Combined astrometric imaging, coronagraphy and deep wide field imaging

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Stuart Shaklan (NASA JPL)  Error budget, mission architecture
Robert Woodruff (LMC)  Optical design for wide field telescope compatible with coronagraphy
Bijan Nemati (NASA JPL)  Numerical simulations, modeling approach
Mark Ammons (UofA)  Lab demo design & operation
Eduardo Bendek (UofA)  Lab demo optical design & operation
Marie Levine (NASA JPL)  System engineering, mission architecture
Joe Pitman (Expl. Sci.)  System engineering

Tom Milster (UofA)  Mask manufacturing, scaling of mask manufacturing to full scale PM
Jim Burge (UofA)  Mask manufacturing, scaling of mask manufacturing to full scale PM
Neville Woolf (UofA)  Exoplanet science, concept definition
Roger Angel (UofA)  Exoplanet science, concept definition
Josh Eisner (UofA)  Exoplanet and star formation/evolution science
Ruslan Belikov (NASA Ames)  Compatibility with coronagraphy
Daniel Eisenstein (UofA)  Extragalactic science enabled with wide field camera
Ann Zabludoff (UofA)  Extragalactic science with wide field camera
Dennis Zaritsky (UofA)  Extragalactic & galactic science with wide field camera
Jay Daniel (L3/Tinsley)  Optics manufacturing
epoch #1 (first observation)
epoch #2 (first observation)
Blue points show the position of background stars at epoch #2 (second observation). The telescope is pointed on the central star, so the spikes have not moved between the 2 observations, but the position of the background stars has moved due to the astrometric motion of the central star (green vectors).
Why is imaging astrometry difficult?

Principle: use background stars around coronagraph target as an astrometric reference

With a 1.4-m telescope in the visible, 0.25 sq deg offers sufficient photons from stars at the galactic pole to provide an astrometric reference at the <50 nano-arcsec after taking into account realistic efficiency, zodi light and pixel sampling.

(1) Light from different stars on the sky travels different paths → small bending of optics produces field distortions

(2) The detector can move between observations (especially when using large mosaics)

(3) Pixels are not perfect and their response changes with time

+ (4) Central star is much brighter than background stars
epoch #1 (first observation)
Optical Layout for simultaneous coronagraphy and astrometry

The telescope is a conventional TMA, providing a high quality diffraction-limited PSF over a 0.5 x 0.5 deg field with no refractive corrector. The design shown here was made for a 1.4m telescope (PECO).

Light is simultaneously collected by the coronagraph instrument (direct imaging and spectroscopy of exoplanet) and the wide field astrometric camera (detection and mass measurement of exoplanets).
Approach & Assumptions

Baseline: 1.4-m telescope, with 0.29 sq deg FOV (0.31 deg radius)

The FOV is chosen to reach performance goal (0.2 μas/ measurement) in a sufficiently stable system (Photon noise limited performance for this FOV is 0.044 μas single measurement at galactic pole, but actual performance is significantly lower due to distortions and detector limits)

Baseline assumes no special requirements on detector or optics, other than a design to support wide field imaging and dots on M1:  
ASTROMETRY DOES NOT DRIVE TELESCOPE OR INSTRUMENT DESIGN
- no special detector requirements (standard errors on flat field, geometry), assumes no calibration beyond what is “standard”
- no component requirement exceeds what has already been demonstrated and manufactured
- assumes no data calibration is done between observations of different stars (pessimistic)
- fraction of primary mirror covered by dots kept small (1%) to avoid loss in sensitivity for general astrophysics and coronagraphy
Dots on primary mirror create a series of diffraction spikes used to calibrate astrometric distortions.

All astrometric distortions (due to change in optics shapes of M2, M3, and deformations of the focal plane array) are common to the spikes and the background stars. By referencing the background star positions to the spikes, the astrometric measurement is largely immune to large scale astrometric distortions.

Instead of requiring ~pm level stability on the optics over yrs, the stability requirement on M2, M3 is now at the nm-level over approximately a day on the optics surfaces, which is within expected stability of a coronagraphic space telescope. (Note: the concept does not require stability of the primary mirror).
Precursors...

**Long-Focus Photographic Astrometry**

A 5-seconds exposure of Castor, enlarged 75 times. The separation of the components is 3774 or 0.198 mm on the plate. The first-order spectra are one magnitude fainter than the central image. Taken December 1, 1939, by K. A. Strand, with the Sprague 24-inch refractor, aperture reduced to 13 inches, Eastman IV G emulsion, Wratten No. 12 (minus-blue) filter.

The age of the fainter component, a compensation for possible magnitude error is provided by using the mean of the measured positions of the two spectral images instead of the central image. As long as the difference in intensity between the images does not exceed half a magnitude, the magnitude error is usually negligible; it is therefore sufficient to have a limited number of gratings, producing first-order spectra which are a whole number of magnitudes fainter than the central image.

For example, in his work with the Sprague refractor, Strand used four gratings, made of duraluminum, giving differences of one, two, three, and four magnitudes, respectively, between the central image and the first-order spectra. The bars are mounted on 10 cm-wide annular frames, cut from sheets of duraluminum, 3 mm thick. The constants of the four gratings are given below.

**Constants of Sprague Objective Gratings**

<table>
<thead>
<tr>
<th>Grating</th>
<th>width of bar</th>
<th>opening</th>
<th>central image</th>
<th>mag. difference</th>
<th>distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.25 mm</td>
<td>11.21 mm</td>
<td>1.51 mag</td>
<td>0.98 mag</td>
<td>0.270 mm = 5.10</td>
</tr>
<tr>
<td>2</td>
<td>7.12 mm</td>
<td>15.06 mm</td>
<td>0.84 mag</td>
<td>2.05 mag</td>
<td>0.275 mm = 5.15</td>
</tr>
<tr>
<td>3</td>
<td>3.98 mm</td>
<td>14.80 mm</td>
<td>0.52 mag</td>
<td>3.01 mag</td>
<td>0.272 mm = 6.08</td>
</tr>
<tr>
<td>4</td>
<td>3.20 mm</td>
<td>19.06 mm</td>
<td>0.24 mag</td>
<td>3.95 mag</td>
<td>0.275 mm = 5.13</td>
</tr>
</tbody>
</table>

Fig. 1.—Monochromatic and broadband direct and coronagraphic PSFs with a square-aperture relucate pupil mask. All images are on a logarithmic gray scale stretching 10 mag fainter than their peaks. The pupil is 128 pixels across, and the grid has a wire spacing of 16 pixels, with 2-pixel-wide wires. (1): Direct PSF for the shortest wavelength of a 20% bandwidth filter with uniform transmission within the bandpass, in the absence of phase errors. The satellite PSFs are at the origin but along the horizontal and vertical axes are fainter than the central core of the PSF by a factor \( e^2 = (g/d)^2 \), where \( g \) is the wire thickness and \( d \) is the wire spacing. The satellite spots off the axes are \( e^4 \) fainter than the corresponding central peak. (2): Coronagraphic PSF at the shortest wavelength of the filter. The off-axis sea of satellite spots is more visible in the coronagraphic image because the core has been suppressed. (3) and (4): Direct and coronagrophic PSFs for the longest wavelength of the filter. (5) and (6): Direct and coronagraphic PSFs for the full bandpass. The length of any particular radial streak in this last pair of images (in resolution elements at the central wavelength of the bandpass) is approximately the fractional filter bandwidth multiplied by the radial distance of the spot at band center. The streaks all point toward the origin, so the smearing has no effect on astrometric precision according to Fraunhofer regime image formation theory. We suggest using the four satellite peaks closest to the core as fiducials for the position of the central occulted star in coronagraphic images.

“Long-focus photographic astrometry”, van de Kamp, 1951
Red points show the position of background stars at epoch #1 (first observation)
Blue points show the position of background stars at epoch #2 (second observation). The telescope is pointed on the central star, so the spikes have not moved between the 2 observations, but the position of the background stars has moved due to the astrometric motion of the central star (green vectors).
Due to astrometric distortions between the 2 observations, the actual positions measured (yellow) are different from the blue point. The error is larger than the signal induced by a planet, which makes the astrometric measurement impossible without distortion calibration.
The measured astrometric motion (blue vectors in previous slide) is the sum of the true astrometric signal (green vectors) and the astrometric distortion induced by change in optics and detector between the 2 observations. Direct comparison of the spike images between the 2 epochs is used to measure this distortion, which is then subtracted from the measurement to produce a calibrated astrometric measurement.
The calibration of astrometric distortions with the spikes is only accurate in the direction perpendicular to the spikes length. For a single background star, the measurement is made along this axis (1-D measurement), as shown by the green vectors. The 2-D measurement is obtained by combining all 1-D measurements (large green vector).
Observation scheme

A slow telescope roll is used to average out small scale distortions, which are due to non-uniformity in the pixel size, (spectral) response, and geometry.

The green vector is what should be measured.

Epoch #1

Epoch #2

Proper motion + Parallax + astrometric signal

diffraction spikes
Observation scheme

A slow telescope roll is used to average out small scale distortions, which are due to non-uniformity in the pixel size, (spectral) response, and geometry.
Science goals and required astrometric accuracy
Science goals

Primary science goal: **Measure planet mass with 10% accuracy (1-\(\sigma\)) for an Sun/Earth analog at 6pc.**
This allows mass measurement of all potentially habitable planets (Earth-like & SuperEarths) imaged by PECO.

SNR>5 detection at R=5 in less than 6 hrs along 20% of the planet orbit, assuming 45% system efficiency, and 1 zodi (no WF errors)

Table 4-2: Stars with Earth-like planets in habitable zones (1 AU equiv) easily detectable with PECO

<table>
<thead>
<tr>
<th>HIP#</th>
<th>dist (pc)</th>
<th>max el ((\lambda/D))</th>
<th>*rad ((\lambda/D))</th>
<th>SNR (1s, tp)</th>
<th>t20% (s, tp)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>71683</td>
<td>1.3</td>
<td>11.5</td>
<td>0.06</td>
<td>0.49</td>
<td>35</td>
<td>Alf Cen A G2 V, V=0</td>
</tr>
<tr>
<td>71681</td>
<td>1.3</td>
<td>6.6</td>
<td>0.04</td>
<td>0.45</td>
<td>44</td>
<td>Alf Cen B K2 IV, V=1.3</td>
</tr>
<tr>
<td>8102</td>
<td>3.6</td>
<td>2.3</td>
<td>0.01</td>
<td>0.08</td>
<td>2750</td>
<td>Tau Cet G8.5 V, V=3.5 **</td>
</tr>
<tr>
<td>16537</td>
<td>3.2</td>
<td>2.2</td>
<td>0.01</td>
<td>0.09</td>
<td>2968</td>
<td>Eps Eri K2 V, V=3.7 **</td>
</tr>
<tr>
<td>3821</td>
<td>6.0</td>
<td>2.3</td>
<td>0.01</td>
<td>0.04</td>
<td>14329</td>
<td>Eta Cas G0 V V=3.5 ***</td>
</tr>
<tr>
<td>2021</td>
<td>7.5</td>
<td>3.1</td>
<td>0.01</td>
<td>0.03</td>
<td>14878</td>
<td>Bet Hyi G0 V, V=2.8</td>
</tr>
<tr>
<td>99240</td>
<td>6.1</td>
<td>2.2</td>
<td>0.01</td>
<td>0.03</td>
<td>19636</td>
<td>Del Pav G8 IV, V=3.6</td>
</tr>
</tbody>
</table>

Table extracted from PECO SRD (http://caao.as.arizona.edu/PECO/PECO_SRD.pdf)
Simulated observations

<table>
<thead>
<tr>
<th>Planetary system characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Star</strong></td>
<td>Sun analog</td>
</tr>
<tr>
<td><strong>Distance</strong></td>
<td>6 pc</td>
</tr>
<tr>
<td><strong>Location</strong></td>
<td>Ecliptic pole</td>
</tr>
<tr>
<td><strong>Orbit semi-major axis</strong></td>
<td>1.2 AU</td>
</tr>
<tr>
<td><strong>Planet mass</strong></td>
<td>1 Earth mass</td>
</tr>
<tr>
<td><strong>Orbit excentricity</strong></td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Astrometric signal amplitude</strong></td>
<td>0.5 μas</td>
</tr>
<tr>
<td><strong>Orbit apparent semi-major axis</strong></td>
<td>200 mas</td>
</tr>
</tbody>
</table>

**Observations**

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>32 (regularly spaced every 57 days)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coronagraph: planet position measurement accuracy in coronagraphic image</strong></td>
<td>2.5 mas per axis (= 3.6 mas in 2D): corresponds to diffraction-limited measurement with 100 photon at 550 nm on PECO</td>
</tr>
<tr>
<td><strong>Coronagraph: Inner Working Angle</strong></td>
<td>130 mas (coronagraph cannot see planet inside IWA)</td>
</tr>
<tr>
<td><strong>Astrometry: accuracy</strong></td>
<td>Variable (to be matched to science requirements)</td>
</tr>
</tbody>
</table>
Combined solution for simultaneous coronagraphy + astrometry

Planet on a 1.2 AU orbit (1.3 yr period), $e=0.2$

orbit orientation on sky: planet outside the coronagraph IWA for 17 out of the 32 observations.
Coronagraphic image measures orbital parameters and stellar mass (with astrometry) -> reduced planet mass error
Combined solution for simultaneous coronagraphy + astrometry

Required single measurement astrometric accuracy = 0.2 μas (1-sigma, 1D)

Coronagraphy allows terrestrial planet confirmation with 2x worse astrometric accuracy

Coronagraphy allows high accuracy mass measurement by measuring directly the Star mass

Astrometry alone (no coronagraphy)
Combined solution for simultaneous coronagraphy + astrometry is very accurate for orbital parameters measurement.
### Combined solution for simultaneous coronagraphy + astrometry

<table>
<thead>
<tr>
<th></th>
<th>Astrometry only</th>
<th>Astrometry + coronagraphy</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallax</td>
<td>0.037 μas</td>
<td>0.035 μas</td>
</tr>
<tr>
<td>x proper motion</td>
<td>0.017 μas/yr</td>
<td>0.012 μas/yr</td>
</tr>
<tr>
<td>y proper motion</td>
<td>0.020 μas/yr</td>
<td>0.013 μas/yr</td>
</tr>
<tr>
<td>Planet mass</td>
<td>0.132 ME</td>
<td>0.098 ME</td>
</tr>
<tr>
<td>Semi-major axis</td>
<td>0.0228 AU</td>
<td>0.0052 AU</td>
</tr>
<tr>
<td>orbital phase</td>
<td>0.653 rad</td>
<td>0.039 rad</td>
</tr>
<tr>
<td>orbit inclination</td>
<td>0.0968 rad</td>
<td>0.0065 rad</td>
</tr>
<tr>
<td>sma projected PA on sky</td>
<td>0.1110 rad</td>
<td>0.0040 rad</td>
</tr>
<tr>
<td>orbit ellipticity</td>
<td>0.098</td>
<td>0.0035</td>
</tr>
<tr>
<td>PA of perihelion on orbit plane (w)</td>
<td>0.648 rad</td>
<td>0.0034 rad</td>
</tr>
<tr>
<td>stellar mass</td>
<td>0.050 $M_{\text{Sun}}$</td>
<td>0.013 $M_{\text{Sun}}$</td>
</tr>
</tbody>
</table>
Exoplanet science with coronagraphy + astrometry + wide field imaging

**Provides a complete picture of a planetary system:**

**CORONAGRAPHY:**
- Planets orbits
- Planet atmospheres (spectra, polarization from coronagraph)
- Rotation periods (time photometry from coronagraph)
- Zodiacal cloud: morphology, spectra, polarization (coronagraph)

**ASTROMETRY:**
- Planet masses

**CORONAGRAPHY + CORONAGRAPHY:**
- Good sensus of planets in a system (astrometry + coronagraphy)
- Immunity from confusion issues between multiple planets, zodi clumps
- Immunity from 1yr period blind spot

**WIDE FIELD IMAGING:**
- Very distant planets, possibly ejected
- Debris disks at large separation (Fomalhaut type disks)
- Occultations of field stars by Kuiper belt objects

**Also: high precision photometry of field stars**
- Microlensing program possible (with pointing to galactic bulge)
  - stable sharp PSF with good astrometry valuable
- Transit observations
Deep wide field imaging science

Wide field + stable diffraction limited PSF is scientifically valuable for many scientific programs, and will be unique in visible:
- Cosmology: weak lensing, type Ia supernovae
- Galactic astronomy
- Planetary astronomy: search for small & distant objects (asteroids, comets, KBOs)

The dots on PM do not significantly impact sensitivity
Loss in sensitivity is due to 3 effects:

- Light absorbed by the dots → 1% loss in throughput

- Light diffracted out of the PSF core by the dots → 1% loss in flux

- Additional background due to diffraction spikes of central star
  - spikes occupy a tiny fraction of the FOV, and are sufficiently stable to be efficiently removed from images by postprocessing
  - for a mV=3.7 central source, over 95% of the field, additional diffracted light is less than 1% of zodi background
  - mean value for additional diffracted background over the field = 6 ph/pix/day (unfiltered), vs 20000 ph/pix/day for zodi
Astrometric error budget analysis and simulations
Approach & Assumptions

Baseline: 1.4-m telescope, with 0.29 sq deg FOV (0.31 deg radius)

The FOV is chosen to reach performance goal (0.2 μas/ measurement) in a sufficiently stable system (Photon noise limited performance for this FOV is 0.044 μas single measurement at galactic pole, but actual performance is significantly lower due to distortions and detector limits)

When detailed simulations are required, a smaller FOV system is used (0.1 deg radius = 0.03 sq deg FOV) to ease computations.

Baseline assumes no special requirements on detector or optics, other than a design to support wide field imaging and dots on M1: ASTROMETRY DOES NOT DRIVE TELESCOPE OR INSTRUMENT DESIGN

- no special detector requirements (standard errors on flat field, geometry), assumes no calibration beyond what is “standard”
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<table>
<thead>
<tr>
<th></th>
<th>Baseline design</th>
<th>Value for simulations</th>
<th>Rationale for flight instrument value</th>
<th>Impact on astrometric accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope diameter (D)</td>
<td></td>
<td>1.4 m</td>
<td>PECO sized, cost constrained</td>
<td>Astrometric accuracy goes as $D^2$,thanks to larger collecting area and smaller PSF size (assuming constant FOV)</td>
</tr>
<tr>
<td>Detector pixel size</td>
<td></td>
<td>44 mas</td>
<td>Nyquist at 600 nm</td>
<td>Little impact as long as sampling is close to or finer than Nyquist</td>
</tr>
<tr>
<td>Field of view (FOV)</td>
<td></td>
<td>0.29 sq deg (0.31 deg radius)</td>
<td>0.03 sq deg (0.1 deg radius)</td>
<td>low WF error across field, 1.6 Gpix detector</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Astrometric accuracy goes as $\text{FOV}^{-0.5}$</td>
</tr>
<tr>
<td>Single measurement time</td>
<td></td>
<td>48 hr</td>
<td>Typical single observation duration for coronagraph</td>
<td>Astrometric accuracy goes as $t^{-0.5}$</td>
</tr>
<tr>
<td>Dot coverage on PM (area)</td>
<td></td>
<td>1%</td>
<td>0.12%</td>
<td>Keeps thoughtput loss moderate in coronograph</td>
</tr>
<tr>
<td>Dot size / pitch (μm)</td>
<td></td>
<td>120 / 932 (black dots)</td>
<td>360 / 2800 (grey dots)</td>
<td>Dot diameter imposed by FOV</td>
</tr>
<tr>
<td>Flat field error after calibration, static (high spatial frequency)</td>
<td></td>
<td>1.02% RMS, 6% peak</td>
<td>Conservative estimate for modern detector after calibration</td>
<td>Negligible effect on background PSF measurement (well averaged with roll)</td>
</tr>
<tr>
<td>Flat field error, dynamic</td>
<td></td>
<td>0.1% RMS per pixel, uncorrelated spatially and temporally between observations</td>
<td>Undetected cosmic ray impacts on detector</td>
<td>Negligible effect on background PSF measurement, but effect on measurement of spikes locations</td>
</tr>
<tr>
<td>Telescope roll</td>
<td></td>
<td>0.33 rad (+/- 10 deg)</td>
<td>1.0 rad (+/- 28 deg)</td>
<td>Manageable PSF elongation at edge of FOV</td>
</tr>
<tr>
<td>Uncalibrated change in optics surface between observations for M2 &amp; M3</td>
<td></td>
<td>40 pm</td>
<td>Wavefront measurement repeatability (optical element removed / reinserted) obtained when testing similar sized optics on ground</td>
<td>Larger change in optics surface reduces astrometric accuracy</td>
</tr>
<tr>
<td>Static optics surface error (M3 mirror)</td>
<td></td>
<td>1.5 nm</td>
<td>WF error and PSD taken from similar existing optical element</td>
<td>Small impact on performance, as background PSFs are almost fixed between observations</td>
</tr>
<tr>
<td>Astrometric accuracy, single measurement, single axis, $m_v=3.7$, galactic pole</td>
<td></td>
<td>0.20 μas</td>
<td>0.63 μas</td>
<td>0.2 μas is required to achieve science goals</td>
</tr>
</tbody>
</table>
# Error budget (baseline system)

<table>
<thead>
<tr>
<th>Error on background stars position (photon noise, zodi, sampling)</th>
<th>Value</th>
<th>Assumption(s)</th>
<th>Mitigation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.128 (\mu)as</td>
<td>Galactic pole pointing</td>
<td>Reduce other terms (\rightarrow) brighter stars can be used with smaller photon noise</td>
</tr>
<tr>
<td>Detector flat field error (static)</td>
<td>0.033 (\mu)as</td>
<td>1% RMS, 6% Peak</td>
<td>Better calibration of detector flat field</td>
</tr>
<tr>
<td>Optical distortions (static)</td>
<td>0.083 (\mu)as</td>
<td>1.5 nm optics for M2 &amp; M3</td>
<td>Better optics</td>
</tr>
<tr>
<td>Detector distortions (static)</td>
<td>0.0153 (\mu)as</td>
<td>0.2% of pixel size</td>
<td>Project interference fringes on detector</td>
</tr>
<tr>
<td>Variation in detector flat field (dynamic)</td>
<td>0.0289 (\mu)as</td>
<td>0.1 % RMS</td>
<td>Calibrate flat field regularly</td>
</tr>
</tbody>
</table>
| Variation of optical distortions (dynamic)                   | 0.0629 \(\mu\)as | 40 pm on surfaces | More stable system
More light into spikes
Correlate distortions to temperature |
| Variation of detector geometry (dynamic)                      | 0.0755 \(\mu\)as | \(~20\) mK uncalibrated variations across FPA | More stable temperature control
Correlate temperature to distortions
Project fringes on detector |
| Photon noise on spikes                                       | 0.0478 \(\mu\)as | Includes zodi photon noise \(mV = 3.7\) star | More light on spikes |
| **TOTAL**                                                     | 0.20 \(\mu\)as | 0.2 \(\mu\)as Obtained with 0.29 sq deg FOV |

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Error budget: overview
(baseline system)

- Photon noise on background stars: 44%
- Detector flat field (static): 3%
- Optical distortions (static): 18%
- Detector distortions (static): 15%
- Variation in detector flat field (dynamic): 6%
- Variation of optical distortions (dynamic): 11%
- Variation of detector geometry (dynamic): 2%
- Photon noise measurement on spikes: 1%
Finite detector sampling, polychromatic PSF

Detector saturation

- Detector calibration
- Unstable optical system and focal plane array
- Poor detector calibration
- Stable optical system and focal plane array

Accuracy floor due to distortions & detector limits

- Shorter observation time
- Longer observation time

1.4m telescope
- 0.1 deg field radius
- (0.03 sq deg)
- Galactic pole
- 2 day observation

Single star photon noise limited error
- Photon noise limit, monochromatic PSF, infinite sampling, no background

Actual measurement error
- Measurement error (1-sigma)

Shorter detector readout
Performance as function of telescope size and FOV (baseline system)

1 % area coverage on PM

\( m_v = 3.7 \) target

Galactic pole observation

2 day per observation

Larger telescope diameter :
- more light in spikes \((D^2)\), finer spikes \((1/D)\) → spike calibration accuracy goes as \(D^2\)
- more light in background stars \((D^2)\), and smaller PSF \((1/D)\) → position measurement goes as \(D^2\)

**Astrometric accuracy goes as** \(D^{-2} \text{ FOV}^{0.5}\)

Number of pixels goes as \(D^{-2} \text{ FOV}\)

**At fixed number of pixels, larger D is better** (FOV can be reduced as \(D^{-4}\))

<table>
<thead>
<tr>
<th>FOV in sq deg</th>
<th>0.03</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D = 1.4 m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.62 (\mu)as</td>
<td>0.34 (\mu)as</td>
<td>0.22 (\mu)as</td>
<td>0.15 (\mu)as</td>
<td>0.11 (\mu)as</td>
</tr>
<tr>
<td><strong>D = 2.0 m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.30 (\mu)as</td>
<td>0.17 (\mu)as</td>
<td>0.11 (\mu)as</td>
<td>0.07 (\mu)as</td>
<td>0.053 (\mu)as</td>
</tr>
<tr>
<td><strong>D = 3.0 m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.14 (\mu)as</td>
<td>0.074 (\mu)as</td>
<td>0.047 (\mu)as</td>
<td>0.033 (\mu)as</td>
<td>0.023 (\mu)as</td>
</tr>
<tr>
<td><strong>D = 4.0 m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.076 (\mu)as</td>
<td>0.042 (\mu)as</td>
<td>0.026 (\mu)as</td>
<td>0.019 (\mu)as</td>
<td>0.013 (\mu)as</td>
</tr>
</tbody>
</table>
Possible Enhancements (requires changes to telescope design)

Main sources of error:
(1) **optical distortions** (static and dynamic)
   Why ? → because of high spatial frequencies in distortions which are not sampled by the spikes (they fall between the spikes)
(2) **variations in detector geometry**

**Apodizing the edges of M1** (mitigates issue #1)
→ beam walk effect on M2 and M3 becomes unable to produce distortions that change rapidly with sky position
→ astrometric calibration by spikes becomes much better

Issues:
   Small loss in throughput (few % ?)
   manufacturing of apodized edge on M1
   many issues to check (chromaticity, compatibility with coatings, etc..)

**Projecting interference fringes on the detector** (mitigates issue #2)
→ allows measurement of detector geometry any time

Issues:
   Takes time away from observation
   Complexity (add laser, fibers)

**Warning:** as instrumental errors become smaller, risk and impact of possible companions on field stars increase
small lens concentrates light on central spot
Collimated white light
movable (translation & rotation) opaque mask with array of small holes simulates field of stars
additional reflection on a movable mirror used to test sensitivity to field distortions
Temperature-controlled environment
lab demo at UofA (Bendek & Ammons)
Mirror with dots + pupil stop
Focal plane detector array
lab demo at UofA (Bendek & Ammons)
Preliminary work on mask manufacturing and lab demo at UofA (Bendek & Ammons)

Laser beam reflected on first mask prototype shows main beam (center) + fainter diffraction spots. Spot spacing increases with wavelength.
Mask prototype for lab experiment:
5um holes, 25um apart

NOTE:
For 1.4 m telescope, 0.3 sq deg FOV:
Dot diameter = 120 um
Dot spacing = 932 um
Surface Statistics:
Ra: 16.03 nm
Rq: 36.93 nm
Rz: 358.76 nm
Rt: 375.54 nm

Set-up Parameters:
Size: 640 X 480
Sampling: 97.76 nm

Processed Options:
Terms Removed:
Tilt
Filtering:
None

Title:
Note:
Conclusions

Coronagraphy, astrometry and wide field imaging can be combined for simultaneous observation without compromising performance. Robust concept, relatively insensitive to common astrometric instrumental errors.

→ rich science for both exoplanet science and general astrophysics
→ could be key to gain support and funding for large (>1m) space mission for spectroscopy of habitable exoplanets

Future work ...

- Lab testbed at UofA: demonstrate performance and algorithms, validate error budget
  - Test with coronagraph at NASA Ames and NASA JPL
  - We are investigating ground-based system and doing science with it
    (Funding from University of Arizona and NASA)

More info ...

Website (includes detailed error budget, algorithms, C source code):
http://www.naoj.org/staff/guyon/
→ research → coronagraphic astrometry
Transit spectroscopy
Transit photometry
Astrometry
Coronagraphy
"General" astrophysics
Backup slides
Combined solution derived from simultaneous coronagraphy and astrometry measurements

**Known variables:**
- **Star location** on the sky (effect of parallax is known except for star distance, aberration of light perfectly known)
- **Observing epochs**
- **Stellar mass** (assumed to be known at the 5% accuracy level)
- **Measurement noise levels** for astrometry (~μas), coronagraphy planet position (few mas) and star mass (~5%)

**Measurements**
- **Astrometry:**
  - star position (nb of variables = 2x #observations)
- **Coronagraphy:**
  - planet position (nb of variables = 2x #observations)

**Solution**
Maximum likelihood solution for 11 free parameters to be solved for:
- star parallax (1 variables)
- proper motion (2 variables)
- star mass (1 variable)
- planet mass (1 variable)
- orbital parameters (6 variables)
Combined solution from simultaneous coronagraphy and astrometry: method adopted to derive measurement accuracy

System defined by 11 free parameters to be solved for:
- star parallax (1 variable)
- proper motion (2 variables)
- star mass (1 variable)
- planet mass (1 variable)
- orbital parameters (6 variables)

Known variables:
- Star location on the sky (effect of parallax is known except for star distance, aberration of light perfectly known)
- observing epochs
- measurement noises on astrometry (~μas), coronagraphy planet position (few mas) and star mass (~5%)

Repeat $N>1$ times

Measurements:
**Astrometry**: star position (2x #observations)
**Coronagraphy**: planet position (2x #observations)
**Stellar mass**: derived from stellar luminosity (1)

Maximum likelihood solution for 11 parameters

Estimate error on each parameter separately (projection off all solutions on a single axis)
Study covariance between parameters
Better estimate of orbital parameters -> better planet mass estimate

This plot shows correlation between semi-major axis and planet mass in astrometry measurement:

- Smaller error in semi-major axis -> better mass estimate
Better estimate of stellar mass -> better planet mass estimate

This plot shows correlation between stellar mass and planet mass in astrometry measurement.

Direct measurement of stellar mass -> better planet mass estimate.
Benefits of simultaneous coronagraphy + astrometry

Coronagraph images provide an accurate measurement of the orbital parameters (more precise than astrometry), but no mass measurement. For a 1 MEarth planet on a 200mas radius orbit around a Sun-like star, a 5mas position measurement accuracy in the coronagraphic image (~1/10 lambda/D in the blue channel of PECO) = 1/40 orbit radius is equivalent to 0.015 uas astrometric precision.

Note: Position measurement in the coronagraphic image is unlikely to be better than ~5mas (even with >> 100 photon) due to unknown residual speckle field and exozodi structures.

Solving for planet orbit and mass using the combined astrometry + coronagraphy measurements is scientifically very powerful:
- **Reduces confusion with multiple planets.** Outer massive planets (curve in the astrometric measurement) will be seen by the coronagraph.
- Astrometry will **separate planets from exozodi clumps.**
- Astrometric knowledge allows to **extract fainter planets from the images, especially close to IWA**, where the coronagraph detections are marginal.
- Mitigates the **1yr period problem** for astrometry (see next slide).
Simultaneous coronagraphic imaging + astrometry mitigates the 1-yr period problem

**Problem:**
Astrometric signal of a planet in a yr period orbit is absorbed in the parallax term. With astrometry only, the mass estimate error grows as the planet period becomes closer to 1yr. The width of this “blind spot” is reduced with a longer mission life.

**Example:**
1 Earth mass planet at 1.01 AU from a Sun mass star (period = 1.015 yr) at ecliptic pole. Star distance is 6pc. Assuming circular orbits (for both the Earth and the target planet). Planet orbit phase = Earth orbit phase + 1 radian, orbit is face-on.
32 observations over 5 yr, regularly spaced

Astrometry only (0.3 uas error / per measurement):  
Mass estimate (unit = Earth) = 3.25485 +/- 4.17483 -> Planet is not detected

Astrometry (0.3 uas error / measurement) + imaging (5 mas / measurement):  
Mass estimate (unit = Earth) = 1.01314 +/- 0.161752
Observation scheme

Telescope pointings assumed to be driven by the coronagraph requirements

Coronagraph observes high priority targets frequently. For PECO (1.4-m diameter telescope), there are 20 high priority targets (= targets around which a super-Earth could be imaged).
Assuming 2-day single pointing, 5 yr total mission, and 70% of observing time devoted to high priority targets, each high priority targets is observed 32 times (2 day per observation).
The large number of revisits is required to mitigate confusion between planets and exozodi structures, multiple planets. It also allows planet brightness and spectra variations to be monitored.

Note: In reality, more time would likely be allocated to the ~7 targets around which PECO can observe an Earth, and less time for the other ~13 targets. When/if PECO detects an Earth candidate, the corresponding target would likely be observed more intensely.

For this study, it is assumed that the target is observed 32 times during a 5 yr mission. Observations last 2 days and are regularly spaced every 57 days.
Star field

Used “Besancon model” of the galaxy, with default 0.7 mag / kpc extinction. [http://model.obs-besancon.fr/](http://model.obs-besancon.fr/)

Star count computed for galactic pole on 1sq deg field. Then, sources are randomly distributed in the 1sq deg field.

In 1 sq deg field:
- 0 star brighter than $m_V = 10$
- 9 stars brighter than $m_V = 11$
- 23 stars brighter than $m_V = 12$
- 149 stars brighter than $m_V = 15$
- 1181 stars brighter than $m_V = 20$
- 5248 stars brighter than $m_V = 25$

Galactic pole is conservative assumption, but for most pointings, star count is within 40% of galactic pole star count.

<table>
<thead>
<tr>
<th>$m_V$</th>
<th>Galactic pole</th>
<th>50% percentile (30 deg galactic latitude)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&lt; 12</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>&lt; 15</td>
<td>149</td>
<td>205</td>
</tr>
<tr>
<td>&lt; 20</td>
<td>1181</td>
<td>1570</td>
</tr>
<tr>
<td>&lt; 25</td>
<td>5248</td>
<td>6861</td>
</tr>
</tbody>
</table>
Using galaxies for astrometry?

At $m_v \sim 20$, galaxies outnumber stars at high galactic latitude. Using background galaxies for astrometry should be possible (See for example “The absolute motion of the peculiar cluster NGC 6791” by Bedin et al. 2006 using HST)

Figure on the right shows galaxy counts next to galactic pole (Weir et al. 1995)
J filter $\sim 400$ to $500$ nm
F filter $\sim 650$ nm

**Sylos Labini et al., 2009**

*Fig. 19.* Differential counts of galaxies, in bins of $\Delta m = 0.25$, as a function of apparent magnitude in the SGC and NGC. A reference line corresponding to $N(m) \sim 10^{2m}$ with $\alpha = 0.5$ is reported.

*Fig. 7.* $r$- and $g$-band galaxy counts in our four fields. The sharp falloffs at the faint end are due to truncation of the catalog, by construction, beyond the reliable classification limit, rather than the intrinsic plate detection limit.
Astrometric error in the photon noise limit

For each star, pixel coordinate errors due to photon noise (star + zodi) and sampling are computed. Estimation uses a 2D polychromatic finely sampled PSF which is moved by a small amount and then binned to the pixel scale. The flux change for each pixel is compared to the noise, and all values are combined with SNR$^2$ weighting.

Simulation on the right shows the single axis astrometric error for a 2 day observation, 0.03 sq deg at galactic pole, Polychromatic PSF, Nyquist sampling detector at 0.6 μm, 80% optical throughput, 90% detector peak QE (0.36 μm effective bandwidth)

mV= 22.5 / sqarcsec zodi

Combined astrometric accuracy = 0.1265 μas
For a 0.25 sq deg (0.5 x 0.5 deg): 0.044 μas

A small number of bright stars ($m_V$) contribute to most of the measurement accuracy:
If only stars fainter than $m_V=17$ are included, accuracy = 0.46 μas
If only stars brighter than $m_V=17$ are included, accuracy = 0.1315 μas

Green points show theoretical 1D astrometric error:
$$\sigma = 0.318 (\lambda/D) / \sqrt{N_{\text{photon}}}$$

Red points show 1D astrometric error when zodi, PSF polychromaticity and pixel sampling are taken into account. The difference between the 2 curves is explained by an offset due to detector sampling and PSF polychromaticity (independent of star magnitude) + an increase in measurement error at the faint end due to zodiacal light photon noise.

Note: At high galactic latitude, extragalactic sources may be used to increase sensitivity?
Simulation description

Simulation assumes:
• 1.4m telescope TMA (Woodruff design)
• 1.5nm surface (3nm WF) optics for M2 and M3, PSD provided by Tinsley
• Circular field of view, 0.2 deg diam (0.03 sq deg) → performance then scaled to 0.29 sq deg to reach goal
• Galactic pole observation (worst case scenario)
• central star is $m_V = 3.7$ (faintest of the 7 PECO targets for which an Earth can be imaged in <6hr, 14th brightest target in the 20 high priority targets list)
• 90% detector peak QE, 80% optical throughput ($0.96^3$ for optics reflectivity x 0.92 due to dots on PM)
• Nyquist sampled detector at 0.6 micron = 44 mas pixels
• Telescope roll = 1 rad (larger angle = better averaging, but more difficult to maintain stability)
• Single epoch observation = 2 day

Distortions in the system are computed with 3D raytracing (code written in C, agreement with Code V results from Woodruff has been checked)
Images produced by Fourier transform, and then distorted according to geometrical optics. Image sizes are 16k x 16k.
Simulation description

Approach:
identify important sources of noise and errors
→ develop numerical simulation tool which accurately includes these noises

<table>
<thead>
<tr>
<th>Description</th>
<th>Included in numerical model?</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon noise and related effects</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N1 Photon noise (field stars)</td>
<td>yes</td>
<td>See Appendix</td>
</tr>
<tr>
<td>N2 Pixel sampling (field stars)</td>
<td>yes</td>
<td>See Appendix</td>
</tr>
<tr>
<td>N3 PSF polychromaticity (field stars)</td>
<td>yes</td>
<td>See Appendix</td>
</tr>
<tr>
<td>N4 Zodiacal light photon noise (field stars)</td>
<td>yes</td>
<td>See Appendix</td>
</tr>
<tr>
<td>N5 Central star and zodi photon noise</td>
<td>yes</td>
<td>Affects image of diffraction spikes</td>
</tr>
<tr>
<td>Astronomical terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N6 Central star proper motion</td>
<td>yes</td>
<td>fitted/removed in final measurements</td>
</tr>
<tr>
<td>N7 Central star parallax motion</td>
<td>yes</td>
<td>fitted/removed in final measurements</td>
</tr>
<tr>
<td>N8 Aberration of light</td>
<td>no</td>
<td>Effect is similar to, but much smaller than N6 + N7</td>
</tr>
<tr>
<td>N9 Companions around field stars</td>
<td>no</td>
<td>Averaged by large number of field stars</td>
</tr>
<tr>
<td>N10 Central star photometric variability</td>
<td>no</td>
<td>Small effect on spikes measurement</td>
</tr>
<tr>
<td>N11 Stellar spots and activity</td>
<td>no</td>
<td>Expected to be smaller than 0.1 µas</td>
</tr>
<tr>
<td>Detector (See section 6.1.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N12 Uncalibrated errors in detector flat field</td>
<td>yes</td>
<td>1% RMS, 6% peak</td>
</tr>
<tr>
<td>N13 Uncalibrated detector spectral response errors</td>
<td>absorbed in N12</td>
<td>Effect is absorbed in N12 numerical model</td>
</tr>
<tr>
<td>N14 Intra-pixel detector sensitivity variations</td>
<td>absorbed in N12</td>
<td>Effect is absorbed in N12 numerical model</td>
</tr>
<tr>
<td>N15 Uncalibrated detector geometry error</td>
<td>yes</td>
<td>Assumes unknown ≈ 20 mK temperature inhomogeneity variation</td>
</tr>
<tr>
<td>N16 Variations in pixel sensitivities over time</td>
<td>yes</td>
<td>Assumed to be at the 0.1% level (excludes calibration)</td>
</tr>
<tr>
<td>N17 Variations in detector geometry over time</td>
<td>yes</td>
<td>Effect is included in N16 numerical model</td>
</tr>
<tr>
<td>N18 Detector saturation</td>
<td>approximated</td>
<td>Field stars brighter than $m_V = 14$ are excluded</td>
</tr>
<tr>
<td>N19 Readout noise</td>
<td>no</td>
<td>exposure time chosen for photon-noise limited imaging</td>
</tr>
<tr>
<td>N20 Uncalibrated detector non-linearity</td>
<td>no</td>
<td>No significant contribution expected in final measurement</td>
</tr>
<tr>
<td>N21 Uncalibrated variable errors in readout timing</td>
<td>no</td>
<td>$&lt;0.01$ µas</td>
</tr>
<tr>
<td>Telescope and optics (See section 6.1.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N22 Telescope pointing jitter</td>
<td>no</td>
<td>negligible impact if below diffraction limit</td>
</tr>
<tr>
<td>N23 Telescope roll angle errors</td>
<td>no</td>
<td>fitted and removed from final data</td>
</tr>
<tr>
<td>N24 Uncalibrated primary mirror surface errors</td>
<td>no</td>
<td>negligible impact if below diffraction limit</td>
</tr>
<tr>
<td>N25 Uncalibrated M2 and M3 mirrors surface errors</td>
<td>yes</td>
<td>PSD of manufactured optics used for simulation</td>
</tr>
<tr>
<td>N26 Telescope alignment errors</td>
<td>no</td>
<td>known terms fitted, residual $&lt;&lt;$ µas</td>
</tr>
<tr>
<td>N27 Plate scale error</td>
<td>no</td>
<td>Less than 1e-3 µas</td>
</tr>
<tr>
<td>N28 Local random errors in dot positions and size</td>
<td>no</td>
<td>Non-significant if position error $&lt; 10µm$</td>
</tr>
<tr>
<td>N29 Non-uniformity in dots coverage</td>
<td>no</td>
<td>Removed from measurement thanks to roll</td>
</tr>
</tbody>
</table>
Input errors and instrument characteristics

Dynamic distortions

- Telescope size, pupil mask dots geometry
- Polychromatic PSF
- Monochromatic PSF
- PSF angular derivative
- PSF x derivative
- PSF y derivative
- Optics x and y distortion change
- Angular distortion change
- Total x and y distortion change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux
- Detector flat field error
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface errors
- Surface change

Static distortions

- Variation in pupil mask dots geometry
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface errors
- Surface change

Data simulation

- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- PSF angular derivative
- PSF x derivative
- PSF y derivative
- Optics x and y distortion change
- Angular distortion change
- Total x and y distortion change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux
- Detector flat field error
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface errors

Data analysis

- Binned angular distortion signal
- Residual astrometric distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction
- 2-D astrometric measurement

Target and field stars

- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)
- Central star brightness
- Central star proper motion & parallax
- Field stars (positions and brightnesses)

Simulated data

- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Reference image of spikes (Epoch #2)
- Measured astrometric distortion map in angular direction
- Binned angular distortion signal
- Binned square SNR per pixel for a 1 pixel angular distortion
This series of slides describes in more detail each step of the numerical simulation. A red square is shown in the overall simulation description diagram to indicate which part of the simulation is being described.

Green text label next to boxes show the image or file name used in the source code, to help read the source code.
Input errors and instrument characteristics

**Dynamic distortions**
- Telescope size, pupil mask dots geometry
- Monochromatic PSF
- Polychromatic PSF
- Telescope size, pupil mask dots geometry
- PSF angular derivative
- PSF x derivative
- PSF y derivative
- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Measured astrometric distortion map in angular direction
- Binned angular distortion signal
- SNR^2-weighted binning
- SNR-weighted convolution by anisoplanatism sized kernel
- Binned angular distortion measurement
- Residual astrometric distortion error map in angular direction
- Roll averaging
- Roll averaged residual distortion error map in angular direction
- Roll averaged astrometric measurement
- Optimized weighting of all 1-D measurements

**Static distortions**
- Detector flat field error
- Total x and y distortion change
- Angular distortion change
- Optics x and y distortion change
- Static angular distortion
- Total x and y distortion change
- Central star brightness
- Central star proper motion & parallax
- Field stars (positions and brightnesses)
- Field stars position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)
- Target and field stars

**Data simulation**
- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- FPS angular derivative
- FPS x derivative
- FPS y derivative
- Optics x and y distortion change
- Static angular distortion
- Total x and y distortion change
- Central star brightness
- Central star proper motion & parallax
- Field stars (positions and brightnesses)

**Data analysis**
- Sum of two or more images
- Difference between two images
- Product of two images (pixel by pixel)
- Image name (used throughout this document)
- Operation performed on images or data

**Data simulation**
- SNR per pixel for a 1 pixel angular distortion
- FPS angular derivative
- FPS x derivative
- FPS y derivative
- Optics x and y distortion change
- Static angular distortion
- Total x and y distortion change
- Central star brightness
- Central star proper motion & parallax
- Field stars (positions and brightnesses)

**Simulated data**
- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Measured astrometric distortion map in angular direction
- Binned angular distortion signal
- SNR^2-weighted binning
- SNR-weighted convolution by anisoplanatism sized kernel
- Binned angular distortion measurement
- Residual astrometric distortion error map in angular direction
- Roll averaging
- Roll averaged residual distortion error map in angular direction
- Roll averaged astrometric measurement
- Optimized weighting of all 1-D measurements
PM mask

Hexagonal pattern dots. Dots cover 1% of PM surface. Dots are assumed to be perfectly placed, all with same size.

[note: for mission, dot diameter = 72 um; spacing = 0.5 mm]

Dots are assumed to be totally black. Dots do not affect coronagraph if they are regularly spaced (no low spatial frequency)
Monochromatic PSF

Central part of PSF is not disturbed by dots

Full field PSF (0.2 deg on a side) shows 2D grid of diffraction orders
Input errors and instrument characteristics

Dynamic distortions
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux

Static distortions
- Detector flat field error
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface errors
- Raytracing

Data simulation
- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- Polychromatic PSF
- Reference image of spikes (Epoch #1)
- Monochromatic PSF
- PSF x derivative
- PSF y derivative
- Total x and y distortion change
- Optics x and y distortion change
- Angular distortion change
- Variation in detector flat field response
- Photon noise due to zodiacal light and field star flux

Simulated data
- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- SNR weighted convolution by anisoplanatism sized kernel
- SNR^2 weighted binning
- Binned angular distortion signal
- Residual astrometric distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction
- Roll averaging
- Optimal weighting of all 1-D measurements

Data analysis
- Sum of two or more images
- Difference between two images
- Product of two images (pixel by pixel)
- Image name (used through this document)
- Operation performed on images or data

Target and field stars
- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)
- Central star proper motion & parallax
- Field stars (positions and brightnesses)
- Central star brightness

Optics x and y distortion

Monochromatic PSF

Polychromatic PSF

PSF angular derivative

PSF x derivative

PSF y derivative

Total x and y distortion change

Optics x and y distortion change

Angular distortion change

Variation in detector flat field response

Photon noise due to zodiacal background and central star flux

Detector flat field error

Mirrors M2 and M3 surface errors

Focal plane array x and y distortion change

Photon noise error due to zodiacal light and field star flux

Field stars (positions and brightnesses)

Central star proper motion & parallax

Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)

Central star brightness

Background stars position measurement errors between 2 epochs (1-D angular coordinate)

Final target and field stars

2-D astrometric measurement

Sum of two or more images

Difference between two images

Product of two images (pixel by pixel)

Image name (used through this document)

Operation performed on images or data

Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)

Central star proper motion & parallax

Field stars (positions and brightnesses)

Central star brightness

Summary
Polychromatic PSF

Computed as incoherent sum of 5000 monochromatic PSFs: 50 individual FFTs x 100 radial stretch steps

Central part of PSF

Full field PSF (0.2 deg on a side) shows thin narrow spikes
Polychromatic PSF

Brightest part of spikes is $\sim 1\times 10^{-8}$ of central PSF peak
Over most of the field, surface brightness is dominated by zodiacal light, not by spikes.

Scattering by PM surface roughness is much fainter than the spikes, as spikes diffract $\sim 1\%$ of starlight.

Central pixel has 17% of total flux
Static distortions

Definition: Any error static through the mission lifetime.

Why do purely static errors matter?

Background PSFs follow different trajectories during the telescope roll for different observation epochs. The trajectories are close (~arcsecond level), so what matters is the differential astrometric distortion over a ~1” distance.

Main errors:
• Distortions due to optical figure of mirrors M2 and M3
• Focal plane array geometry: position of individual detector chip & variations in pixel size across the detector
• Non-calibrated flat field errors

Impact and mitigation:
Static errors are not calibrated by the diffraction spikes:
- lack of absolute reference for spikes makes it impossible to calibrate static errors (where should the spikes be in a perfect system?)
- spikes can only calibrate low order distortions, but relevant static errors are small scale errors
Input errors and instrument characteristics

- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux
- Photon noise error due to zodiacal light and field star flux
- Detector flat field error

Dynamic distortions

- Polychromatic PSF
- Monochromatic PSF
- Total x and y distortion change
- Angular distortion change
- Optics x and y distortion change
- Variation in detector flat field response

Static distortions

- Focal plane array x and y distortion
- Mirrors M2 and M3 surface errors

Data simulation

- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)

Simulated data

- PSF angular derivative
- PSF x derivative
- PSF y derivative
- PSF angular derivative
- Monochromatic PSF
- Polychromatic PSF
- Total x and y distortion change
- Angular distortion change
- Optics x and y distortion change
- Variation in detector flat field response

Data analysis

- Sum of two or more images
- Difference between two images
- Product of two images (pixel by pixel)
- Image name (used through this document)
- Operation performed on images or data

- Binned square SNR per pixel for a 1 pixel angular distortion
- Binned angular distortion signal
- Binned square SNR per pixel
- SNR^2-Weighted binning
- SNR-weighted convolution by anisoplanatism sized kernel
- Binned angular distortion measurement
- Roll averaging
- Roll averaged residual distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction

Field stars

- Central star brightness
- Central star proper motion & parallax
- Field stars (positions and brightnesses)

Background stars position measurement errors between 2 epochs (1-D angular coordinate)

Target and field stars

- 2-D astrometric measurement
- Optimal weighting of all 1-D measurements
Static distortion map due to M2 & M3 optical surfaces

Distortion maps shown below is for 0.46x0.46 deg field. Unit is arcsec; left map is x, right map is y. Distortion map is computed at 220000 positions on the sky with raytracing software written in C (cross-checked with code-V), then interpolation is used to compute the full map. Total number of rays used = 7e11 (122 day CPU of execution time on 2 GHz CPUs)

Distortion amplitude is ~1 mas, dominated by low order modes. The differential distortion over ~1” is much smaller.
**Data simulation**

- Telescope size, pupil mask dots geometry
- Mirrors M2 and M3 surface change
- Focal plane array x and y distortion change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux

**Data analysis**

- Sum of two or more images
- Difference between two images
- Product of two images (pixel by pixel)

**Simulated data**

- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Residual astrometric distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction
- Binned angular distortion signal
- Binned square SNR per pixel for a 1 pixel angular distortion

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- Product of two images (pixel by pixel)
Static distortion map due to uncalibrated focal plane array geometry

Distortion maps shown for 0.2 x 0.2 deg. Due to pixel size non-uniformity residual after ground/in orbit calibration of detector. Spatial frequencies chosen here put most power in between spikes and at ~arcsec separation (worst case)

~2/1000 pixel amplitude = 90 μas
left: x, right: y. Unit = pixel (44 mas)
Total static distortion map

Angular coordinate distortion (perp. to spikes) map shown for 0.2 x 0.2 deg. Unit = pixel (44 mas)

distortion is +/- 1 mas approximately
Input errors and instrument characteristics

Dynamic distortions

- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux
- Detector flat field error

Static distortions

- Focal plane array x and y distortion
- Mirrors M2 and M3 surface errors

Data simulation

- Distortion measurement SNR per pixel for a 1 pixel angular distortion
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- Field stars (positions and brightnesses)

Simulated data

- PSF angular derivative
- PSF x derivative
- PSF y derivative
- Total x and y distortion change
- Angular distortion change
- Optics x and y distortion change
- Monochromatic PSF
- Polychromatic PSF
- sum

Data analysis

- Sum of two or more images
- Difference between two images
- Product of two images (pixel by pixel)
- SNR^2-weighted binning
- Binned angular distortion signal
- Binned angular distortion map in angular direction
- Residual astrometric distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction
- 2-D astrometric measurement

Target and field stars

- Field stars position measurement errors between 2 epochs (1-D angular coordinate)
- Optimal weighting of all 1-D measurements
Static uncalibrated flat field error

1% random error + lines and columns
error is +/-6% peak, 1.02% RMS
Flat field knowledge requirement

- With 0.2 deg diam, 1 rad roll, measurement is done over ~100 stars x 3000 independent positions (separated by more than l/D) on the detector = 3e5 measurements

- 0.2 uas = 1/200000 pixel -> allowed error (if not correlation) is <1/500 pixel ~ 1% error on flat field at small scales (pixel to pixel)

- Astrometric error due to pixel-to-pixel flat field errors is strongly anticorrelated along the PSF track on the detector-> averages closer to 1/N than 1/sqrt(N) -> flat field knowledge errors of a few % should be OK (see next slides)
Detector static errors are expected to be very small in the roll-averaged angular coordinate.

Combined measurement error is ~0 in angular coordinate.

PSF size measurement error is not measured in the radial coordinate (not measured).

Angular coordinate:
- Dark Pixel (flat field error <0)
- Bright Pixel (flat field error >0)

The PSF true trajectory is marked by a green dashed line, while the measured trajectory is marked by a red dashed line.
Dark Pixel (flat field error <0)

Bright Pixel (flat field error >0)

PSF size

combined measurement error is ~0 in angular coordinate

90 deg

radial coordinate (not measured)

angular coordinate

measured trajectory

PSF true trajectory

measurement error
Input errors and instrument characteristics

**Dynamic distortions**
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux

**Static distortions**
- Detector flat field error
- Focal plane array x and y distortion
- Mirrors M2 and M3 surface errors
- Raytracing

**Data simulation**
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux

**Data analysis**
- Binned angular distortion signal
- Binned astrometric distortion map in angular direction
- Roll averaged residual distortion error map in angular direction
- Optimal weighting of all 1-D measurements

**Simulated data**
- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Measured astrometric distortion map in angular direction
- Residual astrometric distortion error map in angular direction
- Roll averaged residual distortion error map in angular direction
- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)

**Target and field stars**
- Field stars (positions and brightnesses)
- Central star proper motion & parallax
- Central star brightness
- Optical distortion change
- Optics x and y distortion change
- Total x and y distortion change
- Statistics
- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- SNR per pixel for a 1 pixel angular distortion
- Roll averaging
- Field stars position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)
Numerical simulation of astrometric error due to flat field errors

Step 1: pre-compute how a single pixel sensitivity error “pulls” the estimated PSF position (= astrometric error kernel for a single pixel error).
This is done at 0.1 l/D step size, over 10 l/D radius: for each 2-D offset (within 10 l/D radius, with 0.1 l/D step) between the PSF center location and the “bad” pixel, compute the error in PSF position measurement in x and y. Computation uses finely sampled PSFs binned down to the detector sampling.

Maps on the right show how a sensitivity error in a single pixel affects the PSF position measurement.

Maps are normalized to the relative pixel sensitivity error. Unit is l/D. Peak value is 0.05: a 1% sensitivity error can move the PSF measured position by 0.0005 l/D = 44 uas

Step 2: For each roll angle and star, compute 2-D PSF position error by summing all errors due to pixels sensitivity errors within a 10 l/D radius of actual PSF position. This computation uses the maps shown above: for each pixel, the fractional offset between the pixel and the PSF is computed, and the corresponding error values (x and y) are derived from bilinear interpolation of the maps computed in step 1.
Flat field errors are strongly anticorrelated with roll angle -> they average as $1/N$ instead of $1/\sqrt{N}$.

Figure on the left shows 1-D astrometric error for a single star as a function of roll angle. The raw error (brown) is $\sim 1e-3$ l/D RMS ($\sim 0.1$ mas). The roll-averaged error (red) goes as $1/\text{roll angle}$.
Astrometric error due to flat field errors is \( \sim 0.5 \text{ uas per star} \) for a 1 rad roll. Error is stronger for stars closer to the optical axis (less roll averaging).
Single star astrometric error due to flat field errors shows no obvious time correlation in this example (1 arcsec / yr proper motion). With smaller proper motion and more distant stars (small parallax), correlation is expected over two timescales: time for proper motion to move star by 1 pixel, and 1 year period due to parallax.
Intra-pixel sensitivity errors are captured in this analysis

Unknown variations of sensitivity within a pixel show the same anti-correlation behavior, and are captured in this analysis.

Example: top half of a pixel less sensitive than bottom half
If PSF is below the pixel, PSF position error is positive
If PSF is above the pixel, PSF position error is negative

A small error in sensitivity between pixels is similar to a larger error within a pixel.

Intra-pixel sensitivity errors can be simulated by the same analysis as shown here, but with a finer sampling.
Dynamic distortions

**Definition:** Any change between observations epochs
These changes introduce errors in the measured position of background stars or on the distortion change measured by the spikes image.

**Description of main error terms:**
- Variation in the optical shape of mirrors M2 and M3 due to thermal and mechanical stresses introduces astrometric distortions that change between the observation epochs
  - Rigid body motion of optics (telescope alignment)
  - Focal plane array geometry: motion and distortion of individual detector chip due to temperature fluctuation and mechanical stress
  - Variations in the flat field response of the detector

**Impact and mitigation:**
Low order components of dynamic errors are calibrated by the diffraction spikes. To measure how distortions change between observations, the motion of the spikes is measured by comparison of the spike images between the different observation epochs.
Errors in this estimate come from
- photon noise (spikes, zodi)
- changes in the pixel response between the 2 epochs
- interpolation between spikes (no signal between spikes)
Time-variable distortions are not perfectly estimated by the spikes -> astrometric error
Time-variable distortions: M2 and M3

Thermal variations in substrate + mirror mounting:
On 150-350mm apertures, better than 0.1nm RMS wavefront insertion repeatability with 0.25 C temperature stability. (Jay Daniel, L-3 Tinsley, private communication)
Assuming 100mK temperature stability -> 40 pm RMS stability

Material creep:
probably slow process (timescale > single observation) which can be tracked during course of mission by averaging distortions over several consecutive observations. -> not included

x and y astrometric distortions due to change in the shape of optics is shown on the left.

Same as static optical distortions, but scaled by 3%. Unit = pixel (44 mas)
Detector array distortions

A temperature change on a 4k detector changes its linear size by 0.0172 pixel / K, assuming Si (CTE=4.2e-6). This is simulated by a low order term in x distortion with +/- 1e-3 pixel and period ~ single 4k detector size. Translation between detector chips not included here - would need to be fitted as a translation for each chip.

Temperature variations have no effect if homogeneous. We assume here a 20mK non-calibrated variation in the homogeneity of the detector temperature between observations.

(NOTE: this error could be mitigated by projecting laser fringes on the detector)

Unit = pixel (44 mas)
Input errors and instrument characteristics

Dynamic distortions
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response

Static distortions
- Detector flat field error
- Focal plane array x and y distortion
- Mirrors M2 and M3 surface errors

Data simulation
- Distortion measurement SNR per pixel for a 1 pixel angular distortion
- PSF angular derivative
- Polychromatic PSF
- Monochromatic PSF
- PSF x derivative
- PSF y derivative
- Total x and y distortion change
- Optics x and y distortion change
- Angular distortion change

Simulated data
- Reference image of spikes (Epoch #1)
- Measured spikes image, (Epoch #2)
- Measured astrometric distortion map in angular direction
- Residual astrometric distortion error map in angular direction

Data analysis
- Binned angular distortion signal
- Binned angular distortion measurement
- Roll averaging
- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)
- Optimal weighting of all 1-D measurements

Target and field stars
- Field stars (positions and brightnesses)
- Central star proper motion & parallax
- Central star brightness

Image name (used through this document)
- Sum of two or more images
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Total angular distortion change

Unit = pixel (44 mas)
Amplitude ~ 1/1000 pixel (44 uas)
Flat field change between epochs

Detector response map changes between observation by 1e-3 (RMS)
This will produce an error in the measurement of spikes displacements.

Even if the spikes are steady (no distortion), a distortion will be measured.
Data simulation

- Distortion measurement SNR per pixel for a 1 pixel angular distortion

- PSF angular derivative

- Polychromatic PSF

- Monochromatic PSF

- PSF x derivative

- PSF y derivative

- PSF angular derivative

- Telescope size, pupil mask dots geometry

- Focal plane array x and y distortion change

- Mirrors M2 and M3 surface change

- Variation in detector flat field response

- Photon noise due to zodiacal background and central star flux

- Detector flat field error

- Focal plane array x and y distortion

- Mirrors M2 and M3 surface errors

Data analysis

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Data analysis

- Binned square SNR per pixel for a 1 pixel angular distortion

- SNR^2-weighted binning

- SNR-weighted convolution by anisoplanatism sized kernel

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- Binned angular distortion measurement

- Residual astrometric distortion error map in angular direction

- Roll averaging

- Roll averaged residual distortion error map in angular direction

- Target and field stars

- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)

- Central star brightness

- Central star proper motion & parallax

- Field stars (positions and brightnesses)

- Background stars position measurement errors between 2 epochs (1-D angular coordinate)

- 2-D astrometric measurement

- Optimal weighing of all 1-D measurements
Spikes image, 0.2 deg FOV

Spike image is computed by:

step 1: compute derivative in x and y for the spikes

step 2: multiply derivative by x and y distortion maps

step 3: add noise terms (photon noise, readout noise, flat field noise)
Central part of the field is blocked by the coronagraph pickup mirror.

The spikes do not extend inward to the coronagraphic field.
Photon noise from spikes and zodiacal light are visible in this frame.

Spikes are I/D wide
The overall size of the spike envelope, the spikes density (spacing between spikes) and brightness can be chosen by design of the dot pattern.
Distortion measurement

Compute SNR for a 1 pixel angular distortion for each pixel -> SNRmap
Compute signal (unit = pixel of angular distortion) for each pixel = difference between ideal spike image and measured spike image, divided by dImage/dDistortion -> Signalmap

To speed up computation, Signalmap and SNRmap are binned to lower resolution (with optimal weights derived from SNRmap)
Distortion measurement

\[ \text{SNR}^2 \]

binned Signal (using \( \text{SNR}^2 \) weighting within each bin)
Value set to zero where SNR is below threshold
Distortion interpolation

Convolve signal \( x \) \( \times \ln\text{SNR}^2 \) by gaussian kernel, with sigma of the kernel \( \sim \) anisoplanatism patch size

Problem: next to a bright spike, the solution will give a flat value with a sharp jump when moving to the next spike.
Estimate for each pixel the effective centroid of the result (different from the pixel location), and the local slope of the distortion \( \rightarrow \) using these 2 quantities, correct for the centroid offset error.
Distortion interpolation

for(ii=0;ii<size*size;ii++)
distarray[ii] = 0.0;

for(ii0=0;ii0<size;ii0++)
for(jj0=0;jj0<size;jj0++)
{
v = 0.0;
vv = 0.0;
x = 0.0;
y = 0.0;
vt = 0.0;
vcent = 0.0;
vycnt = 0.0;

for(kk=0;kk<NBpt;kk++)
{
ii = iarray[kk]-ii0;
jj = jarray[kk]-jj0;
x = 1.0*ii*SLA_pixscale*binfoct; // radian
y = 1.0*jj*SLA_pixscale*binfoct; // radian
r2 = x*x+y*y;
r2 /= SLA_corr_aniso_rad*SLA_corr_aniso_rad;
if(r2<9.0)
{
v += varray[kk]*snr2array[kk]*exp(-r2);
vt += x*snr2array[kk]*exp(-r2);
vcent += snr2array[kk]*exp(-r2);
}
if(vcent > eps)
{
v /= vcent;
vt /= vcent; // effective x
vt /= vcent; // effective y
}
}

for(kk=0;kk<NBpt;kk++)
{
ii = iarray[kk]-ii0;
jj = jarray[kk]-jj0;
x = 1.0*ii*SLA_pixscale*binfoct; // radian
y = 1.0*jj*SLA_pixscale*binfoct; // radian
r2 = x*x+y*y;
r2 /= SLA_corr_aniso_rad*SLA_corr_aniso_rad;
if(r2<9.0)
{
vx += [varray[kk]-v]*snr2array[kk]*(x-xt)*exp(-r2);
vvy += [varray[kk]-v]*snr2array[kk]*(y-yy)*exp(-r2);
vxcent += snr2array[kk]*(x-xt)*exp(-r2);
vycnt += snr2array[kk]*(y-yy)*exp(-r2);
}
if(vxcent>eps)
{
vx /= vxcent;
if(vycent>eps)
{
vx *= vycent;
}
}
}

v -= xt*vx;
v -= yt*vy;
distarray[jj0*size+ii0] = v;
Input errors and instrument characteristics

Dynamic distortions
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux
- Detector flat field error
- Focal plane array x and y distortion
- Mirrors M2 and M3 surface errors

Static distortions
- Polychromatic PSF
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- PSF angular derivative
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Data simulation
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- Data analysis
- Sum of two or more images
- Difference between two images
- Product of two images (pixel by pixel)
- Bi-linear interpolation
- Roll averaging
- Optimal weighting of all 1-D measurements
- 2-D astrometric measurement
- Data simulation
- Simulated data
- Target and field stars
- Field stars position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)
- Central star brightness
- Central star proper motion & parallax
- Field stars (positions and brightnesses)
Distortion interpolation

Sigma = 15"

True distortion

Measured distortion

Unit = pixel
Input errors and instrument characteristics

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- Photon noise error due to zodiacal light and field star flux

**Static distortions**
- Detector flat field error
- Focal plane array x and y distortion
- Mirrors M2 and M3 surface errors
- Raytracing

**Data simulation**
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- Field stars position measurement error for each star, roll angle, and observation epoch
- Excludes dynamic distortions
- Optimal weighing of all 1-D measurements
- 2-D astrometric measurement

**Data analysis**
- Binned angular distortion signal
- Binned square SNR per pixel for a 1 pixel angular distortion
- SNR^2-weighted binning
- SNR-weighted convolution by anisoplanatism sized kernel
- Binned angular distortion measurement
- Residual astrometric distortion error map in angular direction
- Roll averaging
- Optimal weighing of all 1-D measurements
- 2-D astrometric measurement

**Target and field stars**
- Field stars position measurement errors between 2 epochs (1-D angular coordinate)
- Optimal weighing of all 1-D measurements
- 2-D astrometric measurement
Residual distortion after calibration is \( \sim 1e-4 \text{ pix} = 4.4 \, \mu\text{as} \)
This is 10x smaller than original distortion, and residual is mostly free of low order -> will average well with telescope roll.
Input errors and instrument characteristics

Dynamic distortions
- Telescope size, pupil mask dots geometry
- Focal plane array x and y distortion change
- Mirrors M2 and M3 surface change
- Variation in detector flat field response
- Photon noise due to zodiacal background and central star flux

Static distortions
- Detector flat field error
- Focal plane array x and y distortion
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- Roll averaging
- SNR^2-weighted binning
- Signal binning
- SNR-weighted convolution by anisoplanatism sized kernel
- Binned angular distortion signal
- Binned angular distortion
- Bilinear interpolation
- Field star position measurement error for each star, roll angle, and observation epoch (excludes dynamic distortions)

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Astrometric error due to distortion changes (after roll)

Unit = arcsec
RMS ~ μas

This map is obtained by roll-averaging the distortion map in the previous slide.

Error tends to be smaller for stars further out (more averaging thanks to roll)
Final astrometric error

For each star, 1-sigma error is computed as quadratic sum of:
- pixel coordinate error (due to photon noise)
- distortion errors (derived from 2D distortion map)
- flat field error on detector
Then, optimally combine all measurements by weighting according to astrometric SNR$^2$ for each star.

**Final astrometric 1 sigma error in this example:**
0.63 uas per axis (1-sigma) for 0.03 sq deg (= 0.1 deg radius circular field)

0.2 μas per axis would require 0.29 sq deg (= 0.31 deg radius)
Note: scaling to larger FOV needs to be done more carefully - this is just a rough estimate
Detector saturation

Detector readout

poor detector calibration
unstable optical system and focal plane array

good detector calibration
stable optical system and focal plane array

shorter observation time

longer observation time

accuracy floor due to distortions & detector limits

finite detector sampling, polychromatic PSF

zodi background

1.4m telescope
0.1 deg field radius
(0.03 sq deg)
galactic pole
2 day observation

single star photon noise limited error
photon noise limit, monochromatic PSF, infinite sampling, no background
actual measurement error
measurement error (1-sigma)
Existing mechanical positioning accuracy

Key issue for coronagraphic performance is placement accuracy of dots and their size uniformity.

High precision CMM: ~1 um over PECO PM seems possible
Example:
- NIST Moore 48 CMM: typical error is 130nm absolute + 200 nm per m = 0.4 um on PECO PM

http://www.cenam.mx/cmu-mmc/Evento_2007/Presentaciones/John_Stoup-
High_accuracy_CMM_measurements_at_NIST.pdf
Work plan

Astrometry modeling and analysis

Establish realistic astrometric performance estimate

Identify manufacturing challenges

Develop & test data acquisition and data reduction approaches

Validate model, when applicable

Manufacture mask for astrometry lab demo

Mask manufacturing

Balance error budget with manufacturing constraints

Explore scaling to full scale primary mirror

Astrometry laboratory demonstration

Manufacture mask for coronagraph demo

Demonstration with coronagraph

Assess compatibility with high contrast coronagraphy

Requirements on mask quality imposed by coronagraph
UofA astrometry

Dotted mirror fabrication
- mirror #2
- mirror #3 (for coronagraph) or #4 (stitching)
- scaling to full-size PM: technology readiness assessment

UofA astrometry testbed
- design
- procurement of key components, assembly & setup
- mirror #3 test
- mirror #4 test
- mirror #5 test

Coronagraph test at NASA Ames
- design
- procurement of components
- setup
- coronagraphic measurement
- establish requirements for dotted mirror

Modeling & simulations
- astrometry
- Preliminary astrometric error budget
- Final astrometric error budget, requirements & technology readiness assessment