Fundamental trade-offs between IWA, contrast, and tip/tilt error for Segmented Apertures

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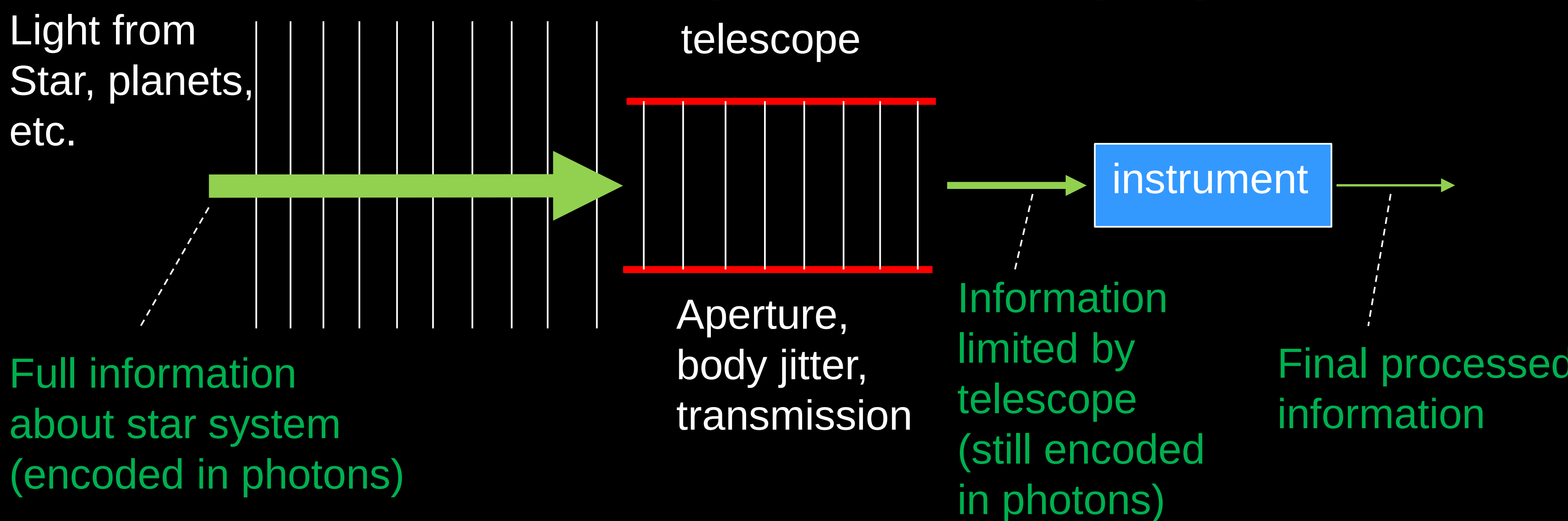
Information-theoretic view of coronagraphic imaging



- Information is lost by
  - Passing through the telescope
  - Passing through the instrument
- As long as mission costs are driven by the telescope, there will be economic pressure to improve instruments (rather than the telescope), until they are close to “lossless”, or “ideal”
  - Corollary: future telescopes will have close to ideal coronagraphs (20 years?)
  - We can predict their instrument performance without knowing the details of the coronagraph

Roadmap to physics-limited performance

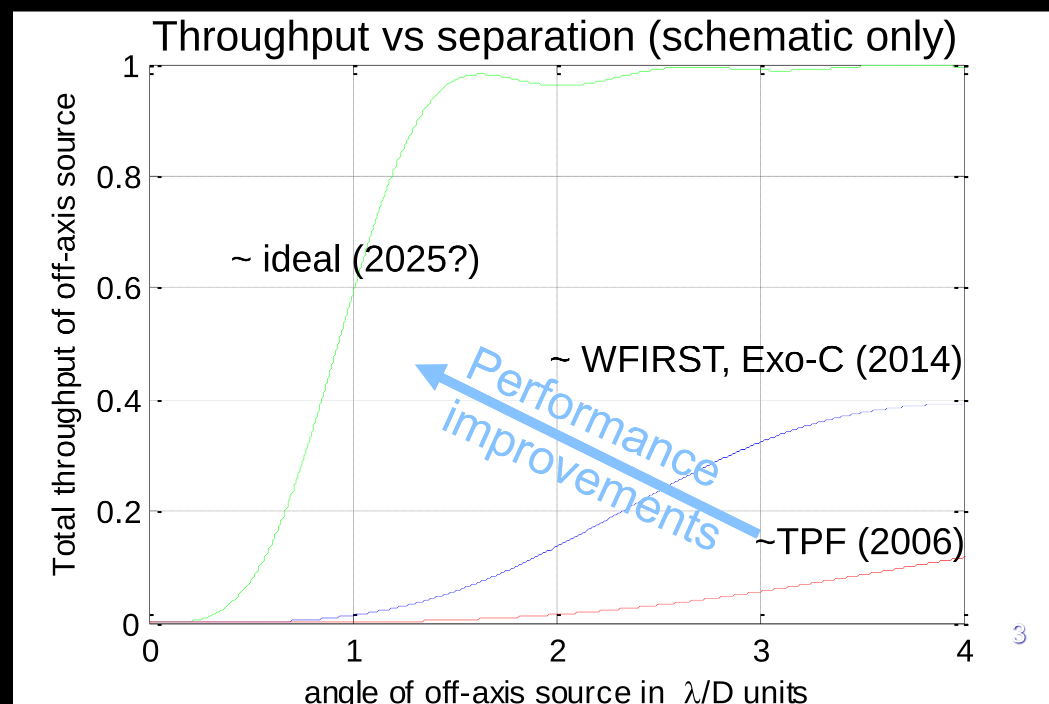
Current coronagraphs

Soluble engineering challenges

Increasing coronagraph performance

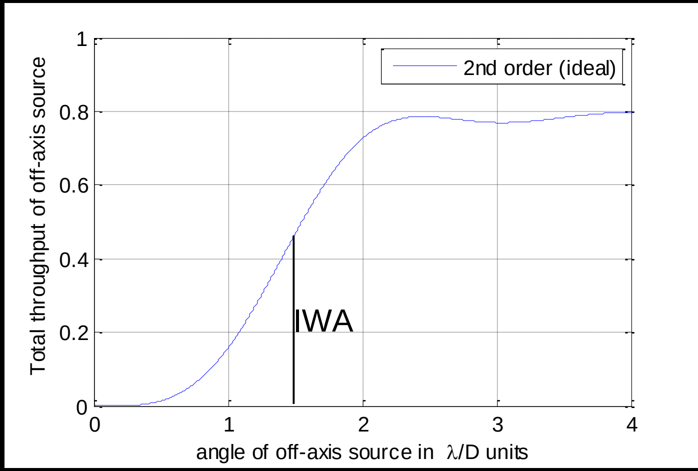
Fundamental information limit due to telescope (“ideal coronagraph”)

- Current top-down approach:
  - Start with many real coronagraph designs
  - Evaluate performance for each one
  - Try to improve them, without knowing how far you can go
- Proposed bottom-up way of thinking:
  - Start with an (abstract) ideal coronagraph limited by fundamental physics only (for a given telescope)
  - Evaluate its performance
  - See how far real coronagraphs are from it and in what ways
  - Try to bridge the gap

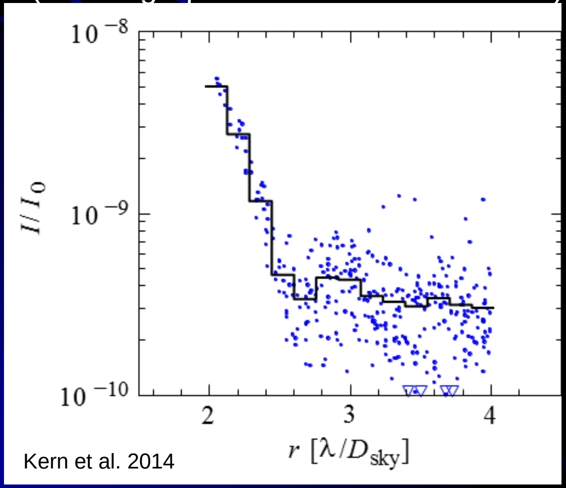


Different ways of looking at coronagraph performance

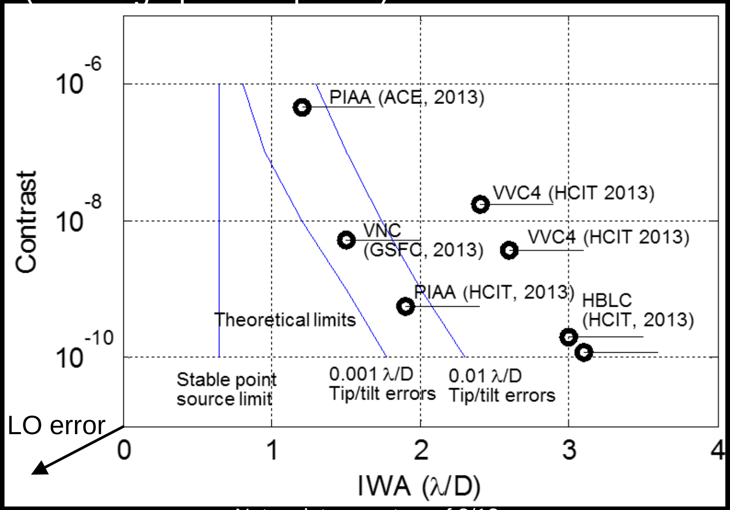
1. Throughput vs angle
(coronagraphs are curves)



2. Contrast vs angle
(coronagraphs are curves or scatters)



3. Contrast vs IWA (vs low order error level)
(coronagraphs are points)



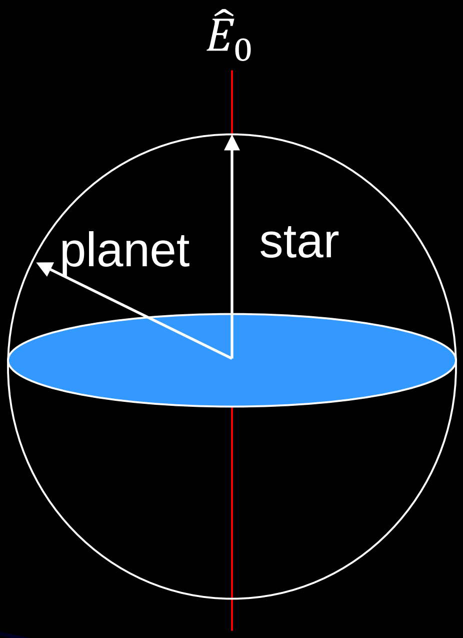
Note: plot current as of 8/13

- LO errors (esp. tip/tilt) is a key parameter coupled to IWA and contrast
- Bandwidth and maximum throughput do not seem to be fundamentally limited (i.e. with sufficiently advanced technology, can be 100%)

Focus on a simpler piece of the problem

- Consider the trade between 3 parameters: IWA, contrast, and low order errors (e.g. telescope jitter)
- Guyon et al. 2006 established that coronagraphic IWA is fundamentally limited, and this limit depends on stellar size and low order errors
- What exactly is this fundamental trade-off between IWA and sensitivity to aberrations? Can we express it with a compact formula?
- How close are existing coronagraphs to this fundamental trade-off? How much room for improvement is there in existing architectures?

Linear algebra representation of coronagraphs



$E_{star}(x,y)$ → Coronagraph (+WFC) → 0

$E_{planets}(x,y)$ → Coronagraph (+WFC) → $E_{planets}(x,y)$

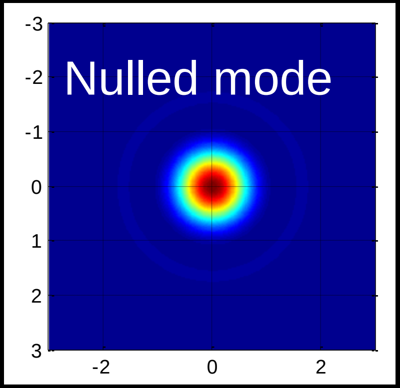
Coronagraph (+WFC)

$$E_{in}(x,y) = \sum a_i \hat{E}_i(x,y) = \begin{matrix} a_0\hat{E}_0(x,y) & \rightarrow \times \lambda_0 \rightarrow & b_0\hat{E}_0(x,y) \\ + a_1\hat{E}_1(x,y) & \rightarrow \times \lambda_1 \rightarrow & + b_1\hat{E}_1(x,y) \\ + a_2\hat{E}_2(x,y) & \rightarrow \times \lambda_2 \rightarrow & + b_2\hat{E}_2(x,y) \\ \ldots & \ldots & \ldots \end{matrix} = E_{CCD}(x,y)$$

a x C = b

(based on Guyon et al. 2006)

"Ideal" (2nd-order) Coronagraph

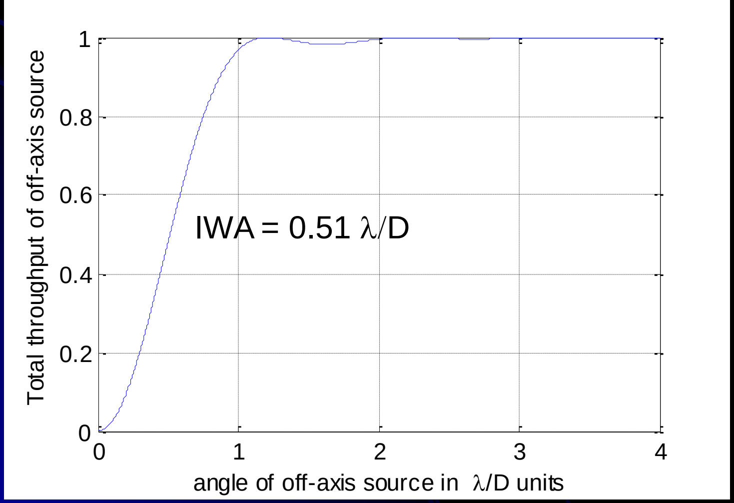


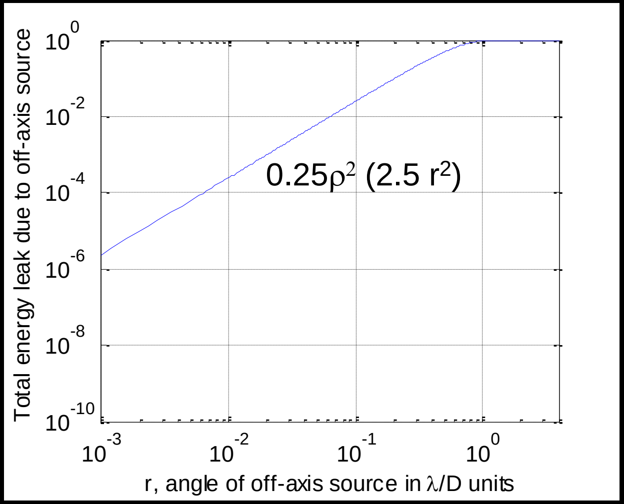
$$\hat{E}_0(\rho) = \frac{2\,J_1(\rho)}{\rho} \text{ (Airy pattern)}$$
$$= 1 - \frac{1}{8}\rho^2 + \frac{1}{192}\rho^4 + o(\rho^6)$$

($\rho = \pi r$, where r is in units of $f\lambda/D$)

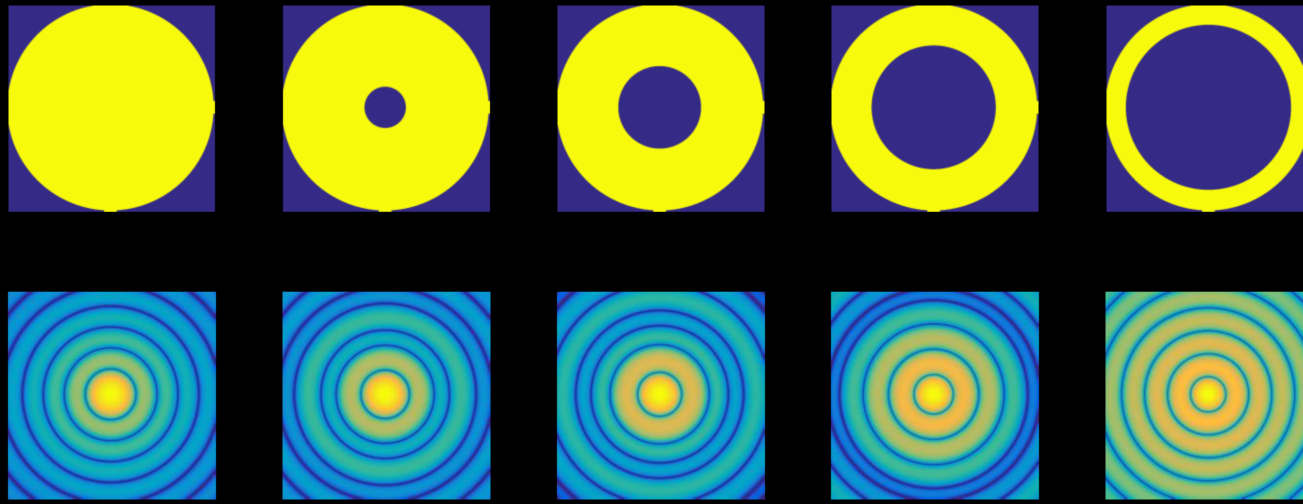
Coronagraph matrix:
$\lambda_0 = 0$
all other $\lambda_i = 1$

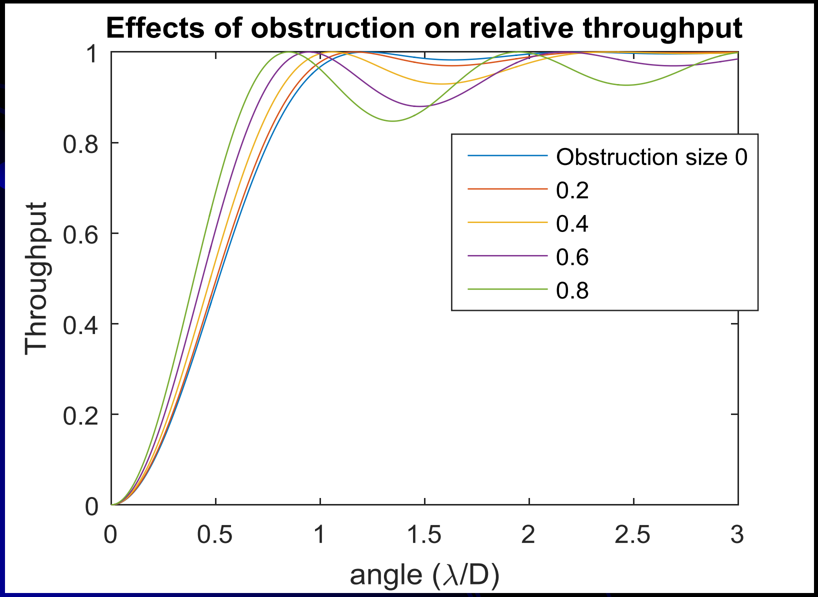
Total throughput for off-axis source: $$\|\Delta E_{CCD}\|^2 = 1 - \hat{E}_0(\rho)^2$$
$$= 1 - \frac{4\,J_1^2(\rho)}{\rho^2}$$
$$= \frac{1}{4}\rho^2 - \frac{5}{192}\rho^4 + o(\rho^6)$$



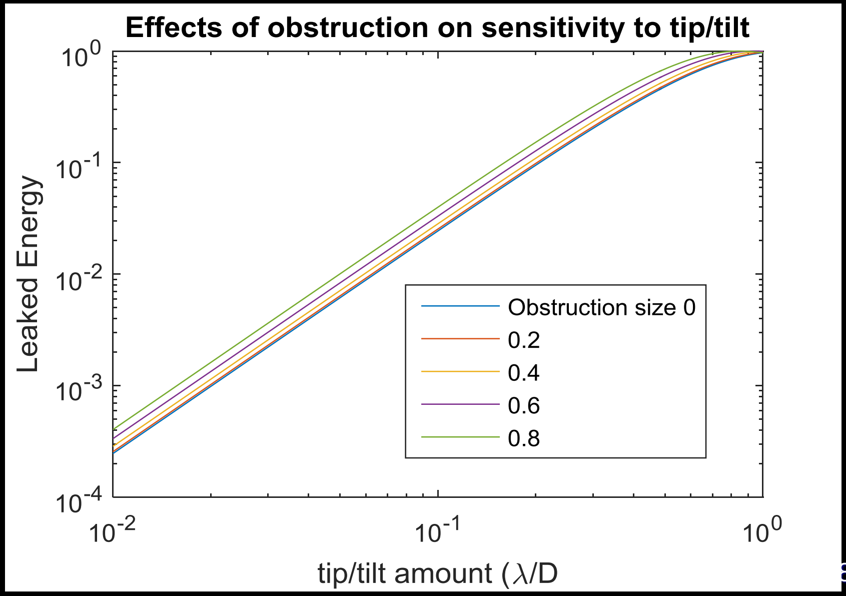


Does obstruction affect ideal coronagraph performance?



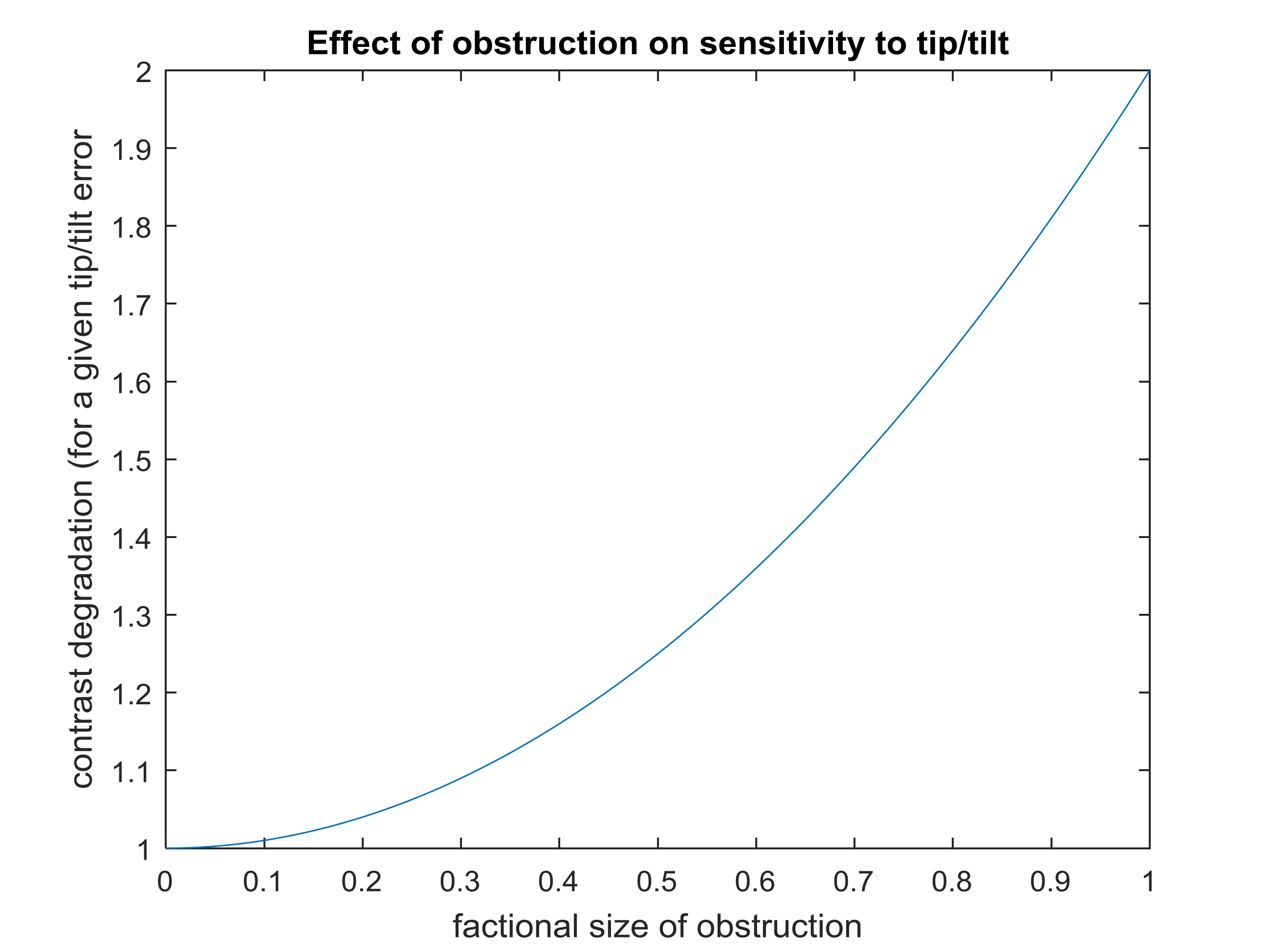


IWA gets more aggressive



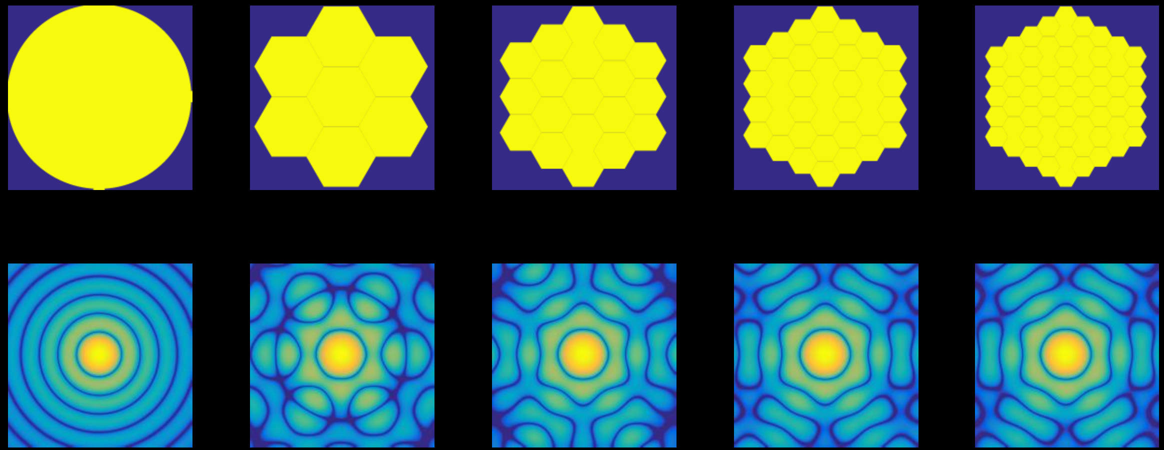
Sensitivity to tip/tilt gets slightly worse

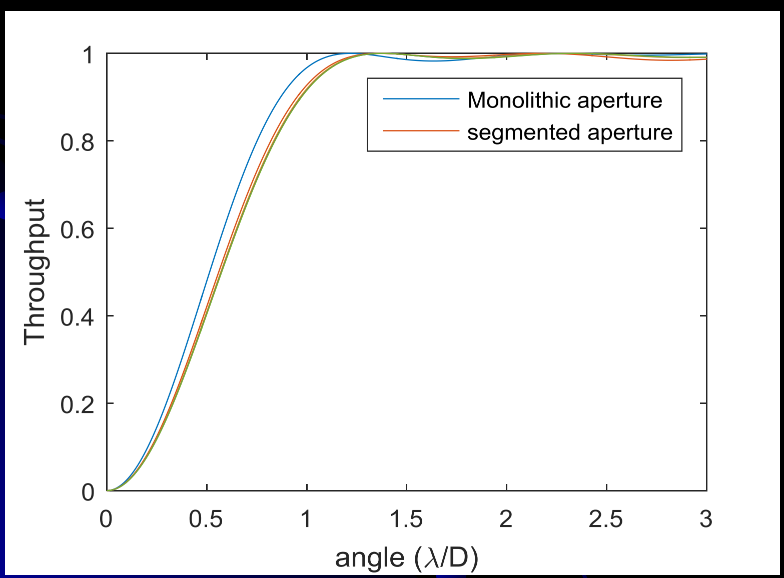
Sensitivity to tip/tilt as a function of obstruction size

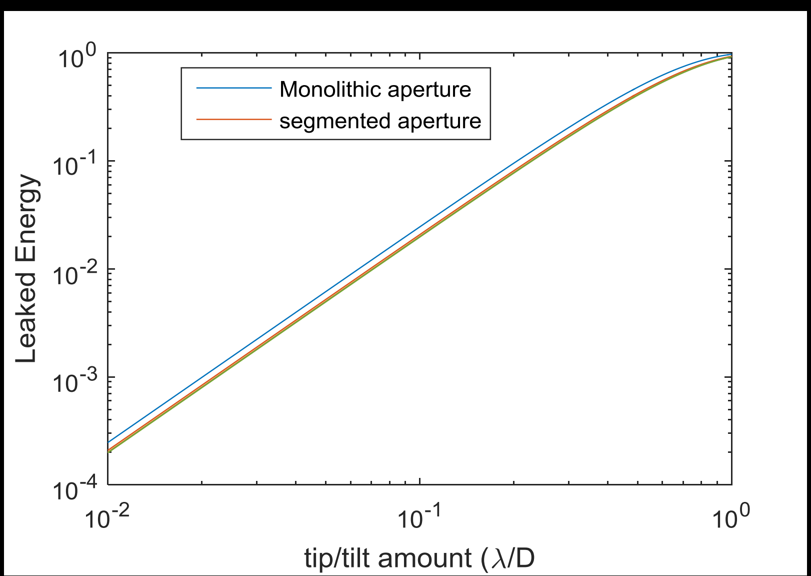


(relation can be derived analytically: contrast degradation = $1 + f^2$)

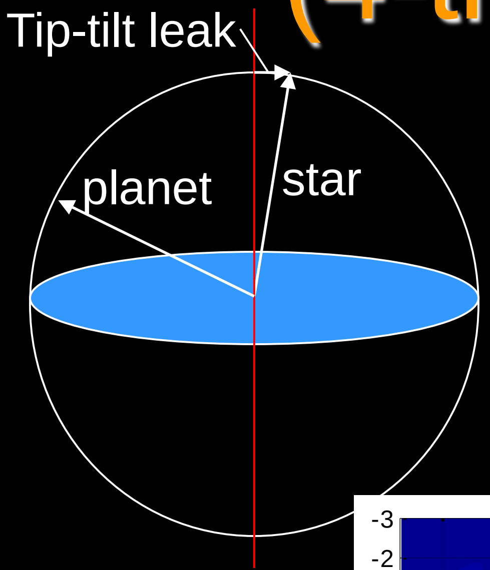
Effects of segmentation

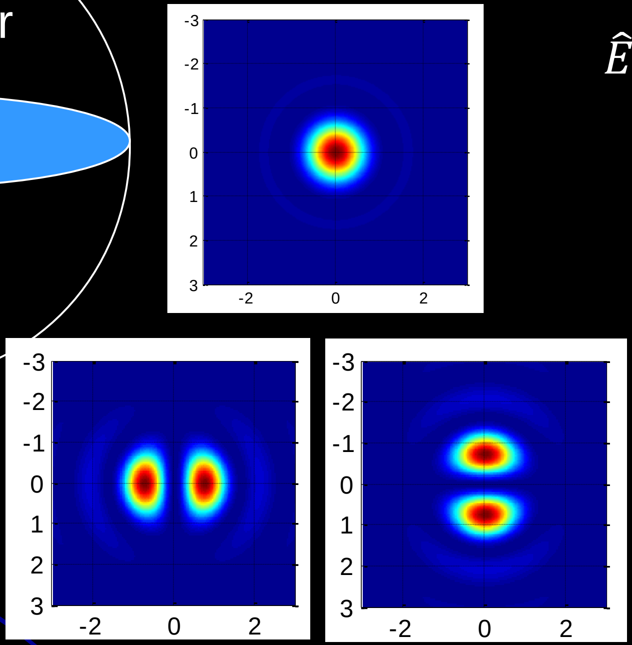






Ideal “tip-tilt insensitive” (4-th order) coronagraph





Nulled modes

$$\hat{E}_0(\rho) = \frac{2\,J_1(\rho)}{\rho} \text{ (Airy pattern)}$$
$$= 1 - \frac{1}{8}\rho^2 + \frac{1}{192}\rho^4 + o(\rho^6)$$

$$\hat{E}_{1,x}(\rho,\phi) = 2\frac{\partial}{\partial x}\hat{E}_0(\rho) = 2\hat{E}_0'(\rho)\cos(\phi)$$

$$\hat{E}_{1,y}(\rho,\phi) = 2\frac{\partial}{\partial y}\hat{E}_0(\rho) = 2\hat{E}_0'(\rho)sin(\phi)$$

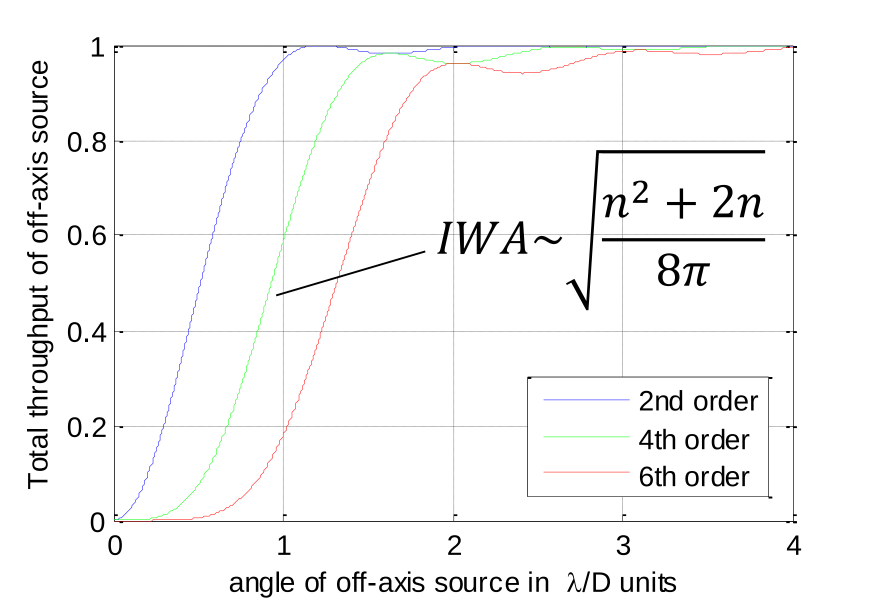
where $\hat{E}_0'(\rho) = 4\frac{J_0(\rho)}{\rho} - 8\frac{J_1(\rho)}{\rho^2}$
$$= -\frac{1}{2}\rho + \frac{1}{24}\rho^3 + o(\rho^5)$$

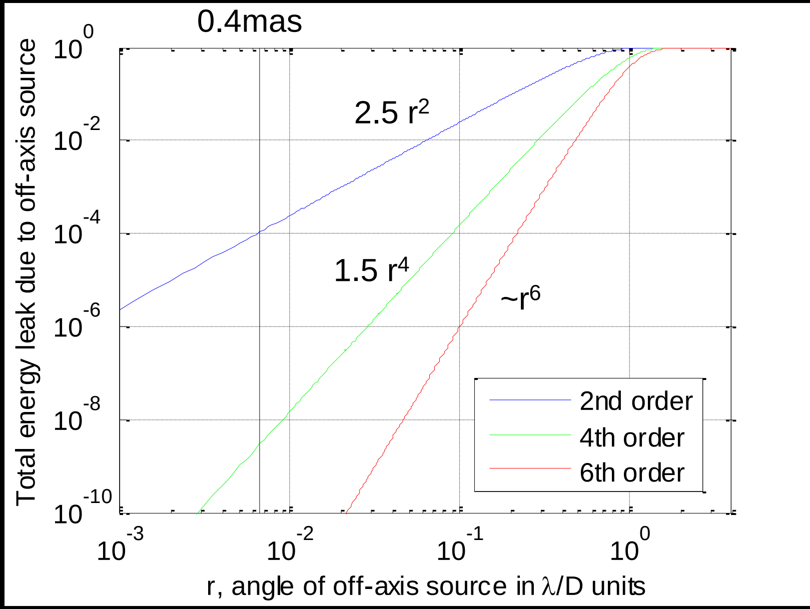
Coronagraph matrix:
$\lambda_0, \lambda_{1,x}, \lambda_{1,y} = 0$
all other $\lambda_i = 1$

Ideal "tip-tilt insensitive" (4-th order) coronagraph

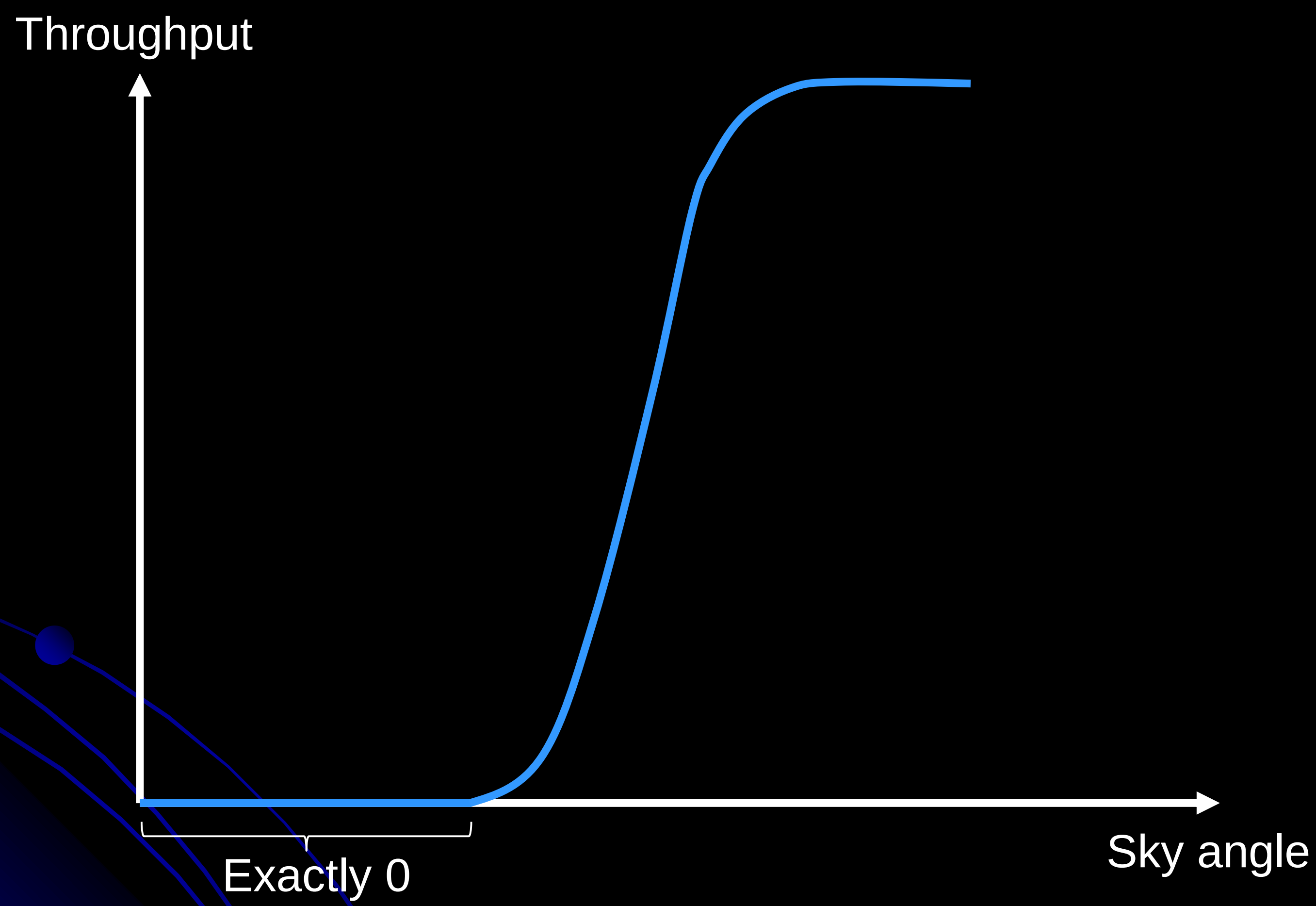
Total throughput for off-axis source (after some algebra):

$$\|\Delta E_{CCD}\|^2 = 1 - \hat{E}_0^2(\rho) - \hat{E}_1^2(\rho)$$
$$= 1 - \frac{4\,J_1^2(\rho)}{\rho^2} - \left(4\frac{J_0(\rho)}{\rho} - 8\frac{J_1(\rho)}{\rho^2}\right)^2$$
$$= \frac{1}{64}\rho^4 + o(\rho^6)$$





Is it possible to have an infinite-order null?



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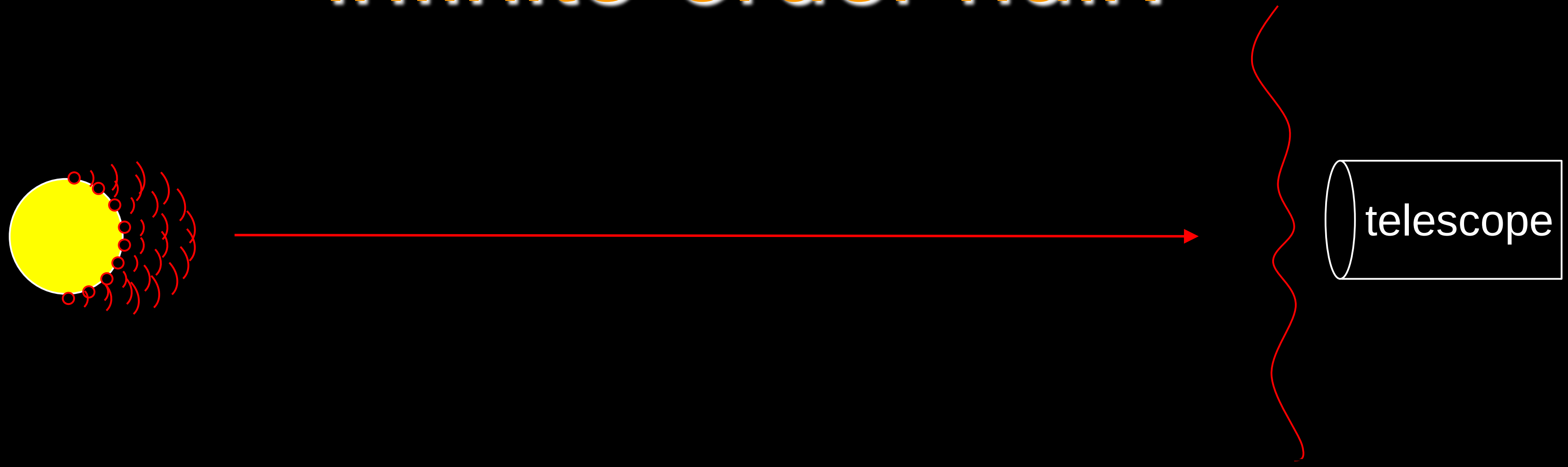


- A star is equivalent to an incoherent array of fibers (arbitrarily many and arbitrarily small)
- Mathematical 0 throughput on star means 0 throughput on each fiber separately and therefore any coherent superposition of them
- Phasing the fibers and controlling their light levels, we can in theory generate an arbitrary field at the aperture of the telescope, (e.g. one that is indistinguishable from a planet).
- Therefore throughput on all planets (and everything else) will also be 0.

IWA, Contrast, and aberration sensitivity trades for ideal coronagraph

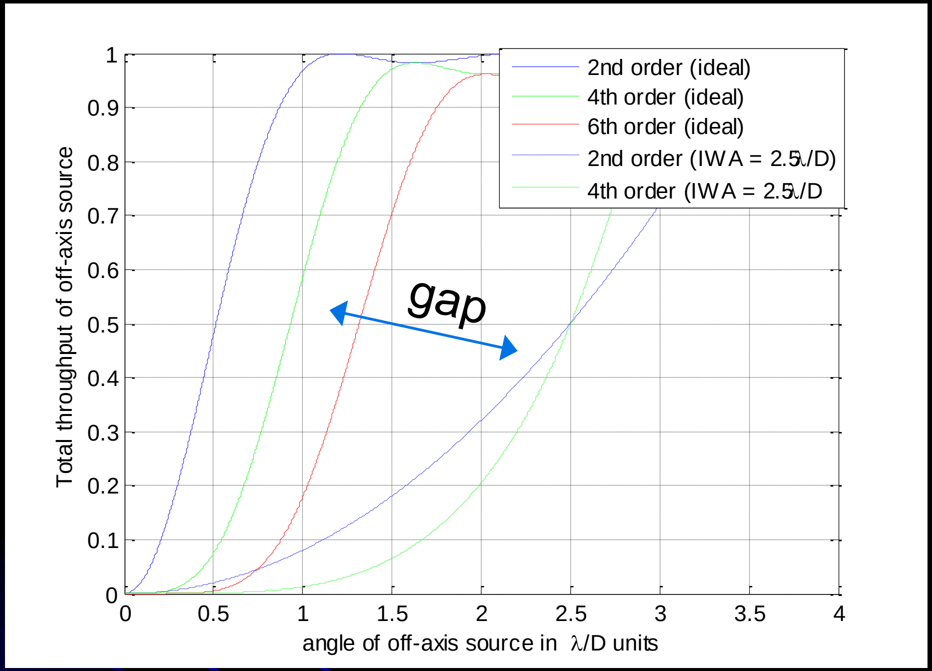
- For an ideal coronagraph of n-th order,
  - $$IWA \sim \sqrt{\frac{n^2+2n}{8\pi}}$$
    - Meaning: "blind spot" area in units of $(\lambda/D)^2$ is equal to the number of blocked modes
    - n-th order ideal coronagraph blocks an additional n/2 modes compared to n-1st order
  - Tip/tilt sensitivity: $Contrast = C\, r^n$, where
    - $C = o(1)$ is a constant
    - *r* is the amount of tip/tilt error in units of λ/D
- Eliminating order *n* leads to fundamental limit:
  - $$Contrast \sim r^{\sqrt{8\pi\, IWA^2+1}-1}$$

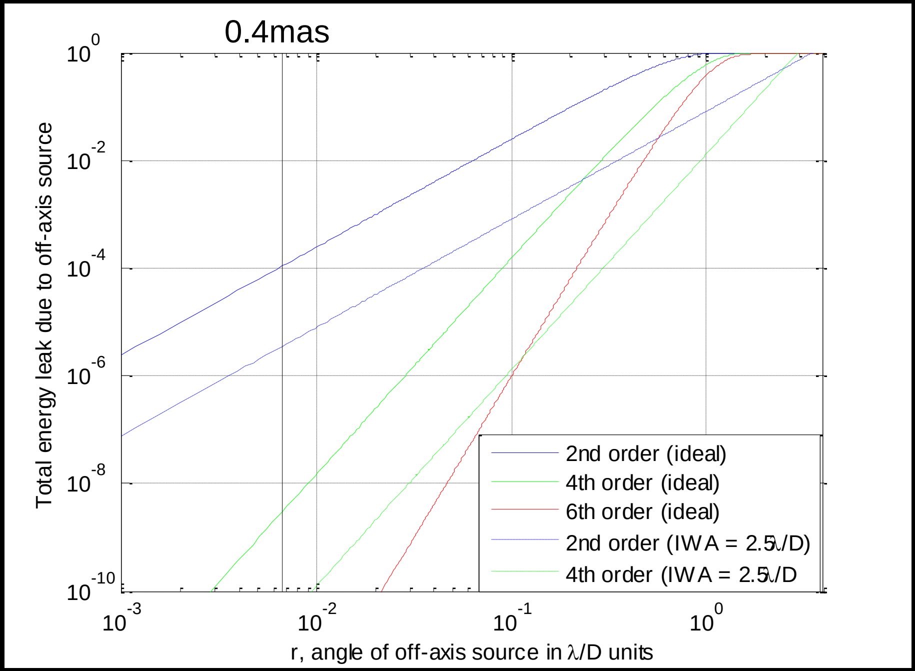
Numerical trade examples
(for D = 2.4m, unobstructed)

IWA (λ/D)	r: tip/tilt error	Contrast	n (order)
1	0.4 mas	3e-9	4
2.2	7mas	1e-10	10

- At 0.4 mas, can in principle achieve 1 l/D IWA (increasing science yield by a factor of 3-10?)
- At 2.2 l/D IWA, can tolerate uncorrected jitter of 7mas

Comparison to "real" coronagraphs





- Substantial gap remains between existing designs and fundamental limits
- Investments in coronagraph technology can bridge this gap, enabling cost savings on telescope

Conclusions

- IWA, contrast, and LO errors are fundamentally coupled, defining a limiting boundary in coronagraph performance space
- These limits are roughly similar for segmented and monolithic telescopes, and do not strongly depend on obstruction.
- Reaching those limits is more challenging for segmented telescopes, but we can probably assume that eventually coronagraphs will be limited by physics rather than engineering.

BACKUP CHARTS

Trade-offs for PIAA