cdot p(\theta)$
    Eg.: phase diversity (x=phase, θ=object),
    deconvolution (Mistral: x=object, θ=PSF)
  - Marginalized inversion
    $p(x,y) = \int p(x,\theta,y) \cdot d\theta$
    Eg.: phase diversity [Blanc JOSAA 03], retinal imaging [Blanco OpEx 11].
- Note: detection also based on posterior likelihood: p(x|y) / p(x=0|y)

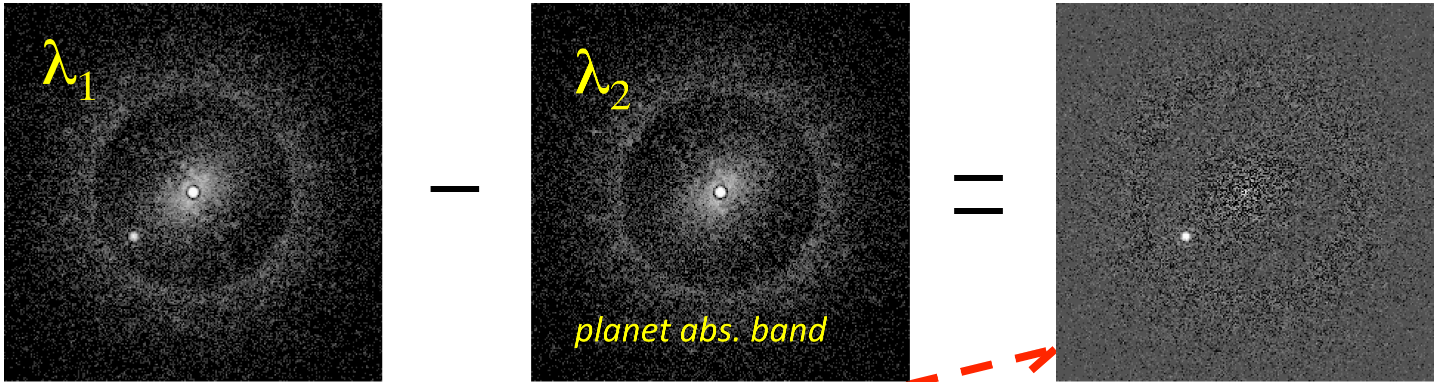
Approaches for planet detection with 1 or 2 λ



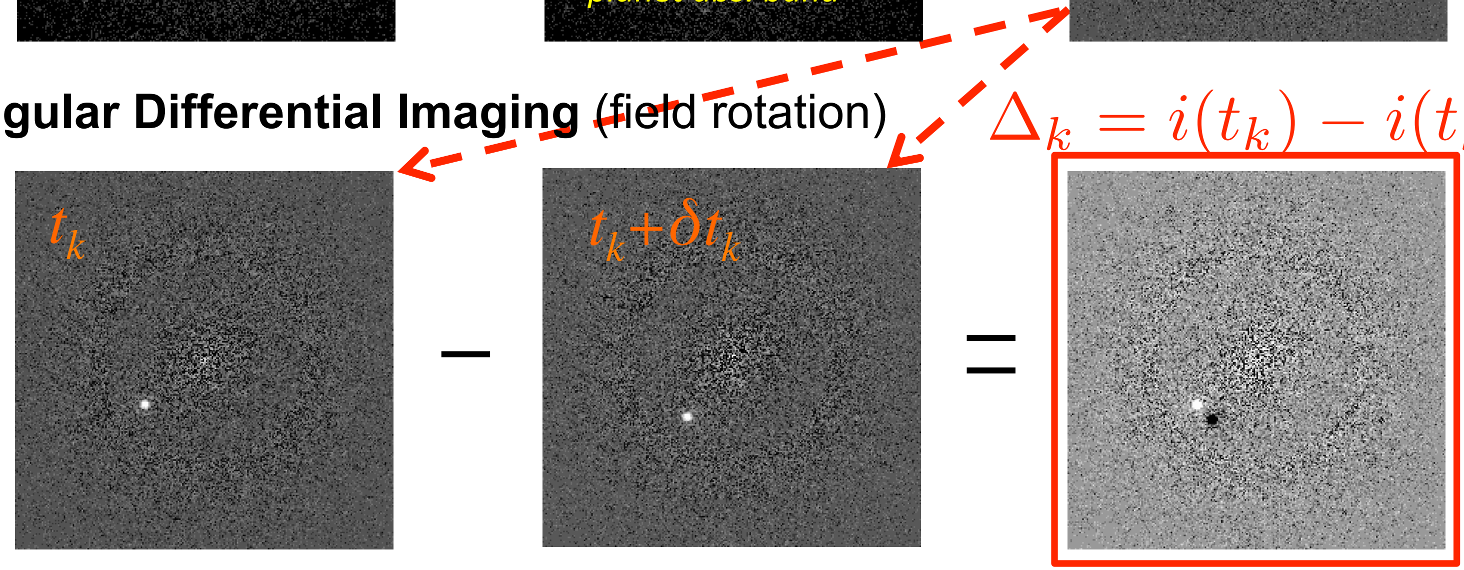
- Context:
  - SPHERE/IRDIS, Infra-Red Dual-beam Imaging and Spectroscopy camera
  - Stabilized pupil: speckles ~fixed, planet rotating with field
- Joint estimation of star residuals and planet(s):
  - Ideally: have a 'compact' data model(x; θ) with few θ (aberrations,…) and invert it.
  - Currently: θ = star residuals (pixel values), to be estimated with x. $\theta(t)$: independent (too many) / fixed / correlated
  - ML method, assumed fixed residuals (MOODS): [Smith, IEEE 09]
  - Empirical method, evolving residuals (LOCI): [Lafrenière ApJ 07]
- Subtraction of star residuals via image differences + ML method: ANDROMEDA [Mugnier-Cornia JOSA 09], [Cornia SPIE 10]

Combination of spectral and angular information

SDI: Spectral Differential Imaging (simultaneous images) $i'(t) = i_{\lambda_1}(t) - i_{\lambda_2}(t)$



ADI: Angular Differential Imaging (field rotation) $\Delta_k = i(t_k) - i(t_k + \delta t_k)$



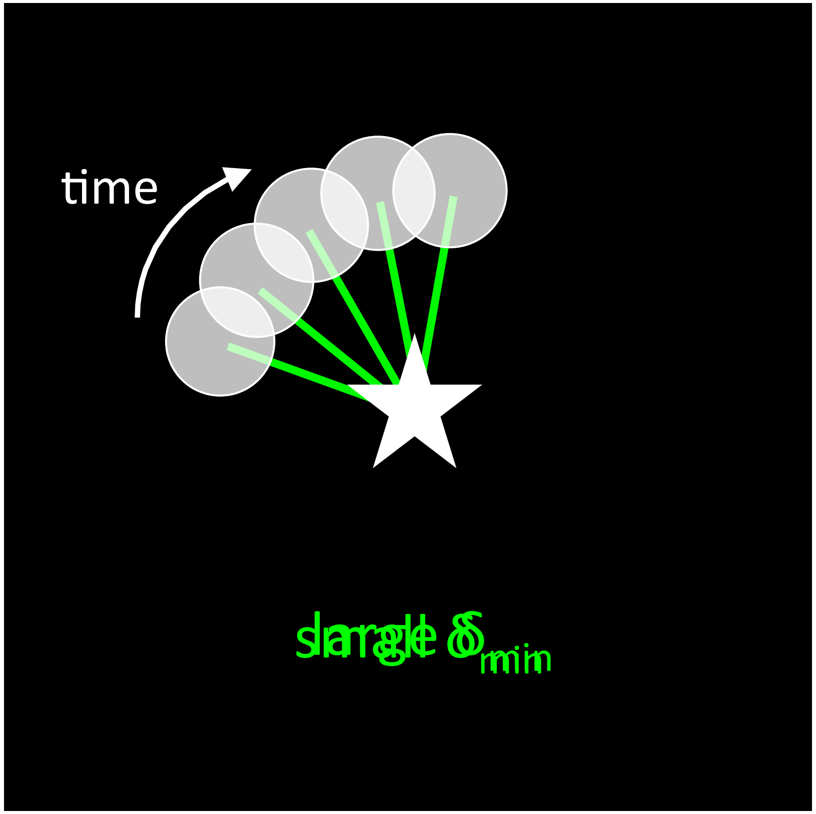
Pseudo-data Δ_k = differences between images: star signal rejected

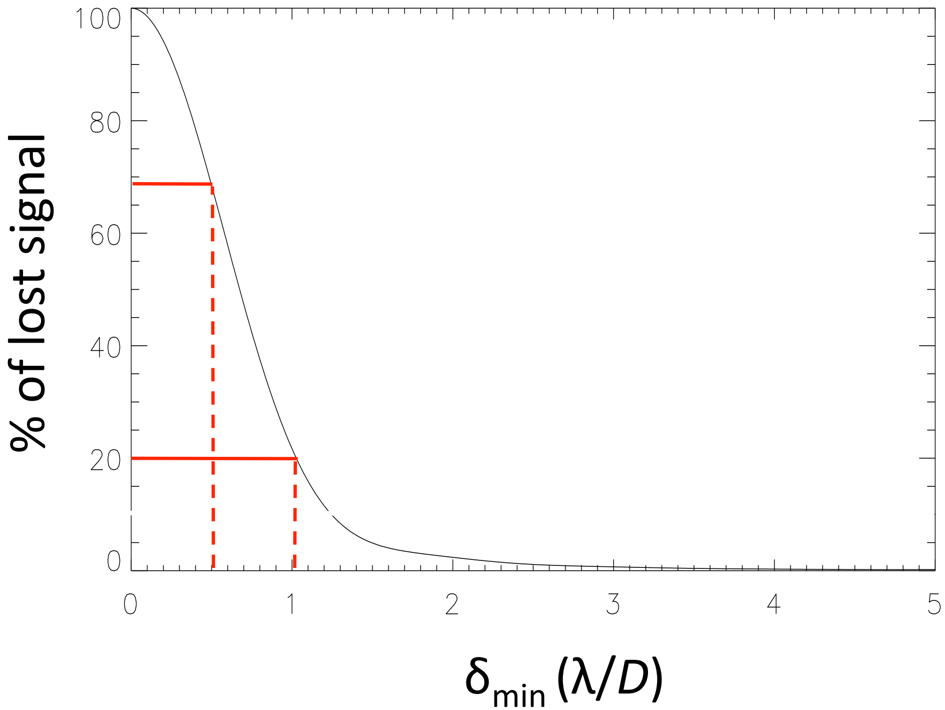
Pre-processing in practice

- Computation of differential images:
  LS minimization of image difference [Cornia SPIE 10],
- Elimination of some instrumental artefacts:
  high-pass filtering of data & planet PSF [Eggenberger Lyot 10].

Minimum angular separation: δ_{min}

Minimum displacement of the planet allowed for ang. subtract.





- small δ_{min}: good suppression of speckles
- large δ_{min}: preserve planet signal in the difference

Optimal δ_{min} depends on instrument stability and obs. conditions

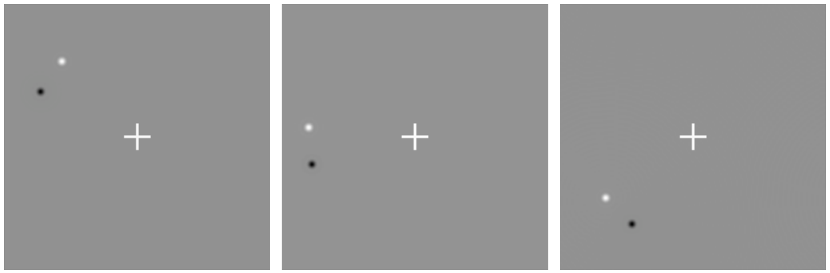
Data formation model

- Data $\Delta(\mathbf{r}, k)$ = image differences: star signal is assumed to be eliminated
- Unknowns: **flux** a and **initial position** $\mathbf{r}_0$ of the planet
- Data model:

$$\Delta(\mathbf{r},k) = \boxed{a} \cdot p(\mathbf{r},k;\boxed{\mathbf{r}_0}) + n(\mathbf{r},k)$$

[$\mathbf{r}$ = position in the image, k = time-related index]

- p: planet "signature" = difference between 2 planetary PSFs at different instants computable as a function of k and $\mathbf{r}_0$



Examples of "signatures" $p(\mathbf{r}, k; \mathbf{r}_0)$ for 3 different values of k

- Noise model $n(\mathbf{r}, k)$: gaussian white, of inhomogeneous variance $\sigma^2(\mathbf{r}, k)$ (good approximation of photon+detector noise at high flux)

Principle of estimation method

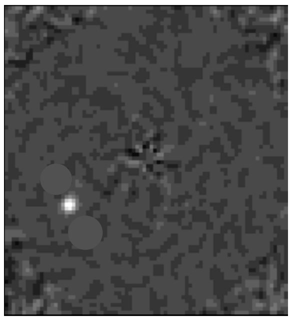
- Search for (a, $\mathbf{r}_0$) that maximizes the log-likelihood, à la [Thiébaut-Mugnier 2005]

$$L(a, \mathbf{r}_0) = -\frac{1}{2} \sum_{\mathbf{r},k} \frac{[\Delta(\mathbf{r},k) - a \cdot p(\mathbf{r},k;\mathbf{r}_0)]^2}{\sigma^2(\mathbf{r},k)}$$

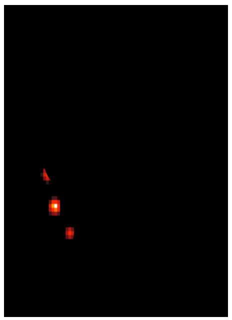
- The optimal flux is analytical for a given $\mathbf{r}_0$: $\hat{a}(\mathbf{r}_0)$
- Reduced log-likelihood:

$$L'(\mathbf{r}_0) = L(\hat{a}(\mathbf{r}_0), \mathbf{r}_0)$$

- maximum in $L'(\mathbf{r}_0)$ = most probable position of the planet



$\hat{a}(\mathbf{r}_0)$



$L'(\mathbf{r}_0)$

Positivity constraint
➔ less false alarms

Mugnier, Cornia et al., JOSA A 2009

Summary of Andromeda

- Inputs: images, field rotation angles, minimum separation for differences
- Noise variance: estimated from data
- Assumptions: white Gaussian noise (room for improvement: not really white)
- Andromeda = ML estimator on differential images under white Gaussian assumption ⇔ **Optimal linear estimator (a.k.a. Hotelling observer…)** even if noise is non Gaussian

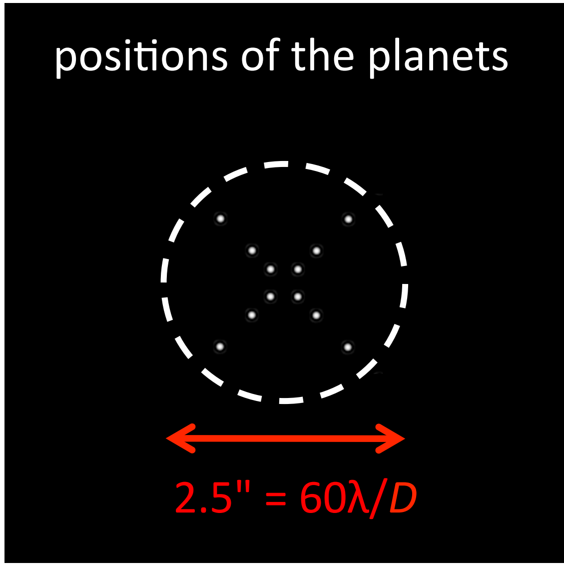
- 2 outputs for 2 tasks:
  - Detect / find planet position: reduced likelihood $L'(\mathbf{r}_0)$
  - Estimate flux: flux map $\hat{a}(\mathbf{r_0})$ + error bars on $\hat{a}$, $\sigma_{\hat{a}}(\mathbf{r_0})$
  - SNR defined as $$\mathrm{SNR}(\mathbf{r}_0) \triangleq \frac{\hat{a}(\mathbf{r}_0)}{\sigma_{\hat{a}}(\mathbf{r}_0)}$$
  - Interpretation: $$L'(\mathbf{r}_0) \propto \left[\mathrm{SNR}(\mathbf{r}_0)\right]^2$$

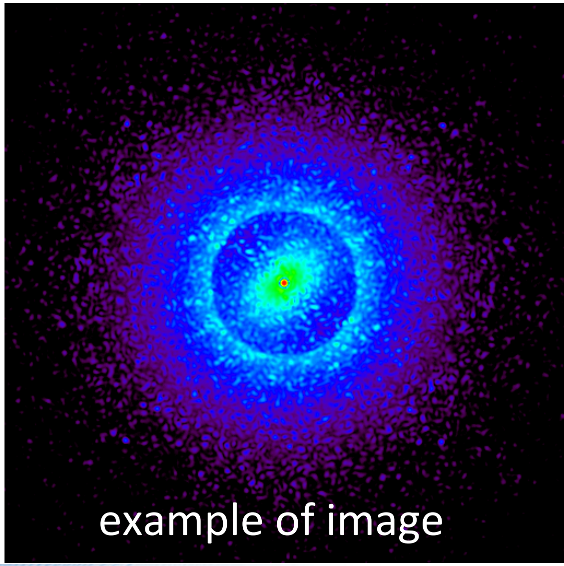
Validation on simulated realistic images

Simulation tool: CAOS-SPHERE environment, specification of test case by D. Mouillet

<u>Simulation conditions:</u>

- varying aberrations and turbulence strength
- 12 planets on 4 rows, separations **0.2", 0.5", 1"**
- star/planet contrast:
  - **10^5** for the single-band images (ADI)
  - **10^6** for the dual band images (SDI+ADI)
- λ = 1.593 μm and 1.667 μm
- G0 star @ 10 pc
- 4h observation time
- seeing: 0.85" ± 0.15"



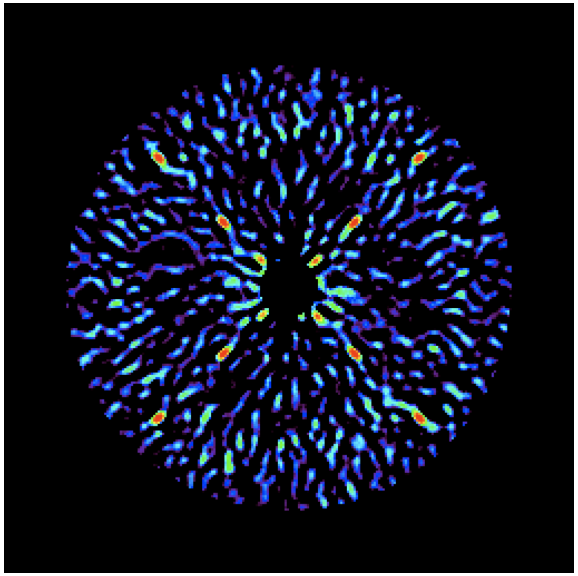


Results of tests on detection: single band images, angular differences only

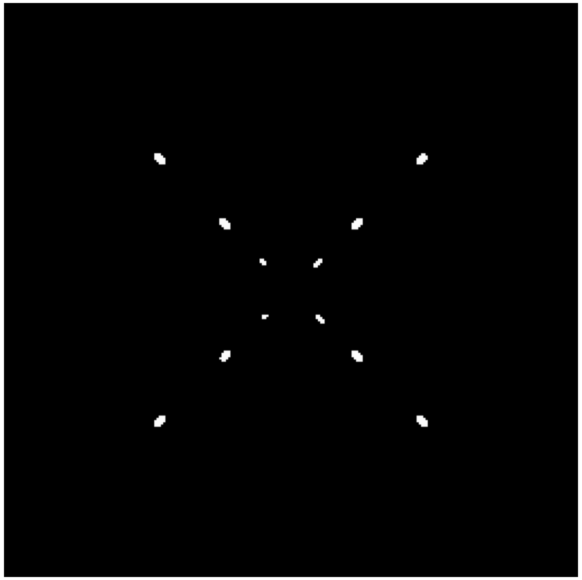
Cornia et al., SPIE 2010

contrast: 10^5

- 3D variance map
- $\delta_{min} = 0.5\ \lambda/D$

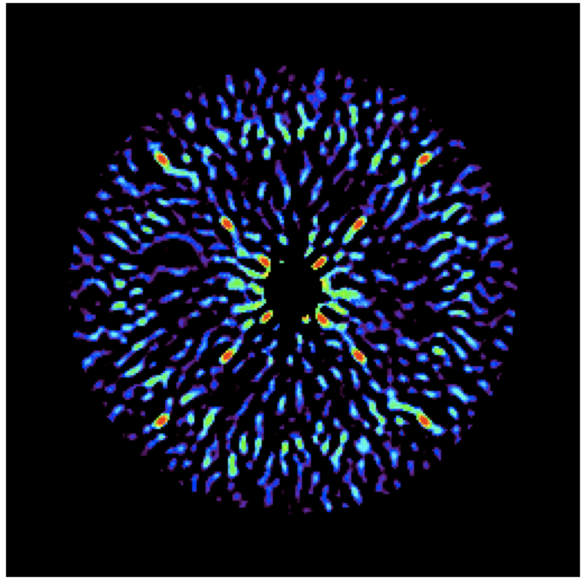


SNR map



SNR map

thresholded at 4σ



flux map

All planets are detected (even at 0.2") and flux is well estimated

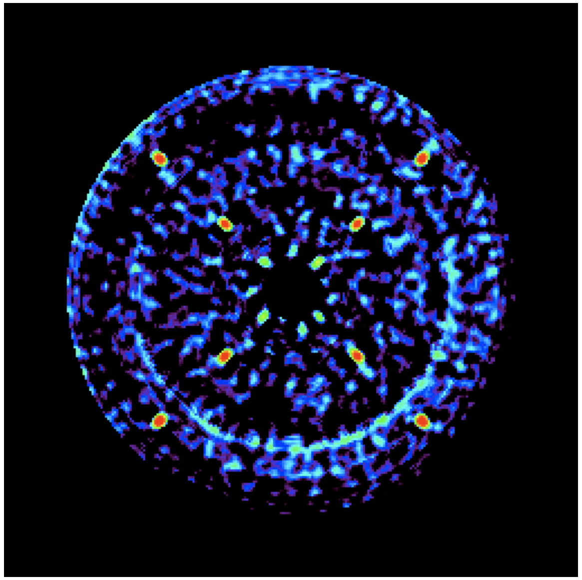
ONERA
THE FRENCH AEROSPACE LAB

Results of tests on detection: double band images, spectral+angular differences

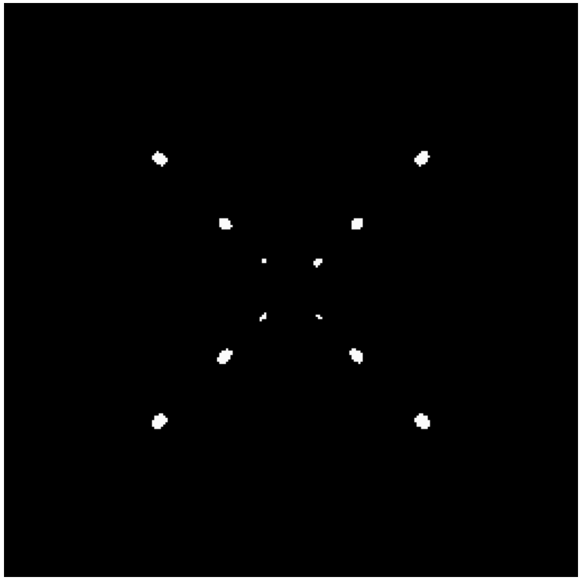
Cornia et al., SPIE 2010

contrast: 10^6

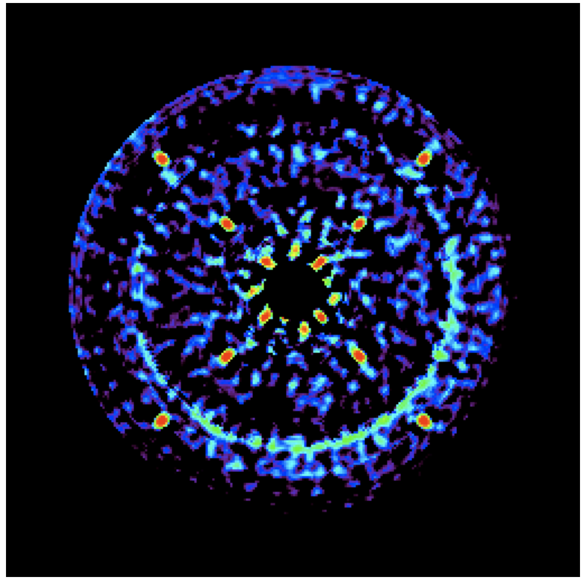
- 2D variance map
- $\delta_{min} = 1\ \lambda/D$



SNR map



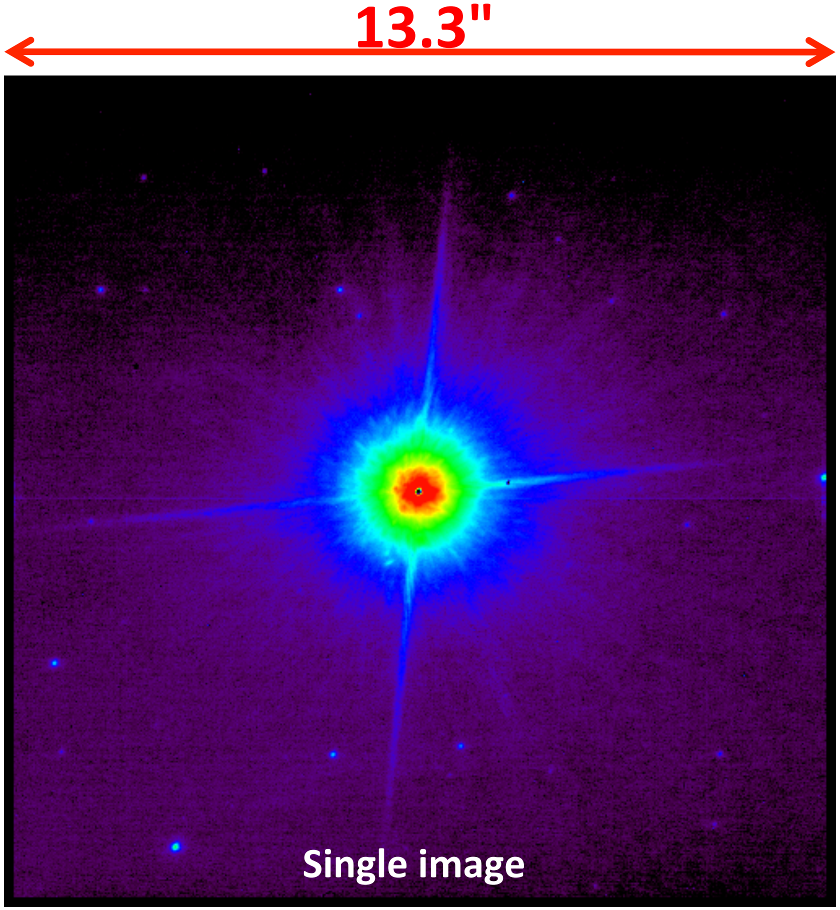
SNR map thresholded at 3σ



flux map

All planets are detected (even at 0.2") and flux is well estimated

Experimental data: observations with NACO



NACO-Large Program (P.I. J-L Beuzit)

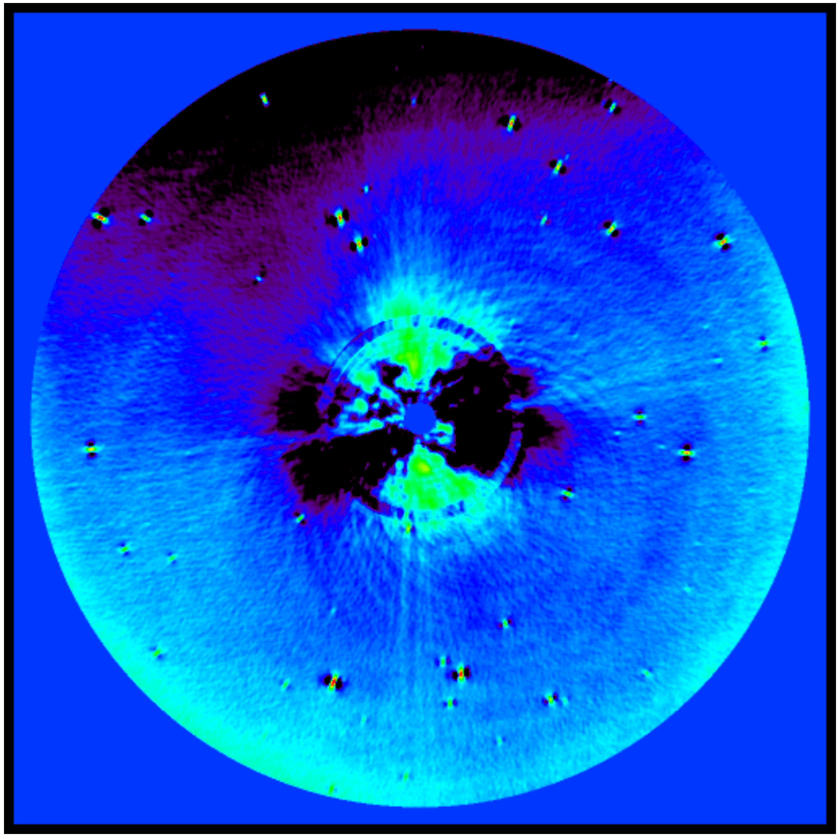
Set of images taken in February 2010
Conditions of observation:

- derotated pupil mode
- saturated images, no coronagraph
- λ = 1.65 µm
- V = 9.25
- 1.8 h observation time
- 319 images (elem. exp. time 6.8 s)
- seeing: 0.81'' ± 0.14''

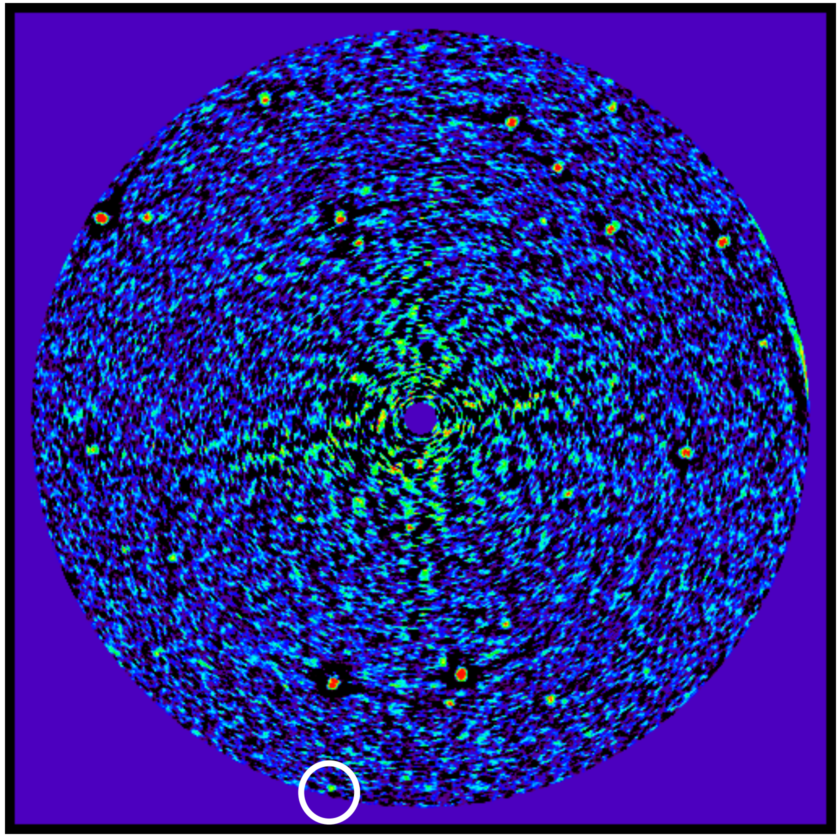
First detection results disappointing:
large scale inhomogeneities (very low spatial frequencies)
➔ reduced by high-pass filtering *both* images and PSF

Results on NACO target (ADI only)

Reconstruction parameters: $\delta_{min} = 0.5\lambda/D$, 2D variance map (bad pixels)



SNR map



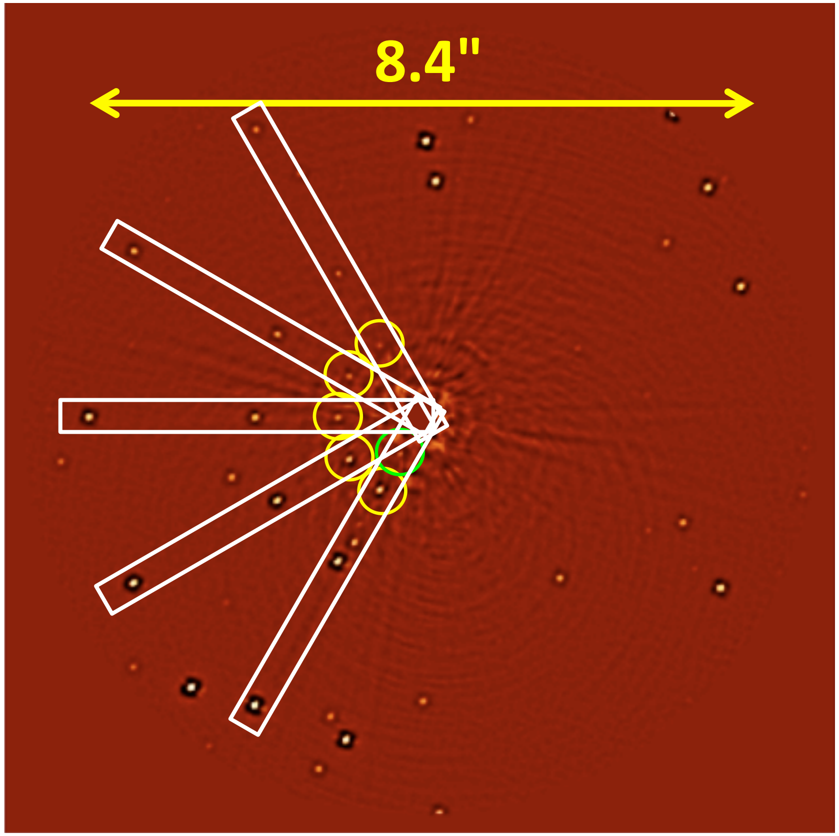
SNR map with 1/16 freqs removed (in images and PSF)
=> Inhomogeneities eliminated, detection improved

Cornia et al., SPIE 2010

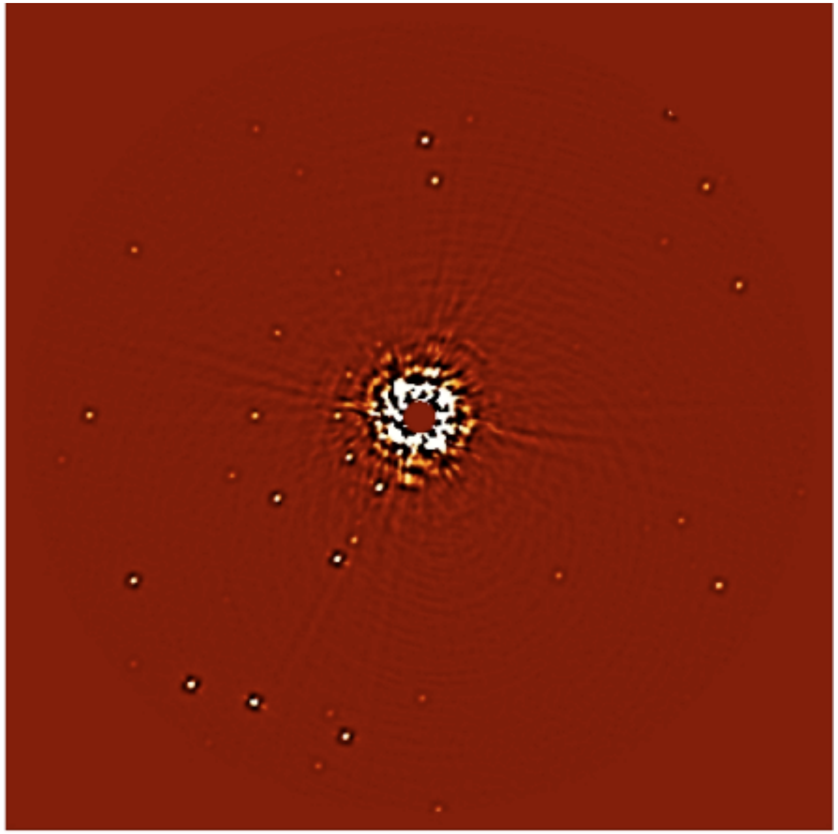
Performance of ANDROMEDA for detection

NaCo data+ fake companions added to the data by Gaël Chauvin

[Eggenberger et al., Lyot 2010]



SNR map

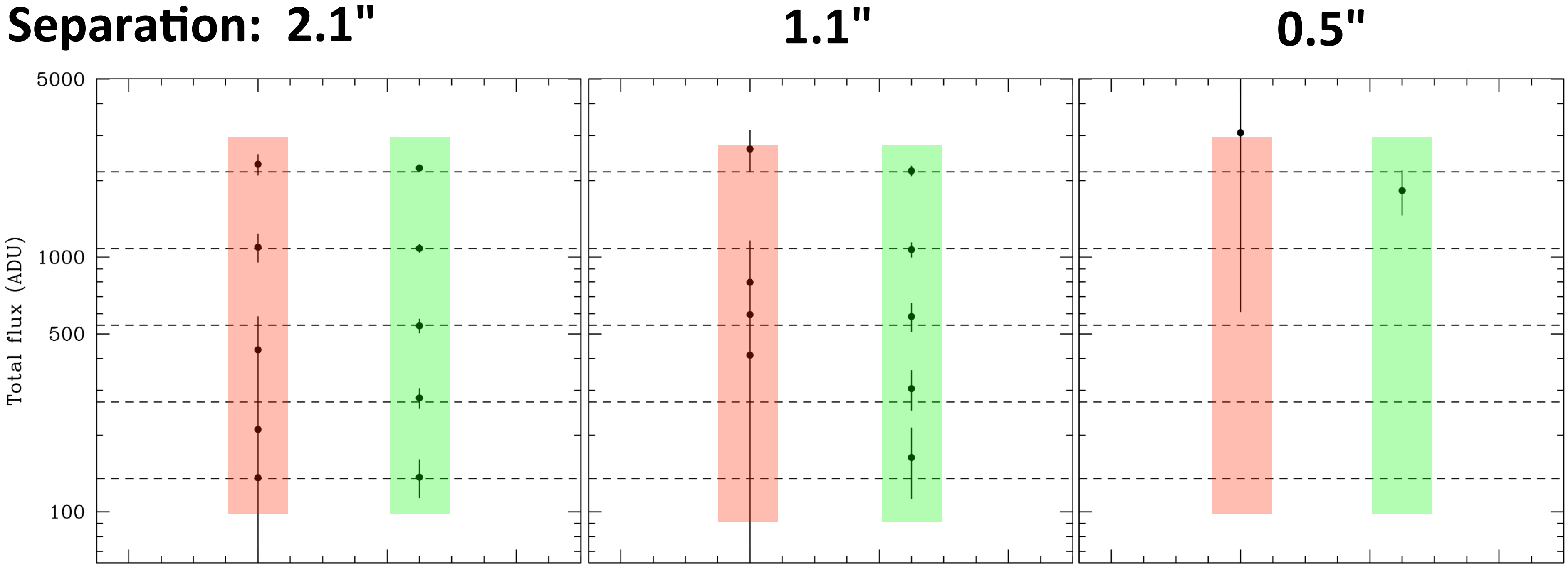


Flux map

1/4 of the spatial frequencies filtered

ONERA THE FRENCH AEROSPACE LAB

Performance of ANDROMEDA for estimation



No filtering – 1/4 of the frequencies filtered

flux well estimated (expected value within 3 sigma)

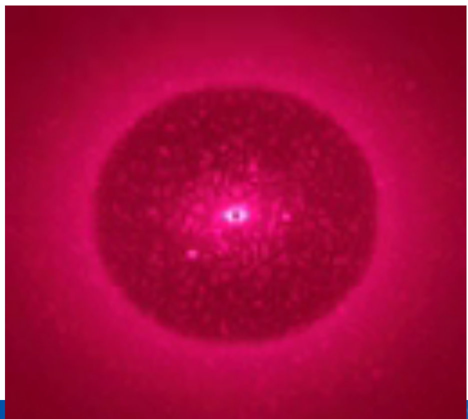
Conclusions

- Method for exoplanet detection and flux estimation with ADI only or SDI+ADI
- Maximum likelihood framework.
  SNR map ‘concentrates’ all planet photons (~deconvolution)
- Validation on simulated SPHERE data:
  - ANDROMEDA meets SPHERE requirements (detection at a contrast 10^6 and separation 0.2")
  - Flux estimation precision limited by speckle and photon noise, then calibration errors – no ‘’flux loss’’
- Application to experimental NACO data:
  high-pass filtering helps eliminate large-scale inhomogeneities
- Included in the SPHERE pipeline
- Perspectives:
  Fine astrometry, better pre-processing(?), non-white (or even non Gaussian?) noise.

Inverse problem approach for exoplanet detection *with multi-spectral data*

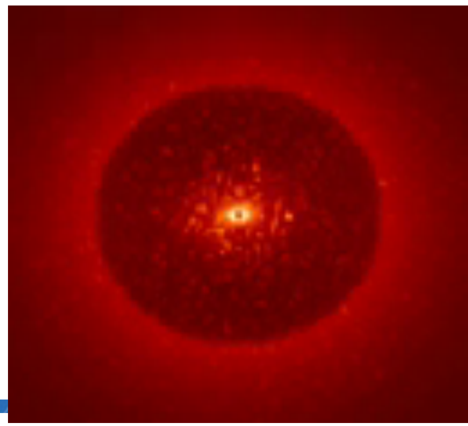
- Marie Ygouf's PhD (ONERA [Châtillon] and IPAG [Grenoble])
- Information redundancy, assuming achromatic aberrations:
  - To 1st order, linear scaling of speckles
    => Spectral 'deconvolution' [Sparks and Ford, 2002]

    = Empirical low-order spectral function fit of the speckle field:
    - ✓ Approximation
    - ✓ A planet perturbs the speckle field suppression
    - ✓ No prior information is used
  - Alternatively, inverse problem approach:
    'Give me a model and I'll invert the Earth' (Archimedes)



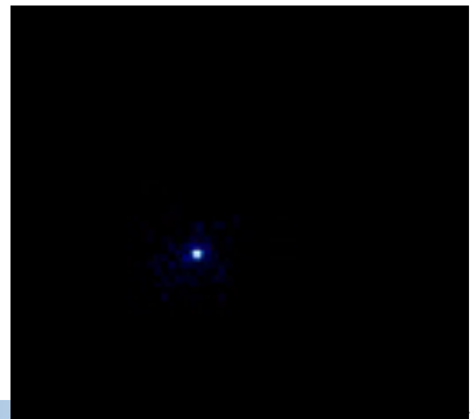
Simulated image

=



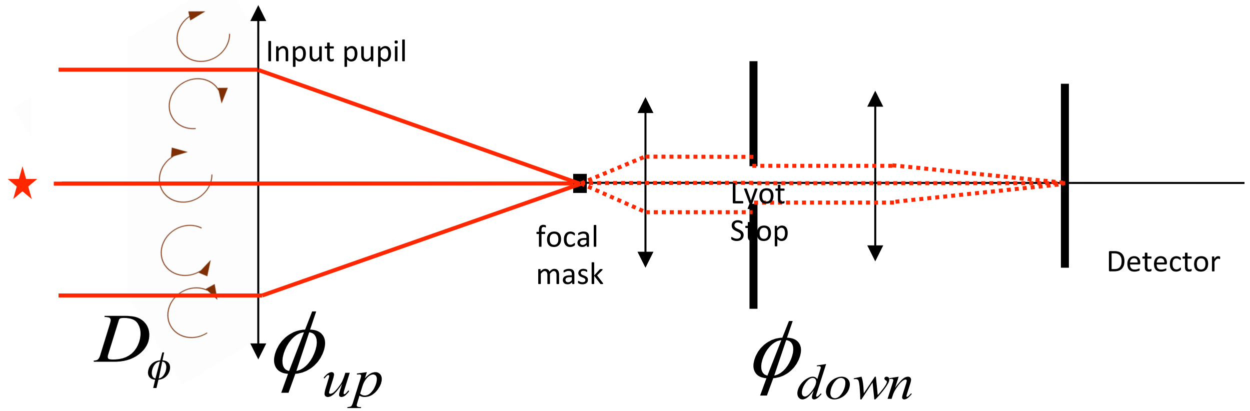
Speckle field

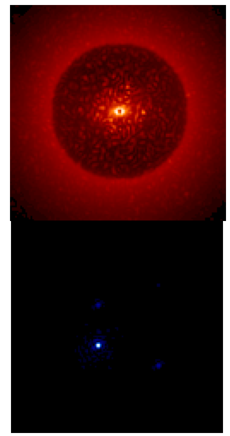
+



Object image

Long exposure coronagraphic imaging model





- Perfect coronagraph:
  subtracts the coherent energy of the incoming wave (projection on flat wave)
- Exact model $h_c = f(\phi_{up}, \phi_{down}, D_\phi)$ [Sauvage et al., JOSA A 2010]

  - ϕ_{down} assumed to be calibrated,
  - D_ϕ can be estimated from telemetry (WFS measurements + DM voltages)
  - ϕ_{up} varies fast and impacts the image

  => Myopic deconvolution: $i(x,y,\lambda) \rightarrow o(x,y,\lambda)$ and $\phi_{up} = (2\pi/\lambda)\,\delta_{up}$

Direct model with possibly extended object

i	Image
o	Object
h	Instrumental response
n	Noise

$$i_\lambda = \underbrace{f_\lambda^* \cdot h_\lambda^c}_{\text{Speckle field / Star image}} + \underbrace{o_\lambda * h_\lambda^{nc}}_{\text{Object image}} + n$$

- f_λ^*: Star flux
- h_λ^c: On-axis instrumental response
- h_λ^{nc}: Off-axis instrumental response

Speckle field
Star image

Object image

- Unknowns:

$\{f_\lambda^*\}$ Star flux

$\delta_u(\rho_x,\rho_y)$ Upstream static aberrations

(Star flux and upstream static aberrations: Nuisance parameters)

$o_\lambda(\alpha_x,\alpha_y)$ Object map: possibly extended objects

Principle of Bayesian inversion

- Direct model:

$$i_\lambda = f_\lambda^* \cdot h_\lambda^c + o_\lambda * h_\lambda^{nc} + n \qquad\qquad h_\lambda^c\left(\delta_u, \delta_d, D_{\delta_r}\right)$$

- Inversion: minimisation of the joint MAP metric:

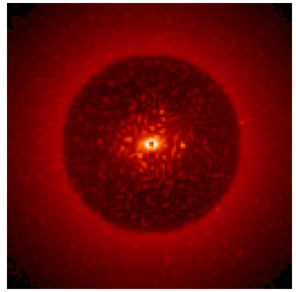
$$J\left(\left\{o_\lambda\right\}, \left\{f_\lambda^*\right\}, \delta_u\right) = \sum_\lambda \sum_{x,y} \frac{1}{2\sigma_n^2(x,y)} \left| i_\lambda - f_\lambda^* \cdot h_\lambda^c(\delta_u) - o_\lambda * h_\lambda^{nc} \right|^2 (x,y) + R_{x,y,\lambda}(o,\phi)$$

Implementation

- Alternate estimation of aberrations and object

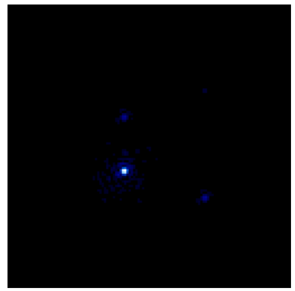
$$\{o_\lambda\}^0 = 0, \{f_\lambda^*\}^0 = 0, \delta_u^0$$

- Aberrations / Speckle field estimation with a known object
  = phase retrieval / wavelength diversity
  [Burke&Devaney, 2010]



$$\Rightarrow \{\hat{f}_\lambda^*\}, \hat{\delta}_u$$

- Object estimation with known aberrations
  = non-myopic multispectral deconvolution

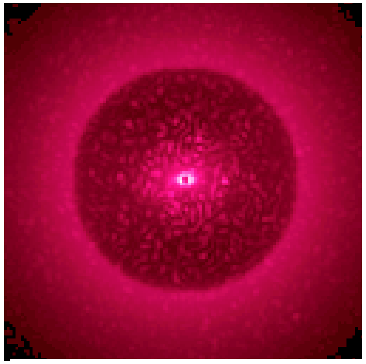


$$\Rightarrow \{\hat{o}_\lambda\}$$

Simulation conditions

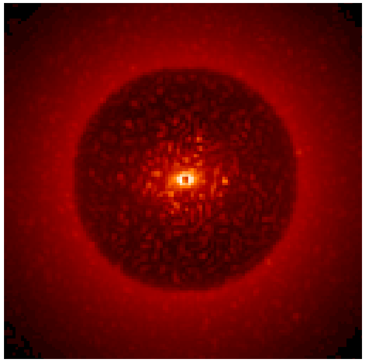
SPHERE
Spectro-Polarimetric High-contrast Exoplanet REsearch

- Typical of a SPHERE-like instrument
  - Simulated images : 128 x 128 px. **~2 x 10^4 object unknowns**
  - **δ_u : ~30 nm (unknown)**, δ_d : ~100 nm (known)
  - δ_r : ~60 nm (known)
  - **Object map / planets (unknowns)** : 10^{-5}, 10^{-6} & 10^{-7}, 8.5 & 17 λ/D
  - Star : ~Mag. 6 on SPHERE-VLT with a 30 min integration time
  - Spectral bandwidth : [950;1647 nm]



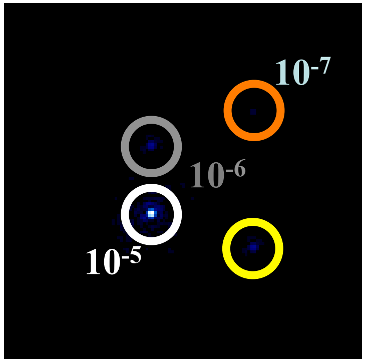
Simulated image

=



Speckle field

+

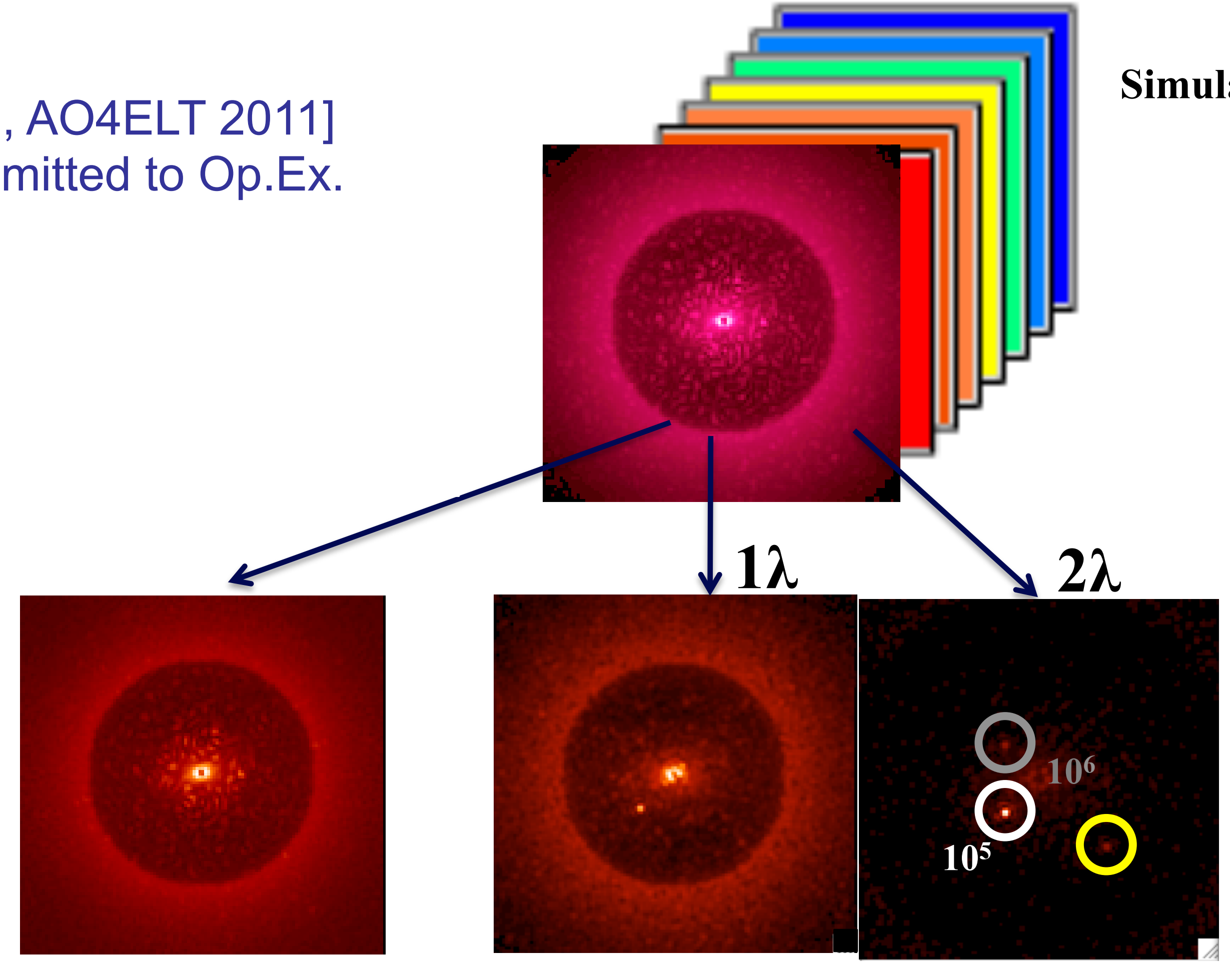


Simulated object image

Validation by simulations

[Ygouf et al., AO4ELT 2011]
+ paper submitted to Op.Ex.

Simulated data cube



950 nm

950, 1650 nm

Speckle field estimation

$f^* \cdot h^c$

Object map estimations $o * h^{nc}$