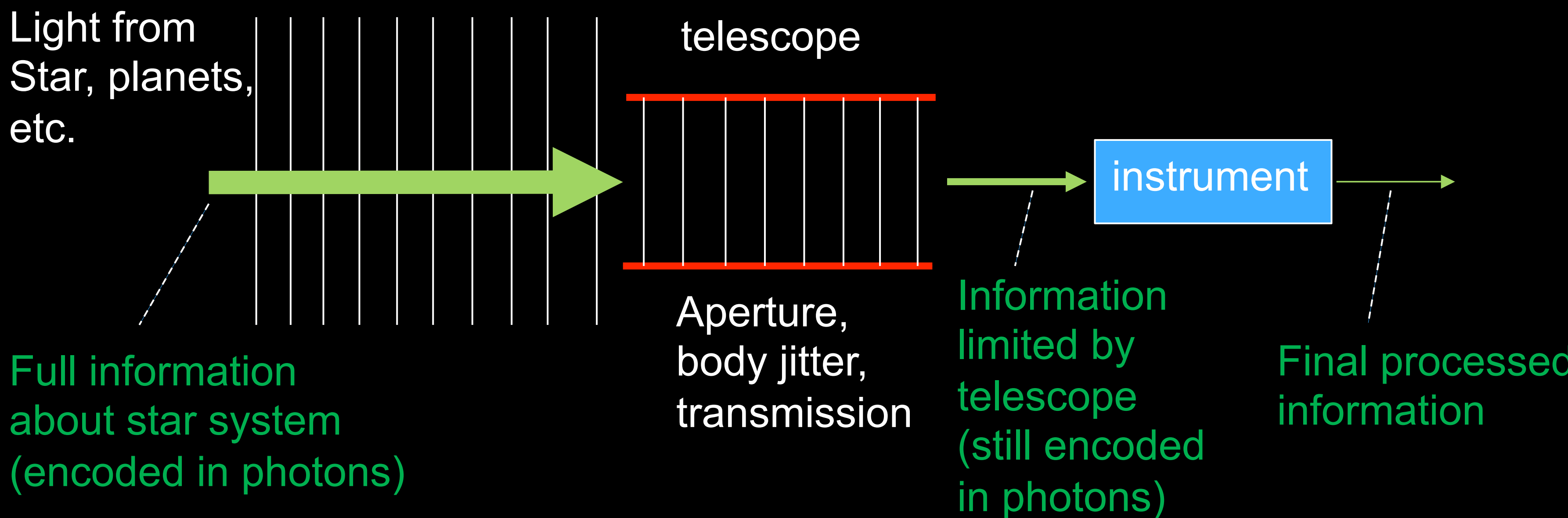
Fundamental trade-offs between IWA, contrast, and tip/tilt error

Ruslan Belikov

ExoPAG 10, Boston, MA, 6/6/2014

Information-theoretic view of coronagraphic imaging



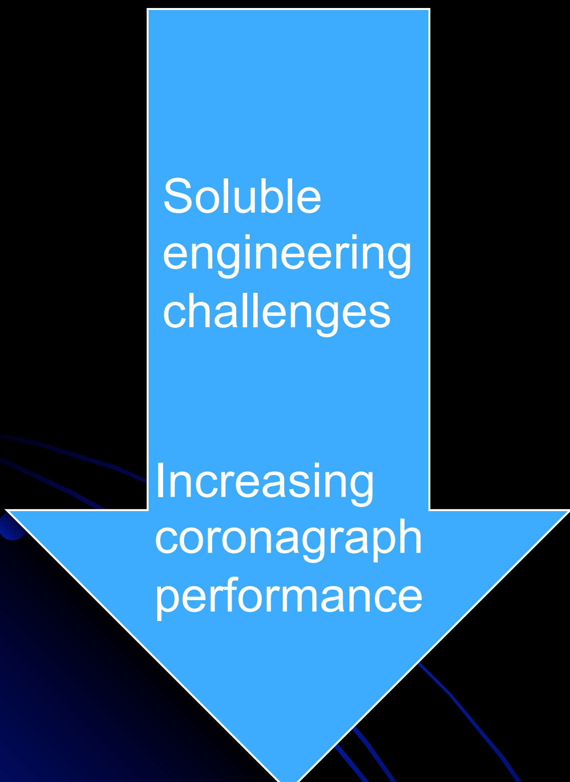
- Information is lost by
  - Passing through the telescope
  - Passing through the instrument
- As long as mission costs are driven by the telescope, there will be economic pressure to improve instruments (rather than the telescope), until they are close to “lossless”, or “ideal”
  - Corollary: future telescopes will have close to ideal coronagraphs (20 years? Maybe even 10?)
    - We can predict their instrument performance without knowing the details of the coronagraph

Proposed bottom-up approach

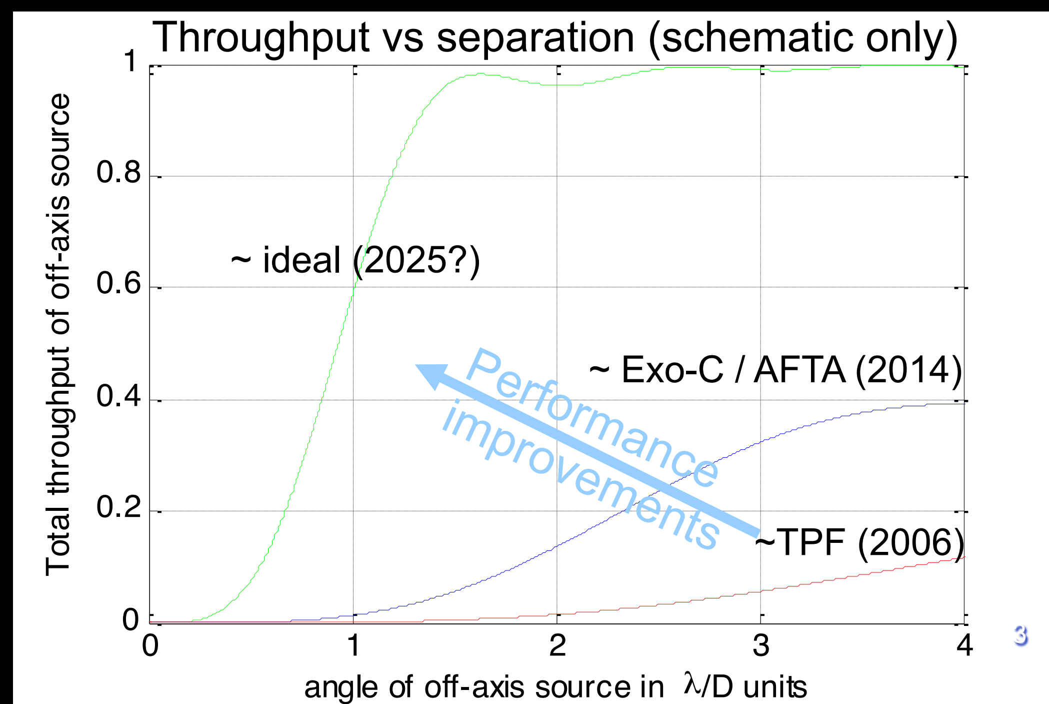
- Current top-down approach:
  - Start with many real coronagraph designs
  - Evaluate performance for each one
  - Try to improve them, without knowing how far you can go

- Proposed bottom-up way of thinking:
  - Start with an (abstract) ideal coronagraph limited by fundamental physics only (for a given telescope)
  - Evaluate its performance
  - See how far real coronagraphs are from it and in what ways
  - Try to bridge the gap

Current coronagraphs

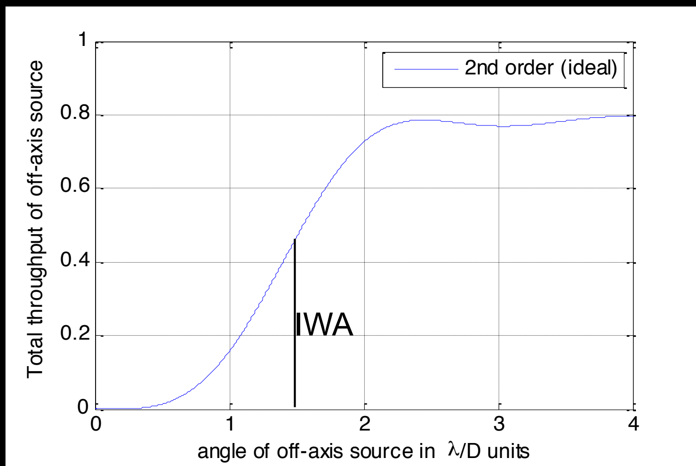


Fundamental information limit due to telescope (“ideal coronagraph”)

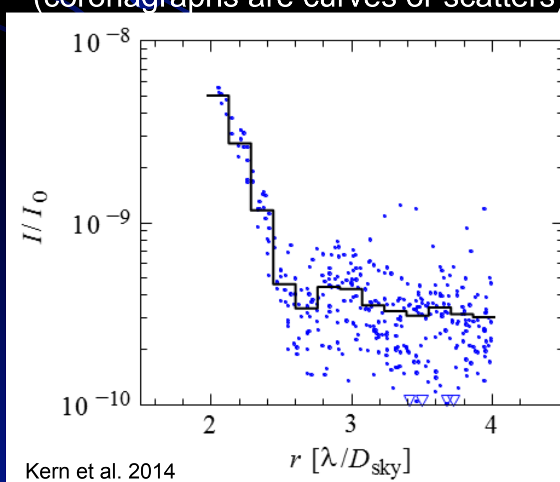


Different ways of looking at coronagraph performance

1. Throughpout vs angle
(coronagraphs are curves)

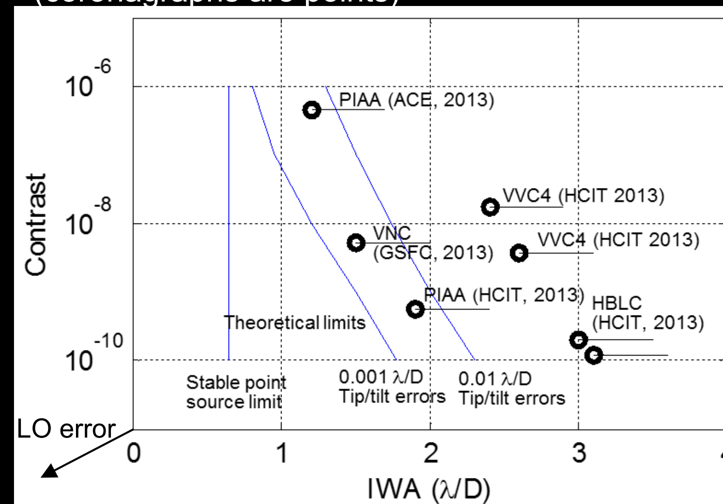


2. Contrast vs angle
(coronagraphs are curves or scatters)



Kern et al. 2014

3. Contrast vs IWA (vs low order error level)
(coronagraphs are points)



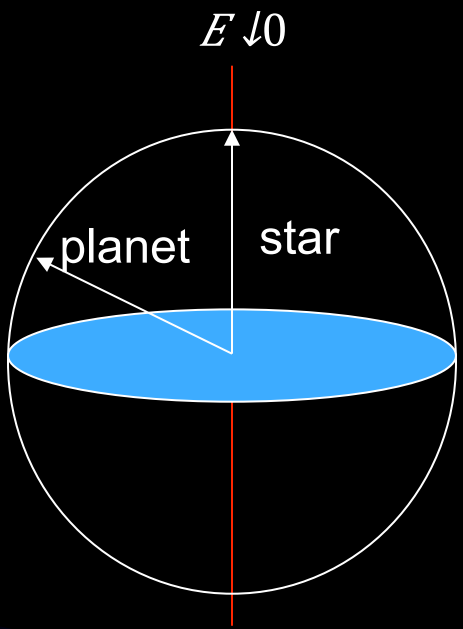
Note: plot current as of 8/13

- LO errors (esp. tip/tilt) is emerging as a key parameter coupled to IWA and contrast
- Bandwidth and maximum throughput do not seem to be fundamentally limited (i.e. with sufficiently advanced technology, can be 100%)

Focus on a simpler piece of the problem

- Consider the trade between 3 parameters: IWA, contrast, and low order errors (e.g. telescope jitter)
- Guyon et al. 2006 established that coronagraphic IWA is fundamentally limited, and this limit depends on stellar size and low order errors
- What exactly is this fundamental trade-off between IWA and sensitivity to aberrations? Can we express it with a compact formula?
- How close are existing coronagraphs to this fundamental trade-off? How much room for improvement is there in existing architectures?

Linear algebra representation of coronagraphs



$E_{star}(x,y) \rightarrow$ Coronagraph (+WFC) $\rightarrow 0$

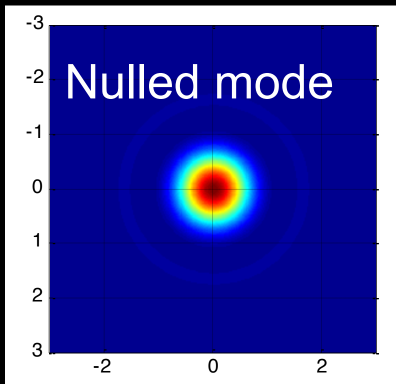
$E_{planets}(x,y) \rightarrow$ Coronagraph (+WFC) $\rightarrow E_{planets}(x,y)$

$E_{in}(x,y) = \sum a_i E_i(x,y) = \begin{matrix} a_0 E_0(x,y) \\ + a_1 E_1(x,y) \\ + \dots \end{matrix} \rightarrow$ Coronagraph (+WFC) $\begin{matrix} \times\lambda_0 \\ \times\lambda_1 \\ \times\lambda_2 \\ \dots \end{matrix} \rightarrow \begin{matrix} b_0 E_0(x,y) \\ + b_1 E_1(x,y) \\ \dots \end{matrix}$

a x C = b

(based on Guyon et al. 2006)

“Ideal” (2nd-order) Coronagraph

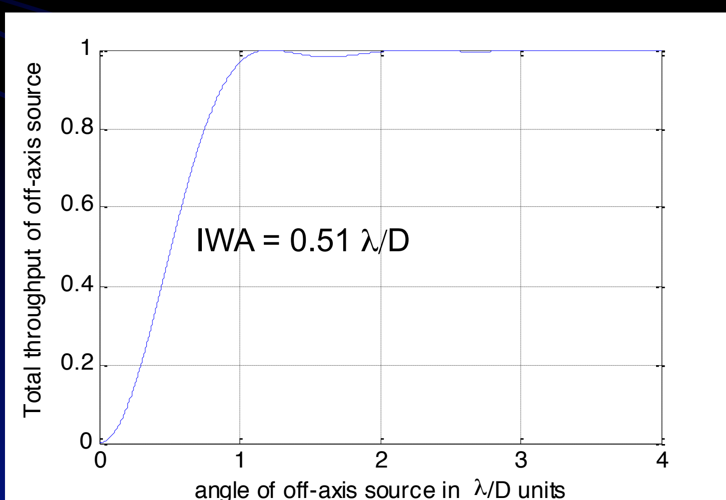


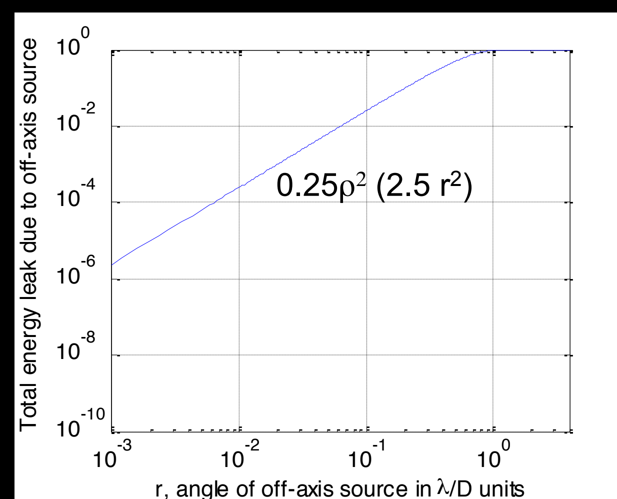
$$E_0(\rho)=2J_1(\rho)/\rho \quad \text{(Airy pattern)}$$
$$=1-\tfrac{1}{8}\rho^2+\tfrac{1}{192}\rho^4+o(\rho^6)$$

($\rho=\pi r$, where r is in units of $f\lambda/D$)

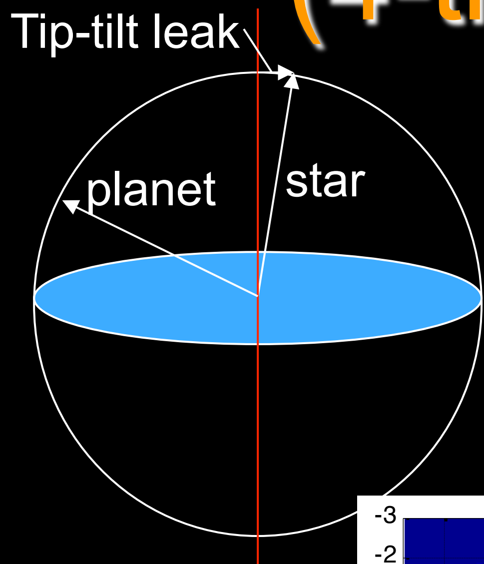
Coronagraph matrix:
$\lambda_0=0$
all other $\lambda_i=1$

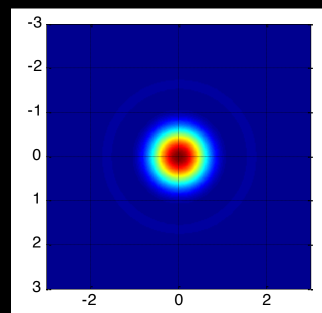
Total throughput for off-axis source: $\|\Delta E_{CCD}\|^2=1-E_0(\rho)^2$
$$=1-4J_1^2(\rho)/\rho^2$$
$$=\tfrac{1}{4}\rho^2-\tfrac{5}{192}\rho^4+o(\rho^6)$$





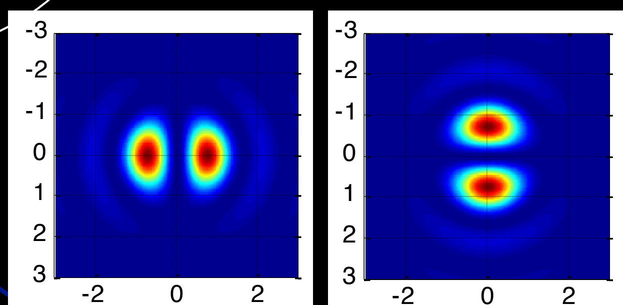
Ideal "tip-tilt insensitive" (4-th order) coronagraph





$E_0(\rho)=2J_1(\rho)/\rho$ (Airy pattern)

$=1-\frac{1}{8}\rho^2+\frac{1}{192}\rho^4+o(\rho^6)$



Nulled modes

$E_{1,x}(\rho,\phi)=2\partial/\partial x\,E_0(\rho)=2E_0'(\rho)\cos(\phi)$

$E_{1,y}(\rho,\phi)=2\partial/\partial y\,E_0(\rho)=2E_0'(\rho)\sin(\phi)$

Coronagraph matrix:
$\lambda_0, \lambda_{1,x}, \lambda_{1,y}=0$
all other $\lambda_i=1$

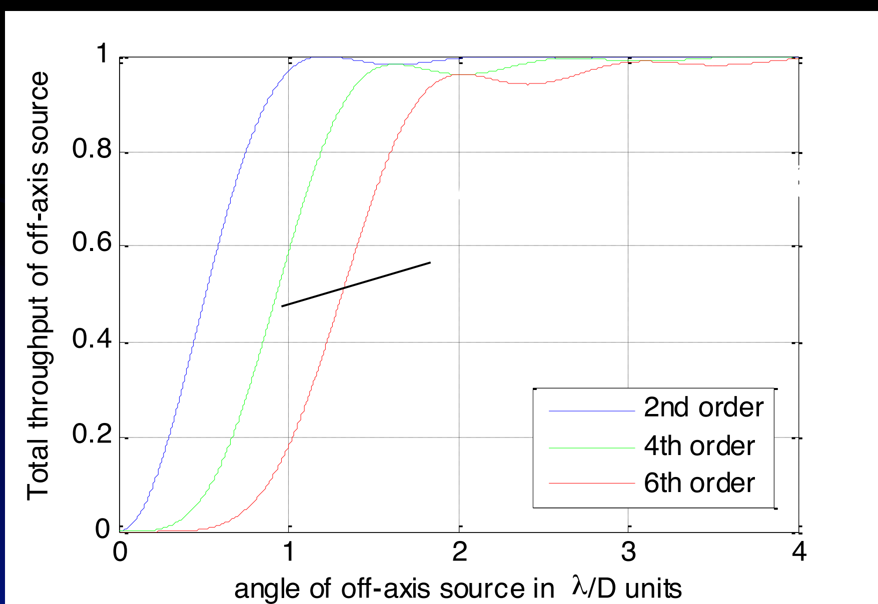
where $E_0'(\rho)=4J_0(\rho)/\rho-8J_1(\rho)/\rho$

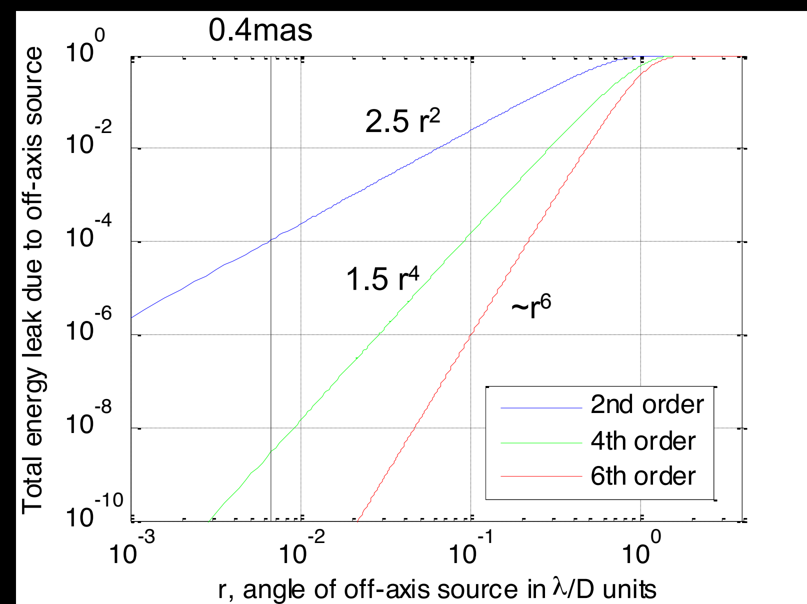
$=-\frac{1}{2}\rho+\frac{1}{24}\rho^3+o(\rho^5)$

Ideal “tip-tilt insensitive” (4-th order) coronagraph

Total throughput for off-axis source (after some algebra):

$$\|\Delta E_{CCD}\|^2 = 1 - E_0^2(\rho) - E_1^2(\rho)$$
$$= 1 - 4J_1^2(\rho)/\rho^2 - \left(4J_0(\rho)/\rho - 8J_1(\rho)/\rho^2\right)^2$$
$$= 1/64\,\rho^4 + o(\rho^6)$$





IWA, Contrast, and aberration sensitivity trades for ideal coronagraph

- For an ideal coronagraph of n-th order,
  - $IWA \sim \sqrt{n^2+2n}/8\pi$
    - Meaning: “blind spot” area in units of $(\lambda/D)^2$ is equal to the number of blocked modes
    - n-th order ideal coronagraph blocks an additional n/2 modes compared to n-1st order
    - Have not yet simulated ideal coronagraphs for obstructed apertures. However, I believe everything holds with the change $D \sim D_{primary} - D_{obstruction}$, i.e. IWA increases by ~40% for AFTA pupil
  - Tip/tilt sensitivity: $Contrast = C\, r^n$, where
    - $C = o(1)$ is a constant
    - *r* is the amount of tip/tilt error in units of λ/D
- Eliminating order *n* leads to fundamental limit:
  - $Contrast \sim r^{\sqrt{8\pi IWA^2+1}-1}$

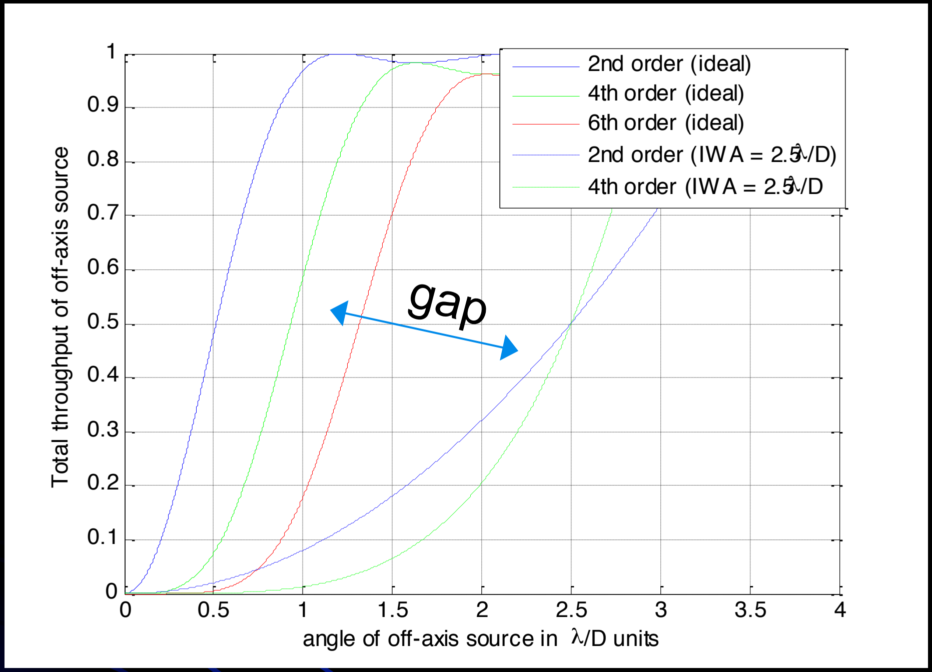
Numerical trade examples
(for D = 2.4m, unobstructed)

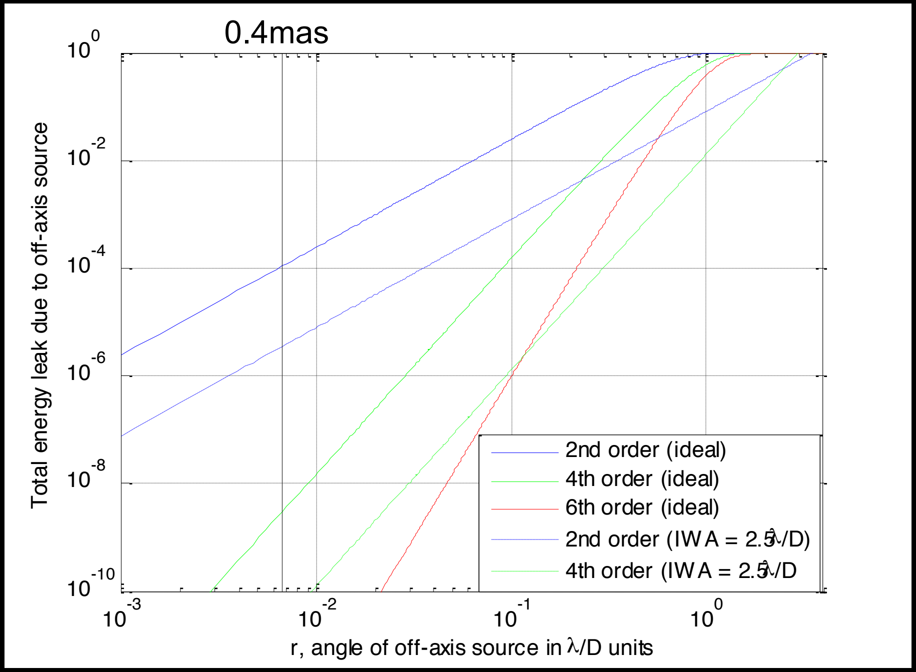
IWA (λ/D)	r: tip/tilt error	Contrast	n (order)
1	0.4 mas	3e-9	4
2.2	7mas	1e-10	10

- For AFTA, ideal coronagraph IWAs are likely 40% higher (1.4 and 3 λ/D)

- At 0.4 mas, can in principle achieve 1 l/D IWA (increasing science yield by a factor of 3-10?)
- At 2.2 l/D IWA, can tolerate uncorrected jitter of 7mas

Comparison to “real” coronagraphs





- Substantial gap remains between existing designs and fundamental limits
- Investments in coronagraph technology can bridge this gap, enablilng cost savings on telescope

Conclusions

- Eventually, future telescopes will have “ideal” coronagraphs (as long as cost is driven by the telescope)
- IWA, contrast, and LO errors are fundamentally coupled (there exists a fundamental limiting surface)
- Existing coronagraph designs are still some ways off from fundamental limits, and technology development can bridge this gap, resulting in major cost savings on the telescope

BACKUP CHARTS

Trade-offs for PIAA